Lesson 4

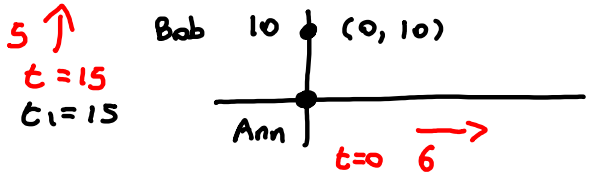
Read Chapter 3

lines and circles

Tricky version

Bob is standing 10 feet North of Ann, when Ann starts moving East at 6 feet/sec. 15 seconds later Bob , starts moving North at 5 feet/sec. When is the distance between Ann and Bob 50 feet ?

$t = 0$



Ann

$x_A(t) = 6t$

$y_A(t) = 0$

valid for $t \geq 0$

$(6t, 0)$

Bob

$x_B(t) = 0$

$y_B(t) = 10 + 5(t-15)$

valid for $t \geq 15$

$(0, 10+5(t-15))$

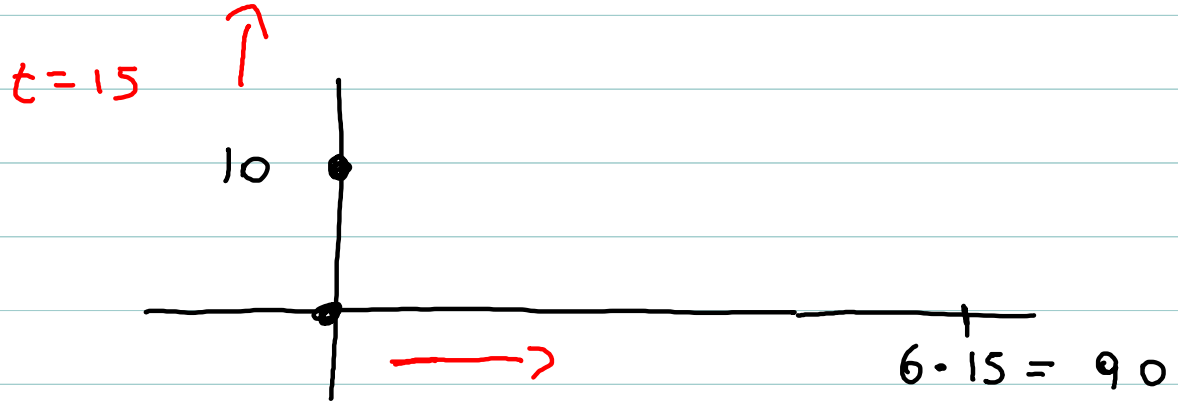
We want $d((6t,0)\ (0, 10+5(t-15))) = 50$

$$50 = \sqrt{(6t-0)^2 + (0-(10+5(t-15))^2}$$

do the algebra .... $t = 5, 5.66$

Can I accept these solutions ? No DISCARD BOTH

There is no time $t \geq 0$ when the distance between Ann and Bob is 50 ft?



Ann

$x_A(t) = 6t$

$y_A(t) = 0$

for $t \geq 0$

Bob

$x_B(t) =$

$y_B(t) =$

for t

Ann at $(6t, 0)$ Bob at $(0, 10)$

$$50 = \sqrt{(6t)^2 + 10^2}$$

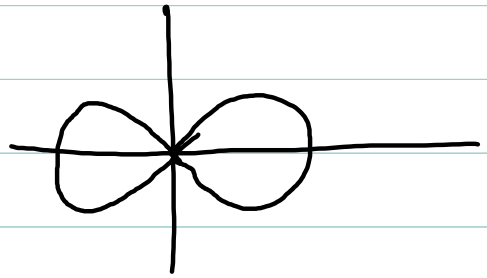
$$2500 = 36t^2 + 100$$

$$2400 = 36t^2$$

$$\pm\sqrt{\frac{2400}{36}} = t$$ discard negative solution

$$\sqrt{\frac{2400}{36}} \approx 8.16 \text{ sec}$$

Chapter 3:



Equation:

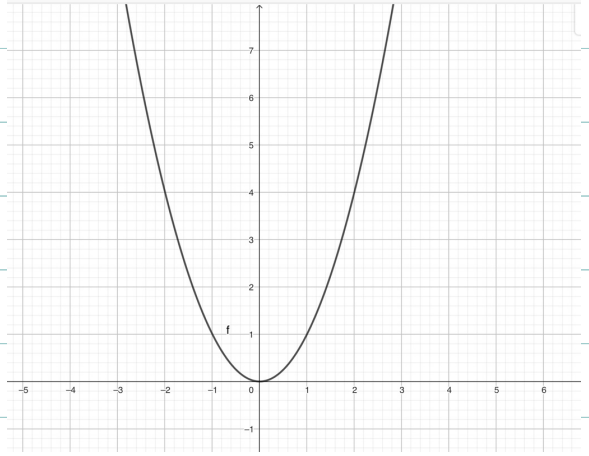
$$(x^2+y^2)^2 = (x^2-y^2)$$

Graph of a function

Function

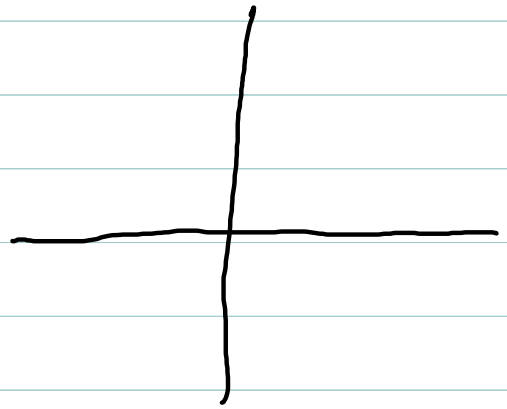
$y =$ expression in x

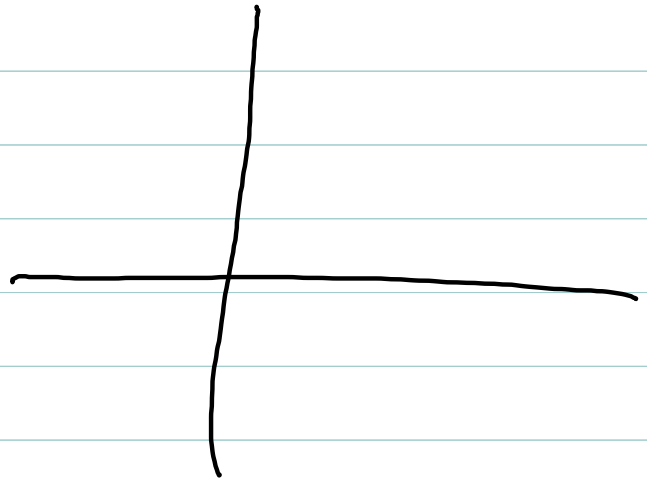
Example of function

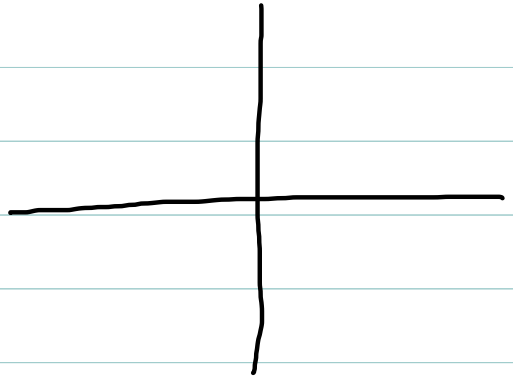


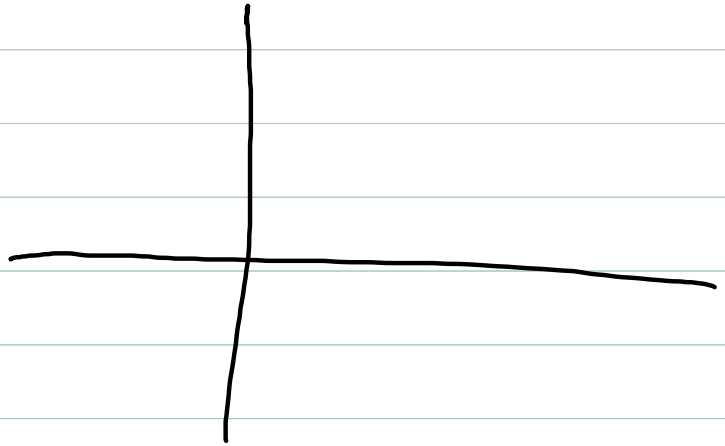
$$y = x^2$$

Is every line the graph of a function?









Lines equations

$ax+by+c=0$

$y = mx + b$

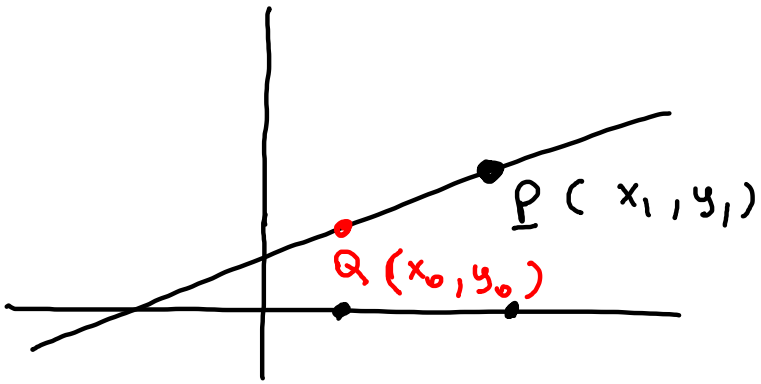
$y = y_1 + m(x - x_1)$

} for a line that is non vertical

slope of a line

$y = mx + b$

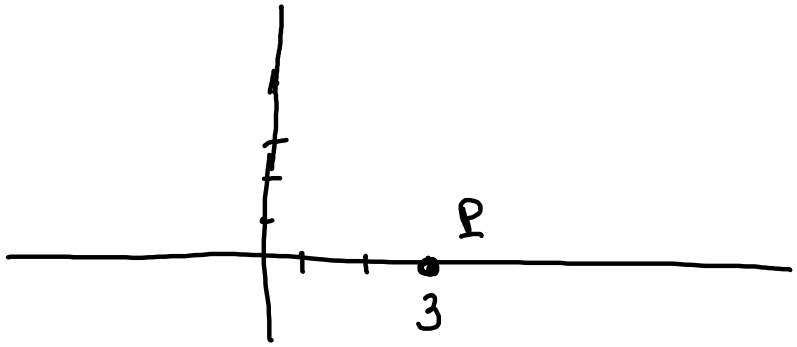
$y = y_1 + m(b - x_1)$



Useful facts about lines

1. Two lines $L_1 : y = m_1x + b_1$ and $L_2 : y = m_2x + b_2$ are parallel iff $m_1 = m_2$.
2. Two lines $L_1 : y = m_1x + b_1$ and $L_2 : y = m_2x + b_2$ are perpendicular iff $m_1 = -\frac{1}{m_2}$
3. The slope of the line through the points (x_0, y_0) and (x_1, y_1) is $m = \frac{y_1 - y_0}{x_1 - x_0}$
4. The equation of a line through $P(x_0, y_0)$ with slope m is $y = y_0 + m(x - x_0)$
5. The equation of the line through point P=(x_0, y_0) and Q=(x_1, y_1) is $y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$ if $x_1 - x_2 \neq 0$ and $y = x_1$ if $x_1 = x_0$.

Find the point Q on the line L: y=2x+1 that is closest to the point P(3,0).



Equation of a circle

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

The circle has center (x_0, y_0) and radius r.
A point $P(x_1, y_1)$

- is on the circle if : $(x_1 - x_0)^2 + (y_1 - y_0)^2 = r^2$
- is inside the circle if:$(x_1 - x_0)^2 + (y_1 - y_0)^2 < r^2$
- is out side the circle if $(x_1 - x_0)^2 + (y_1 - y_0)^2 > r^2$

Find the center and radius of the circle

$$3x^2 + 18x + 3y^2 - 6y + 6 = 0$$

$$x^2 + ax + y^2 + by + c = 0$$

is the equation of a circle with center at $x_0 = -\frac{a}{2}$, $y_0 = -\frac{b}{2}$ and radius $r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4} - c}$