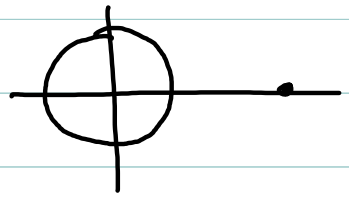
Lesson 6

Read Chapter 4 (no uniform motion yet)

Linear modeling

Lines and Circles word problems

Dave's silo problem



Find the tangent to to the circle $(x-3)^2+(y+2)^2=5$ through the point $Q(0,8)$

$$\begin{cases} (x-3)^2 + (y+2)^2 = 5 \\ \dfrac{3-x}{2+y} = \dfrac{8-y}{-x} \end{cases}$$

Expand squares

cross multiply

$$\begin{cases} x^2 - 6x + 9 + y^2 + 4y + 4 = 5 \\ -3x + x^2 = 16 + 8y - 2y - y^2 \end{cases}$$

simplify

simplify

$$\begin{cases} x^2 + y^2 = 6x - 4y - 8 \\ x^2 + y^2 = 16 + 6y + 3x \end{cases}$$

Keep

$$\begin{cases} x^2 + y^2 = 6x - 4y - 8 \\ 6x - 4y - 8 = 16 + 6y + 3x \end{cases}$$

solve for y (or x)

$$\begin{cases} x^2 + y^2 = 6x - 4y - 8 \\ 3x - 2y = 10y \end{cases}$$

$$\begin{cases} x^2 + \left(\frac{3x-2y}{10}\right)^2 = 6x - 4\left(\frac{3x-2y}{10}\right) - 8 \\ y = \frac{3x-2y}{10} \end{cases}$$

$$\begin{cases} x^2 + \frac{9x^2 - 2\cdot 3\cdot 2y x + 2y^2}{100} = 6x - \frac{12x - 9y}{10} - 8 \\ y = \frac{3x-2y}{10} \end{cases}$$

$$\underbrace{109}_{a}\,x^2 \underbrace{- 62y}_{b}\,x \underbrace{+ 416}_{c} = 0 \quad \begin{cases} x \simeq 4.95 \\ x = 0.77 \end{cases}$$

$$y = \frac{3x - 24}{10}$$

For $x = 4.9545$ $y \approx -0.91$ R_1

For $x = 0.7703$ $y \approx -2.17$ R_2

tangent 1 line through $Q(0,8)$ $R_1(4.95, -0.91)$

$$y = 8 + \frac{8 - (-0.91)}{0 - 4.95} x; \quad y = 8 - 1.8x$$

tangent 2 : line through $Q(0,8)$ $R_2(0.77, -2.17)$

$$y = 8 + \frac{8 + 2.17}{-0.77} x; \quad y = 8 - 13.21x$$

Find the equation of the line tangent to to the circle $(x-3)^2+(y+2)^2=5$ and parallel to the line $4x-2y+10=0$

Video with solution in Canvas

Clue words: LINEAR , CONSTANT RATE

Goal: find the equation of a line and use it to answer questions in the problem

Ex: ch 4 #2 average sale price of home in Seattle and Port Townsend

$y = y_0^s + m_s(x - x_0^s)$ For seattle

x_0^s -> year i.e 1970

y -> sale price

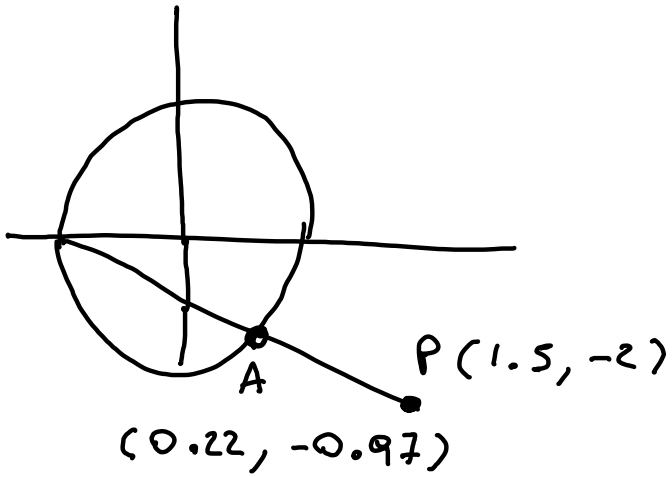
$y = y_0^p + m_p(x - x_0^p)$

Similar to ch 4 #3

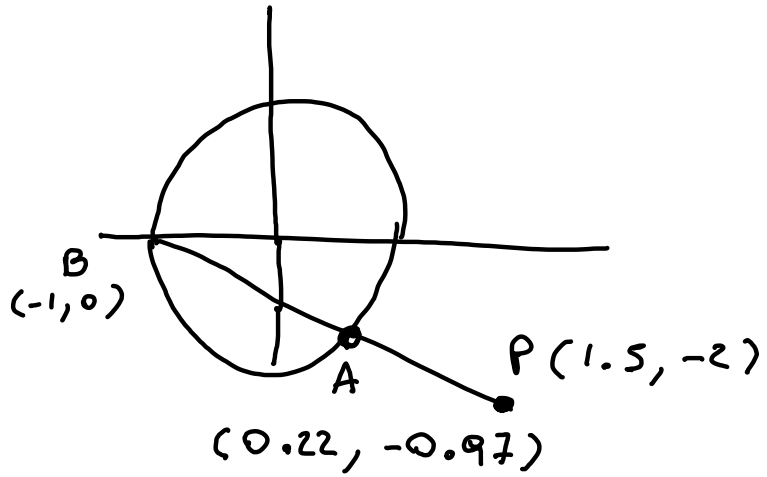
A crop dusting airplane flying a constant speed of 120mph is first spotted 2 miles South and 1.5 miles East of the center of circular irrigated field. The irrigated field has radius 1 mile. The plane flies in a straight line to a point 1 mile West of the center of the irrigated field. Impose a coordinate system with the origin at the center of the field.

Find the location A where the crop duster enters the airspace above the field

When does the plane first enter the airspace above the field ?
(Assume time t=0 corresponds to when the plane is first spotted)



How much time does the plane spend flying over the irrigated field?



How close does the plane get to the center of the field?