Assignment 4.

Given April 1, due April 8.

Objective: To practice using optimization methods.

Consider the function

$$f(x,y) = 100(y - x^2)^2 + (1 - x)^2,$$

which has its minimum at $x = y = 1$.

(1) Write an expression for the *gradient* of f at a point (x, y) and for the *Hessian* matrix of f.

(2) Write a code that uses unsafeguarded Newton's method to find the minimum of a function of two variables. (Note that this is usually a *bad* thing to do. Unless you build in some safeguards, Newton's method may take you to a maximum or a saddle point or it may just diverge. For this example, however, it should be ok.) Try your code on the function $f(x, y)$ for several different initial guesses. Comment on the rate of convergence. Make a 3-D surface plot of $f(x, y)$ and plot the Newton iterates on the same graph. This can be done by evaluating f at uniformly spaced points in x and y and using the MATLAB command `mesh(x,y,f)`. Then enter the command `hold on` to keep the surface plot on the screen, and plot each of the Newton iterates by typing `plot3(xn,yn,fn,'x')`, where arrays xn and yn contain the Newton iterates and fn contains the function values at these points. You can look at your plot from different angles by using the `view` command in MATLAB. Turn in a listing of your code and one plot showing the function $f(x, y)$ and the Newton iterates.

(3) If you have access to the MATLAB optimization toolbox, run the demo `bandem` to see an interesting illustration of the performance of different methods on this problem. Nothing to turn in here, just enjoy.