Assignment 6.

Given April 29, due May 6.

Objectives: To explore basic Monte Carlo and the central limit theorem.

Consider a sphere of pure absorber of radius $R = .5$. We have a uniform isotropic source of neutrons and seek to determine the average number of collisions c that a neutron undergoes and the average pathlength ℓ that it travels before either colliding with an atom and being absorbed or leaving the sphere.

To choose the radius r from which a particular history originates, note that for a uniform source the probability that a neutron is born at a radius less than or equal to r is the ratio of the volume of a sphere of radius r to that of radius R: $F(r) = (r/R)^3$. Hence if ξ_1 is a uniformly distributed random variable between 0 and 1, then

$$r = R\xi_1^{1/3}$$

will give the radial coordinate for uniformly distributed particles. In this problem, only the radial coordinate will be required because the distance to the surface of the sphere will be independent of the other spatial coordinates. Since the source is isotropic, the cosine μ of the angle between a radius through the particle position and the direction in which the particle moves is uniformly distributed between -1 and $+1$:

$$\mu = 2\xi_2 - 1,$$

where ξ_2 is uniformly distributed between 0 and 1. The distance to the surface of the sphere is then

$$d = [R^2 - r^2(1-\mu^2)]^{1/2} - r\mu.$$

The probability density function for the distance that a particle travels between collisions is exponential, and so the distance to the first collision is estimated by

$$d_c = -\log(\xi_3),$$

where ξ_3 is also uniformly distributed between 0 and 1. If $d_c \leq d$, then the particle collides and is absorbed — the total number of collisions increases by 1 and the total pathlength travelled increases by d_c. If $d_c > d$, then the particle leaves the system — no new collisions occur and the total pathlength increases by d. In either case, the particle history terminates after just one step.

(1) Do Monte Carlo simulations with $N = 10,\ 100,\ 1000$, and 10000 particles to estimate the average number of collisions that a particle undergoes and the average distance that a particle travels. For each computation give an error estimate based on the standard deviation.

(2) Run 500 batches of 25 particles each and plot histograms of the estimated values of c and ℓ. (In MATLAB, the command **hist** plots a histogram.) According to the central limit theorem the values of c should be normally distributed with mean .2927 and standard deviation $.455/\sqrt{N}$, while the values of ℓ should be normally distributed with mean .2927 and standard deviation $.2210/\sqrt{N}$. On the same graphs, plot a normal distribution with the mean and standard deviation predicted by the central limit theorem. (Remember to multiply the normal density function by a constant so that the area under the curve is the same as the area under your histograms.) Is it a good match to your histograms?