Project overviews

- Powers of higher nerves 3 students
  - *Fast* computer code
  - Computational evidence for a conjecture on graph algorithms (Evasiveness Conjecture)
- Topology of matching complexes 2 students
  - Behavior of graphs under iterated matching operation
  - Classification of matching complexes with certain topology

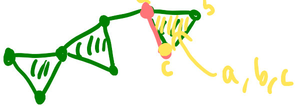
Simplicial complexes

X = simplicial complex

$\{a,b,c\} \in X \Rightarrow \{a,c\} \in X \Rightarrow \{c\} \in X$

Combinatorial: Collection of sets closed under taking subsets

Topological: Object in $\mathbb{R}^n$ made by gluing **simplices** together

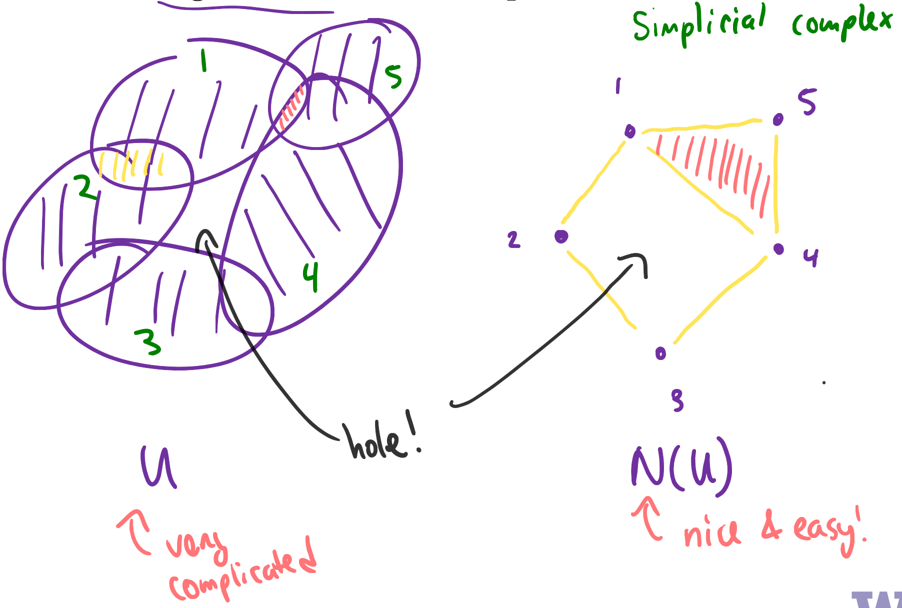


like the integers

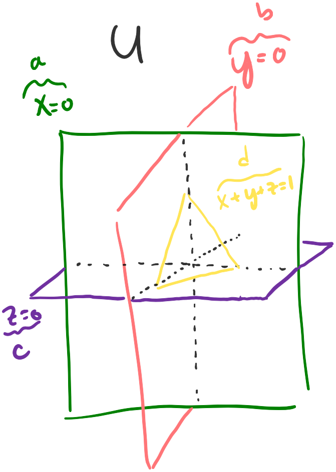
Algebraic: Face ring ← imo most difficult

Q: How do these props interact?

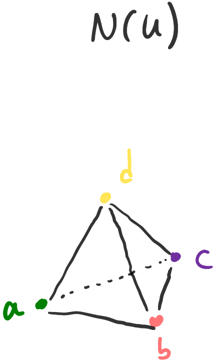
Powers of higher nerves - Example 1



Powers of higher nerves - Example 2



Planes in $\mathbb{R}^3$



Hollow tetrahedron

Powers of higher nerves

Known: If U is "good," then $N(U)$ has the "same topology" as U.

Our project:

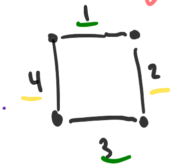
- Higher nerves – finer intersection data
- Computer code for higher nerves
- Computational evidence related to the Evasiveness Conjecture

graph algorithms

Matching complexes

Graph: Vertices and edges. Each pair of vertices either has an edge connecting them or not.

Matching: Set of edges such that no two share an endpoint.

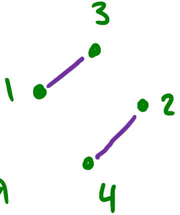


G

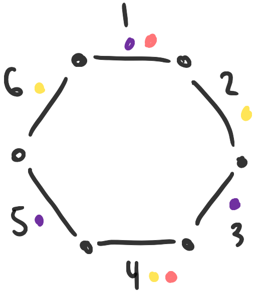
simplicial complex!

$\{24, 13, 1, 2, 3, 4, \emptyset\}$

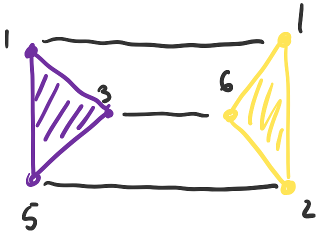
$M(G)$



Matching complexes – another example



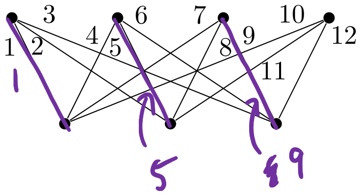
G



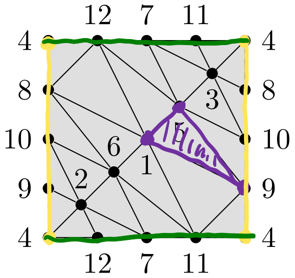
M(G)

Example: C_7 ??

Matching complexes – one more



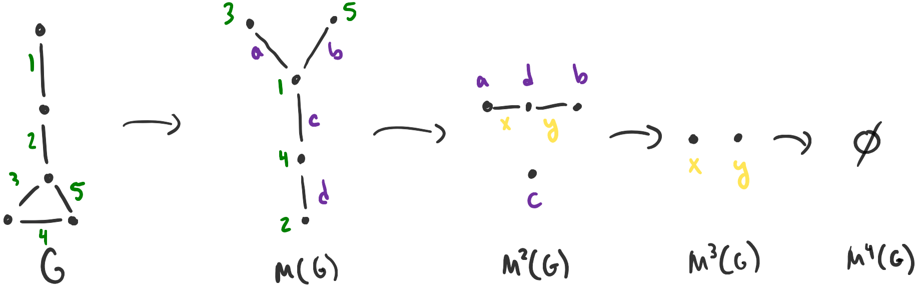
$K_{4,3}$



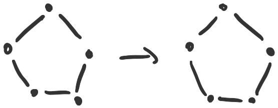
$M(K_{4,3})$

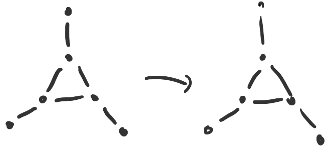
torus!

Iterated matching graphs



Two special graphs:





- A handful of others dissipate like G above
- ALL OTHERS EXPLODE!

Two-dimensional Buchsbaum matching complexes

Every maximal matching has three edges

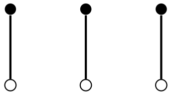
Complicated algebraic notion

One example:

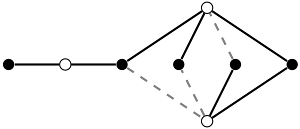
Torus is 2-dim & Buchsbaum!

matching complex of $K_{4,3}$

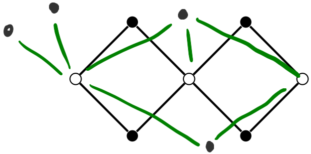
Two-dimensional Buchsbaum matching complexes



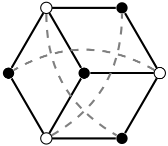
(a) $\mathcal{B}_1$



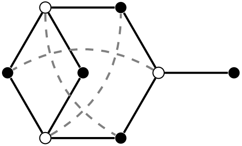
(b) $\mathcal{B}_2$



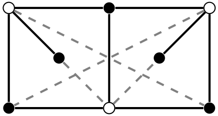
(c) $\mathcal{B}_3$



(d) $\mathcal{B}_4$

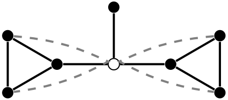


(e) $\mathcal{B}_5$

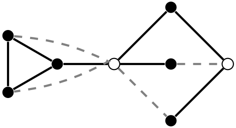


(f) $\mathcal{B}_6$

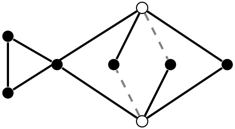
Two-dimensional Buchsbaum matching complexes



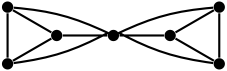
(a) $\mathcal{B}_7$



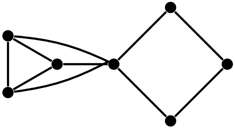
(b) $\mathcal{B}_8$



(c) $\mathcal{B}_9$



(d) Exceptional graph E_1



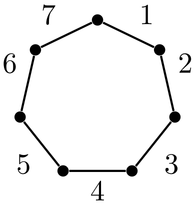
(e) Exceptional graph E_2

Some graphs whose matching complexes are two-dimensional and Buchsbaum.

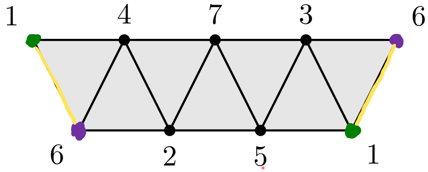
A few other families too!

The end

Thanks for your attention!



C_7



Möbius strip!

M_7