

Lesson 3

Still on Chapter 2

Motion problems



Recall

If an object is moving with constant velocity v and at time t_1 it is at (a, b) then the position of the object at time t is:

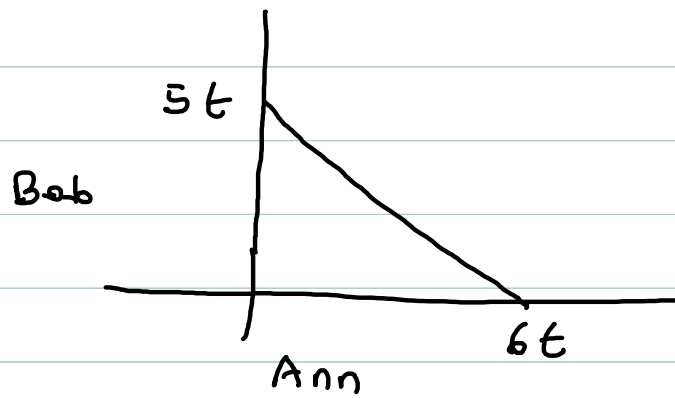
- ▶ $x(t) = a + v(t - t_1), \quad y(t) = b$ object is moving along horizontal line $y = b$.
- ▶ $x(t) = a, \quad y(t) = b + v(t - t_1)$ if object is moving along vertical line $x = a$.

Example

Ann and Bob start moving at the same time from the same location. Ann moves East at 6 feet/sec. Bob moves North at 5 feet/sec.

What is the distance between Ann and Bob 10 sec later ?

When is the distance between Ann and Bob 50 feet ?



When is distance
between Ann and Bob
equal to 50 feet?

Ann $(6t, 0)$

Bob $(0, 5t)$

Algebra question

Is $\sqrt{x^2} = x$?

What if Ann and Bob start at different times from different locations ?

Bob is standing 10 feet North of Ann, when Ann starts moving East at 6 feet/sec. 3 seconds later Bob , starts moving North at 5 feet/sec. When is the distance between Ann and Bob 50 feet ?

$$50 = \sqrt{36t^2 + (10 + 5(t-3))^2}$$

① square both sides

$$2500 = 36t^2 + (10 + 5(t-3))^2$$

② simplify inside \rightarrow

$$2500 = 36t^2 + (5t - 5)^2$$

③ Expand square \rightarrow : recall $(a+b)^2 = a^2 + 2ab + b^2$
 $(a-b)^2 = a^2 - 2ab + b^2$

$$2500 = 36t^2 + 25t^2 - 50t + 25$$

④ move 2500 to the right and simplify

$$61t^2 - 50t - 2475 = 0$$

⑤ Use quadratic formula

recall the solutions of $ax^2 + bx + c = 0$

are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ if $b^2 - 4ac \geq 0$

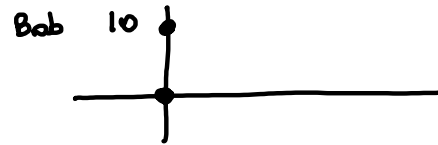
$$t = \frac{50 \pm \sqrt{50^2 + 4 \cdot 61 \cdot 2475}}{2 \cdot 61} \approx -5.97, 6.79$$

For our problem, the negative solution would be a time before Ann and Bob start moving so it does not make sense so

$$t = 6.79 \text{ sec}$$

Tricky version

Bob is standing 10 feet North of Ann, when Ann starts moving East at 6 feet/sec. 15 seconds later Bob, starts moving North at 5 feet/sec. When is the distance between Ann and Bob 50 feet ?



Ann

$$\begin{aligned}x(t) &= 6t \\y(t) &= 0\end{aligned}$$

valid for $t \geq 0$

Bob

$$\begin{aligned}x(t) &= 0 \\y(t) &= 10 + 5(t-15)\end{aligned}$$

valid for $t \geq 15$

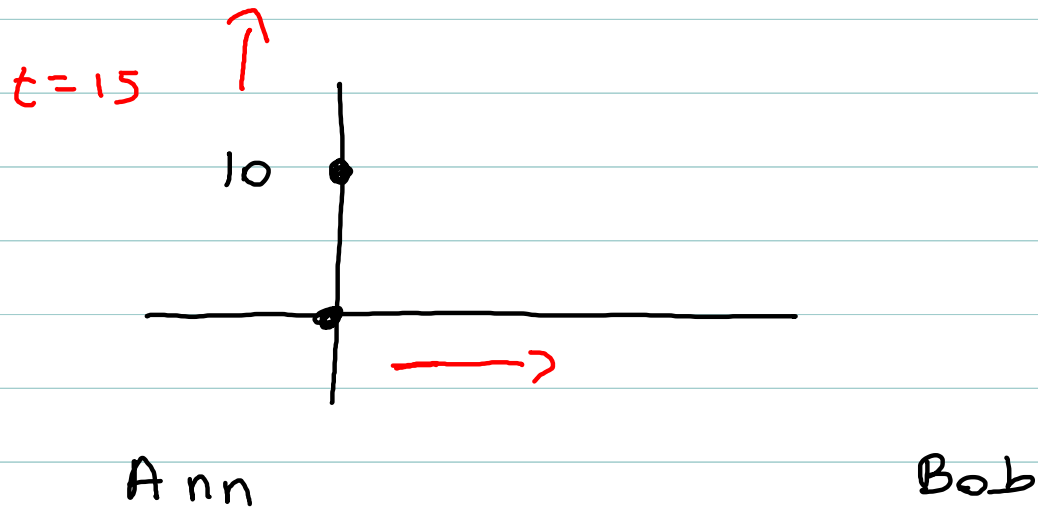
We want $d((6t, 0), (0, 10 + 5(t-15))) = 50$

$$\sqrt{(6t-0)^2 + (0-(10+5(t-15)))^2} = 50^2$$

do the algebra $t = 5, 5.66$

Can I accept these solutions ?

There is no time t_0 when the distance between Ann and Bob is 50 ft?



$$x(t) = 6t$$

$$y(t) = 0$$

$$50 = \sqrt{(6t)^2 + 10^2}$$

$$2500 = 36t^2 + 100$$

$$2400 = 36t^2$$

$$t = \sqrt{\frac{2400}{36}} = t \quad \text{discard negative solution}$$

$$\sqrt{\frac{2400}{36}} \approx 8.16 \text{ sec}$$