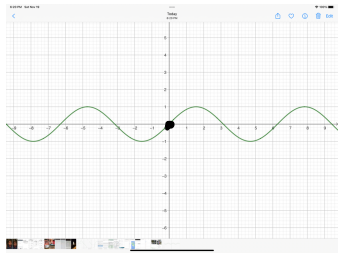
Lesson 26

Read Chapter 19 and 20

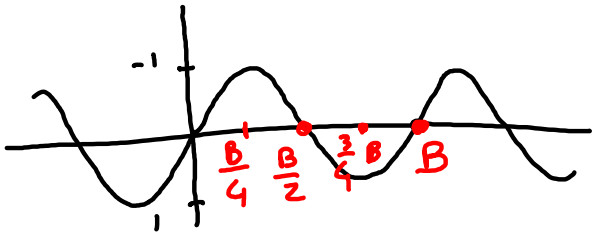
Sinusoidal equations

Sinusoidal modelling problems

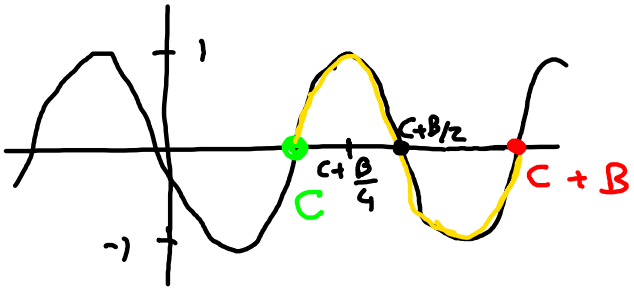
Graph $f(x) = A\sin(\frac{2\pi}{B}(x - C)) + D$ $A, B > 0$



① h scaling of $\frac{B}{2\pi}$

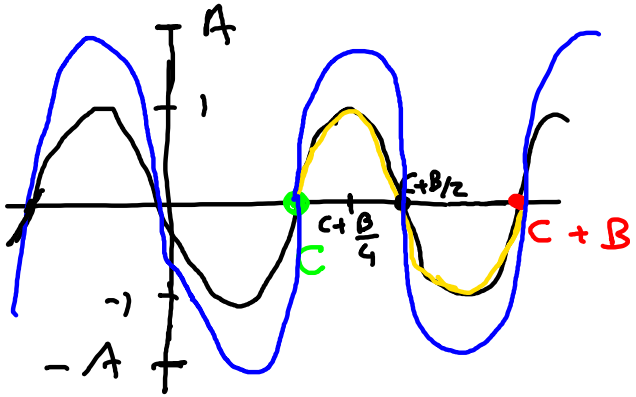


②



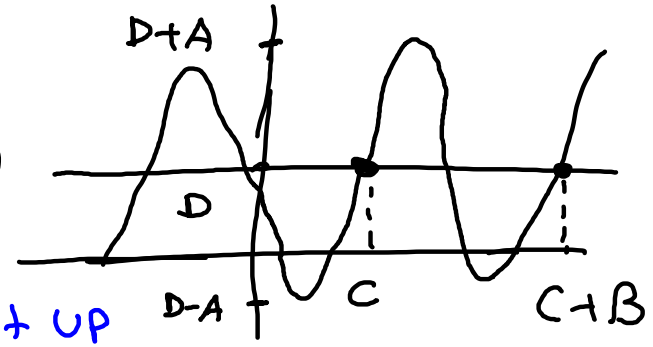
shift right C units (if $C \geq 0$)

③



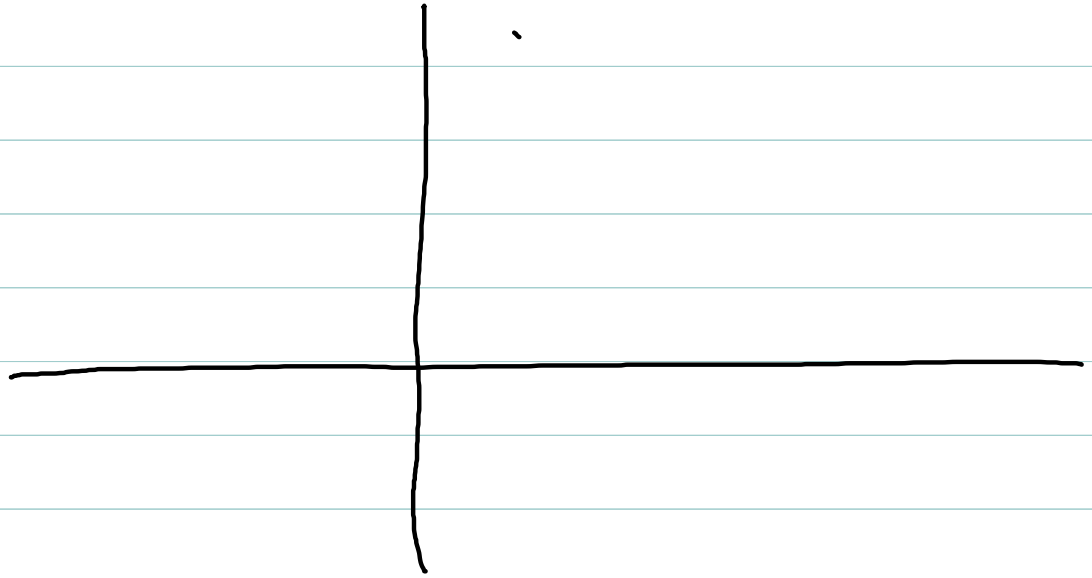
vertical scaling

④



shift up D units ($D > 0$)

Graph $f(t) = \underbrace{3}_{A} \sin \underbrace{\frac{2\pi}{5}}_{B}\left(t - \underbrace{\frac{7}{4}}_{C}\right) + \underbrace{2}_{D}$



1) Draw $y = D$

2) Draw points (C, D) $\left(C + \frac{B}{4}, D\right)$ $\left(C + \frac{B}{2}, D\right)$ $\left(C + \frac{3}{4}B, D\right)$ $(C + B)$

3) Draw points $\left(C + \frac{B}{4}, D + A\right)$, $\left(C + \frac{3}{4}B, D - A\right)$

4) Draw basic S shape and repeat

Sinusoidal functions

$$f(x) = A\sin(\frac{2\pi}{B}(x - C)) + D \quad A, B > 0$$

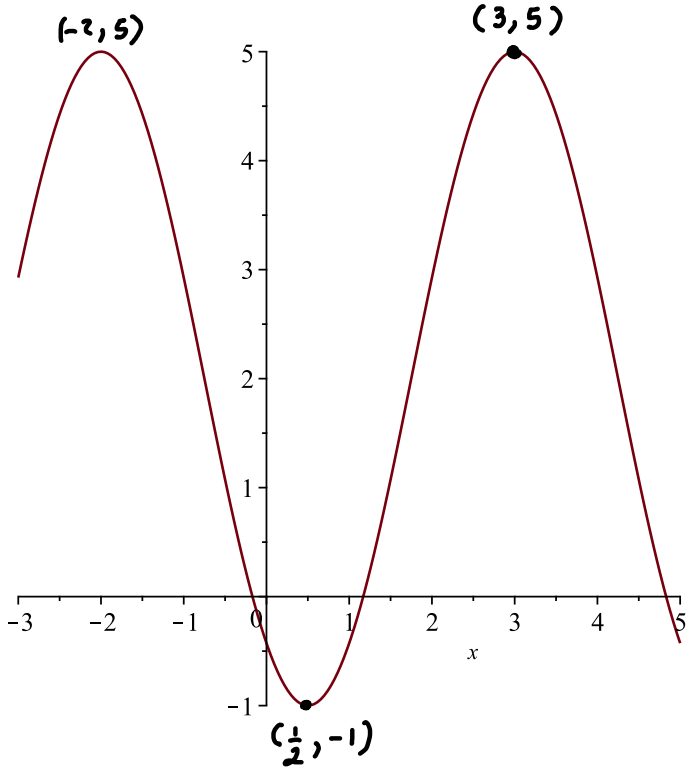
A: amplitude. Half total height $= \frac{y_{max} - y_{min}}{2}$.

B: period. Horizontal distance between two consecutive peaks or valleys, or double the horizontal distance between one peak and the next valley or one valley and the next peak.

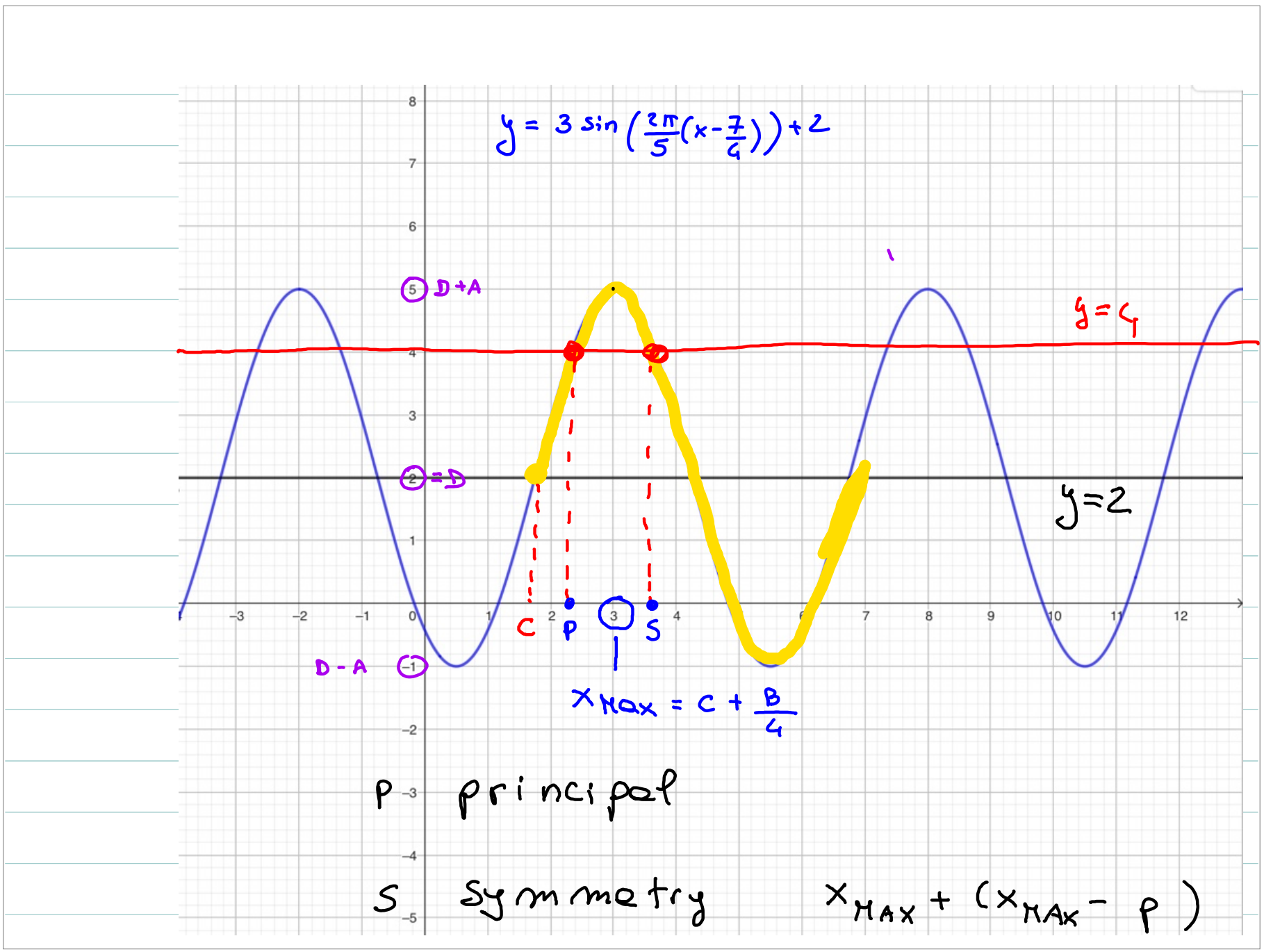
C: phase shift. x-coordinate of max - $\frac{B}{4}$ or x-coordinate of a point half way (vertically) between a valley and a peak.

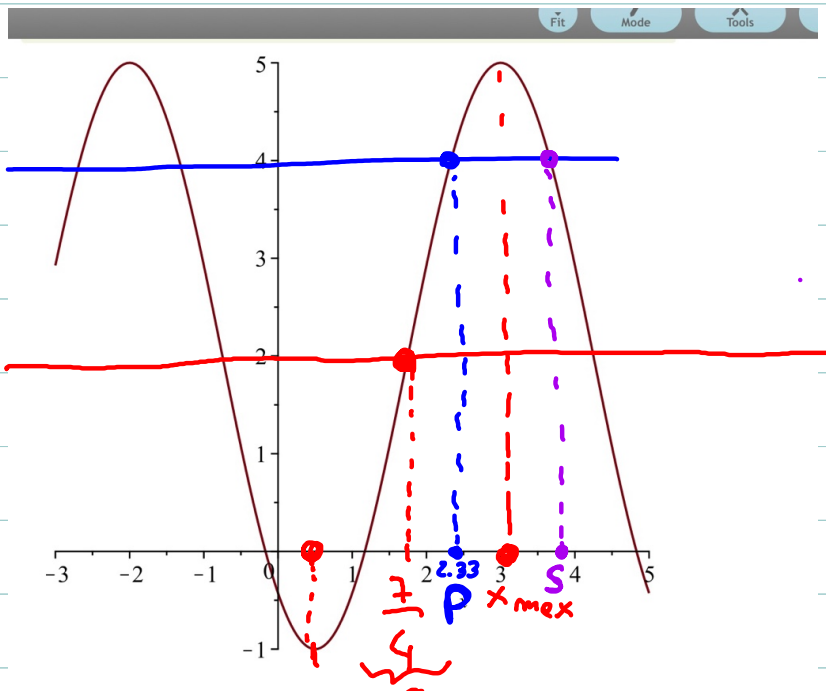
D:mean. Half way vertical point $= \frac{y_{max} + y_{min}}{2}$.

Find a formula for the sinusoidal function below



solve $3\sin(\frac{2\pi}{5}(t-\frac{7}{4}))+2=4$


$y = 3\sin\left(\frac{2\pi}{5}(x-\frac{7}{4})\right)+2$
D+A
y=4
y=2
=D
D-A
C
P
S
$x_{MAX} = c + \frac{B}{4}$
P principal
S Symmetry
$x_{MAX} + (x_{MAX} - p)$



principal solution always between

$$c - \frac{B}{4} \text{ and } c + \frac{B}{4}$$

symmetry solution

$x_{MAX} + x_{MAX}$ - principal

$c + \frac{B}{4} + c + \frac{B}{4}$ - principal

$2c + \frac{B}{2}$ - principal

Other way to find symmetry solution

Solve $3\sin\left(\frac{2\pi}{5}\left(t-\frac{7}{4}\right)\right)+2=4$

$\sin\left(\underbrace{\frac{2\pi}{5}\left(t-\frac{7}{4}\right)}_{\theta}\right)=\frac{2}{3}$ $\qquad \frac{5}{2\pi}\sin^{-1}\left(\frac{2}{3}\right)+\frac{7}{4}=p$

$\sin(\theta)=\frac{2}{3}$

symmetry is $\theta=\pi-\sin^{-1}\left(\frac{2}{3}\right)$

$\frac{2\pi}{5}\left(t-\frac{7}{4}\right)=\pi-\sin^{-1}\left(\frac{2}{3}\right)$ solve for t

$t-\frac{7}{4}=\frac{5}{2\pi}\left(\pi-\sin^{-1}\left(\frac{2}{3}\right)\right)$

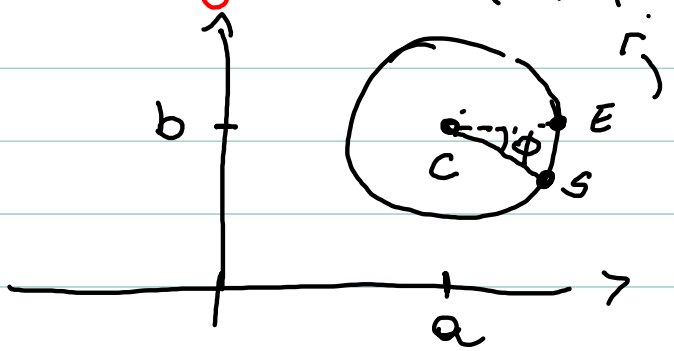
$t=\frac{7}{4}+\frac{5}{2}-\frac{5}{2\pi}\sin^{-1}\left(\frac{2}{3}\right)$

$\underbrace{\frac{7}{4}+\frac{7}{4}+\frac{5}{2}}_{2c+B/2}-\left(\frac{5}{2\pi}-\frac{7}{4}\sin^{-1}\frac{2}{3}\right)$

How to solve $A\sin(\frac{2\pi}{B}(x-C))+D=V$

- Do some algebra first: $\sin(\frac{2\pi}{B}(x-C))=\frac{V-D}{A}$
- Use arcsin : $\frac{2\pi}{B}(x-C)=\arcsin(\frac{V-D}{A})$
- Do some more algebra to solve for x:
  $x=C+\frac{B}{2\pi}\arcsin(\frac{V-D}{A})$.
- $x_1=C+\frac{B}{2\pi}\arcsin(\frac{V-D}{A})$ is the principal solution.
  $C-\frac{B}{4}\leq x_1\leq C+\frac{B}{4}$
- All values x_1+kB ,$k=0,\pm1,\pm2,\pm3,\cdots$ are also solutions.
- The symmetry solution is $x_2=x_{\text{max}}+(x_{\text{max}}-\text{principal})$, where x_{max} is the x coordinate of the first max to the right of C.
- All values x_2+kB $k=0,\pm1,\pm2,\pm3,\cdots$ are also solutions.

WebAssign hw (ch 19 #3)



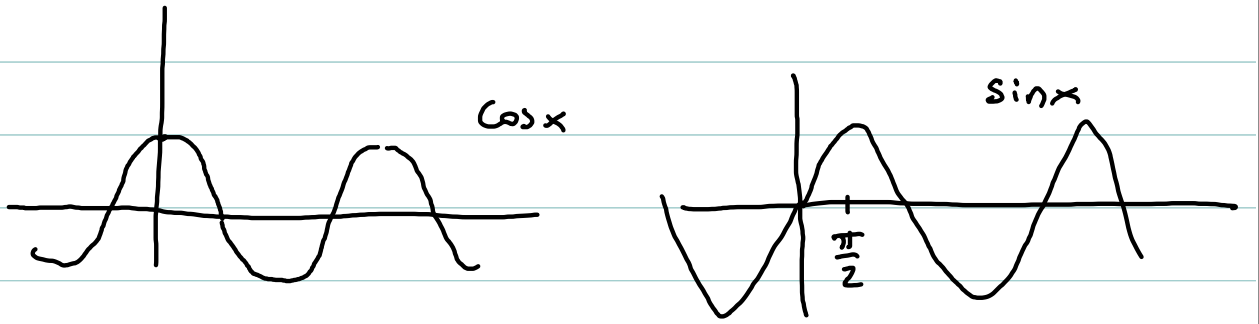
$C(x_0, y_0)$

Angular speed ω

The height of the rotating object above the x axis is a sinusoidal function

See Video on Week 10 module of canvas

Note :



$\cos x = \sin\left(x + \frac{\pi}{2}\right)$

$\cos x = \sin\left(-x + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - x\right)$