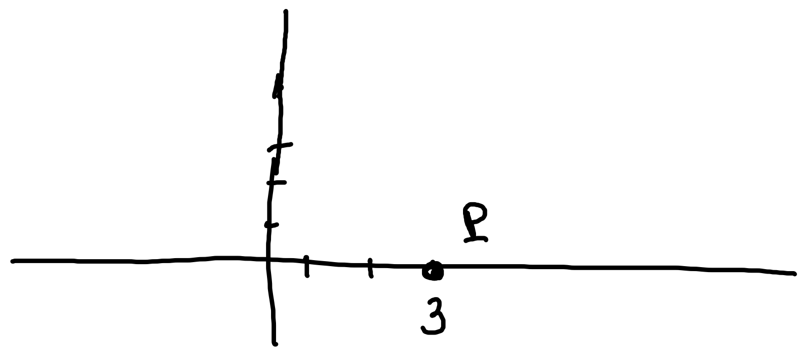
Lesson 5

Read Chapter 3

circles

Find the point Q on the line L: $y=2x+1$ that is closest to the point P(3,0).



Equation of a circle

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

The circle has center (x_0, y_0) and radius r.
A point $P(x_1, y_1)$

- is on the circle if : $(x_1 - x_0)^2 + (y_1 - y_0)^2 = r^2$
- is inside the circle if:$(x_1 - x_0)^2 + (y_1 - y_0)^2 < r^2$
- is out side the circle if $(x_1 - x_0)^2 + (y_1 - y_0)^2 > r^2$

Find the center and radius of the circle

$$3x^2 + 18x + 3y^2 - 6y + 6 = 0$$

$$x^2 + ax + y^2 + by + c = 0$$

is the equation of a circle with center at $x_0 = -\frac{a}{2}$, $y_0 = -\frac{b}{2}$ and radius $r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4} - c}$

Intersection of a line and a circle

Find the intersection of the unit circle and the line $y = x + \frac{1}{2}$

$$\begin{cases} x^2 + y^2 = 1 \\ y = x + \frac{1}{2} \end{cases}$$

$$\begin{cases} y = x + \frac{1}{2} \\ x^2 + \left(x + \frac{1}{2} \right)^2 = 1 \end{cases}$$

$$x^2 + x^2 + x + \frac{1}{4} - 1 = 0$$

$$2x^2 + x - \frac{3}{4} = 0 \quad ; \quad x = \frac{-1 \pm \sqrt{1+6}}{4}$$

If $x = \frac{-1+\sqrt{7}}{4}$ $y = \frac{-1+\sqrt{7}}{4} + \frac{1}{2}$ $P = \left(\frac{-1+\sqrt{7}}{4}, \frac{1}{4} + \frac{\sqrt{7}}{4}\right)$

if $x = \frac{-1-\sqrt{7}}{4}$, $y = \frac{-1-\sqrt{7}}{4} + \frac{1}{2}$ $Q \left(\frac{-1-\sqrt{7}}{4}, \frac{1}{4} - \frac{\sqrt{7}}{4}\right)$

Tangent to a circle

Fact: if a line L is tangent to a circle at P, then the line is perpendicular to the radius CP.

Find the tangent to to the circle $(x-3)^2+(y+2)^2=5$ at the point $P(1,-1)$

Find the tangent to to the circle $(x-3)^2+(y+2)^2=5$ through the point $Q(0,8)$

$$\begin{cases} (x-3)^2 + (y+2)^2 = 5 & Q \\ \dfrac{3-x}{2+y} = \dfrac{8-y}{-x} & Q \end{cases}$$

Expand squares

cross multiply

$$\begin{cases} x^2 - 6x + 9 + y^2 + 4y + 4 = 5 \\ -3x + x^2 = 16 + 8y - 2y - y^2 \end{cases}$$

Simplify

Simplify

$$\begin{cases} x^2 + y^2 = 6x - 4y - 8 \\ x^2 + y^2 = 16 + 6y + 3x \end{cases}$$

Keep

$$\begin{cases} x^2 + y^2 = 6x - 4y - 8 & Q \\ 6x - 4y - 8 = 16 + 6y + 3x & L \end{cases}$$

solve for y (or x)

$$\begin{cases} x^2 + y^2 = 6x - 4y - 8 \\ 3x - 24 = 10y \end{cases}$$

$$\begin{cases} x^2 + \left(\frac{3x-24}{10}\right)^2 = 6x - 4\left(\frac{3x-24}{10}\right) - 8 \\ y = \frac{3x-24}{10} \end{cases}$$

$$\begin{cases} x^2 + \frac{9x^2 - 2\cdot 3\cdot 24x + 24^2}{100} = 6x - \frac{12x - 96}{10} - 8 \\ \text{"} \end{cases}$$

$$\underbrace{109}_{a} x^2 \underbrace{- 624}_{b} x \underbrace{+ 416}_{c} = 0$$

$x \simeq 4.95$

$x = 0.77$

$$y = \frac{3x - 24}{10}$$

For $x = 4.9545$ $\quad y \approx -0.91$ $\quad R_1$

For $x = 0.7703$ $\quad y \approx -2.17$ $\quad R_2$

tangent 1 line through $Q(0,8)$ $\quad R_1(4.95, -0.91)$

$$y = 8 + \frac{8-(-0.91)}{0-4.95} \cdot x \qquad \boxed{y = 8 - 1.8x}$$

tangent 2 : line through $Q(0,8)$ $\quad R_2(0.77, -2.17)$

$$y = 8 + \frac{8+2.17}{-0.77} x \qquad \boxed{y = 8 - 13.21x}$$

$P\ (x_1\ \ y_1)\qquad Q\ (x_2\ \ y_2)$

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Find the equation of the line tangent to to the circle $(x-3)^2+(y+2)^2=5$ and parallel to the line $4x-2y+10=0$

Video with solution in Canvas