Assignment 4.

Due Mon., Nov. 16.

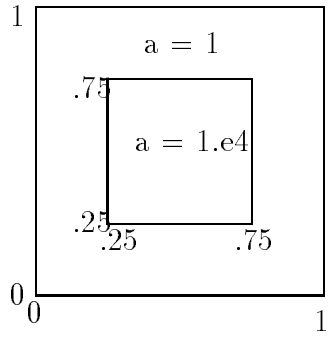
Objectives: To implement the CG algorithm and use it to solve the steady-state diffusion equation; to discover the importance of diagonal scaling.

Use the finite difference method described in the text to write down a system of linear algebraic equations for an approximate solution to the boundary value problem:

$$-\frac{\partial}{\partial x}\left(a\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(a\frac{\partial u}{\partial y}\right) = f, \quad \text{in } (0,1)\times(0,1),$$

$$u(x,0) = u(x,1) = u(0,y) = u(1,y) = 0,$$

where $f(x,y) = 1$, and $a(x,y) = 10^4$ for $(x,y) \in (.25,.75)\times(.25,.75)$, and $a(x,y) = 1$ otherwise, as pictured below:



Solve the system of linear equations using the conjugate gradient method, first with no preconditioner and then using the diagonal of the coefficient matrix as a preconditioner. Plot residual norm (divided by the norm of the initial residual) vs iteration number for each method. (Use a log scale for the residual norm; the MATLAB command *semilogy* will produce a such a plot.)

Try a few different mesh sizes, say, $h = 1/32$, $h = 1/64$, and $h = 1/128$. How is the number of iterations required to achieve a certain level of accuracy related to the mesh size?