Assignment 1.

Due Mon., Oct. 12.

Objectives: To think about iterative methods for solving linear systems; to discover some properties of simple iteration and ORTHOMIN(1).

(1) Suppose you wish to solve (or approximately solve) an n by n nonsingular system of linear equations $Ax = b$, but the matrix A is not given to you directly. Instead you have a black box which when given any vector y produces the vector Ay. Try to devise a method for solving the linear system (or generating a good approximate solution) that is different from the methods covered so far in class. Discuss properties of the matrix A that will ensure that your method converges to the true solution and try to say something about the rate of convergence. [Of course, you could determine the entries of A by applying A to each of the standard basis vectors ξ_j, $j = 1, \ldots, n$, where ξ_j has a 1 in position j and 0's elsewhere, but the purpose of this problem is to think about methods that do *not* require this.]

(2) Use the Jordan form of a matrix to show that for any fixed $k \geq 0$

$$\lim_{j \to \infty} (|||e_{k+j}|||/|||e_k|||)^{1/j} = \rho(I - M^{-1}A),$$

where $|||\cdot|||$ is any norm and e_{k+j} is the error at step $k + j$ of the simple iteration algorithm.

(3) Write down a problem $Ax = b$ for which ORTHOMIN(1) converges to the true solution but (unpreconditioned) simple iteration does not. Write down a problem for which (unpreconditioned) simple iteration converges to the true solution but ORTHOMIN(1) does not. Can you find a problem for which both methods converge but simple iteration converges faster (i.e., reduces the 2-norm of the residual faster) than ORTHOMIN(1)? Explain your answers.