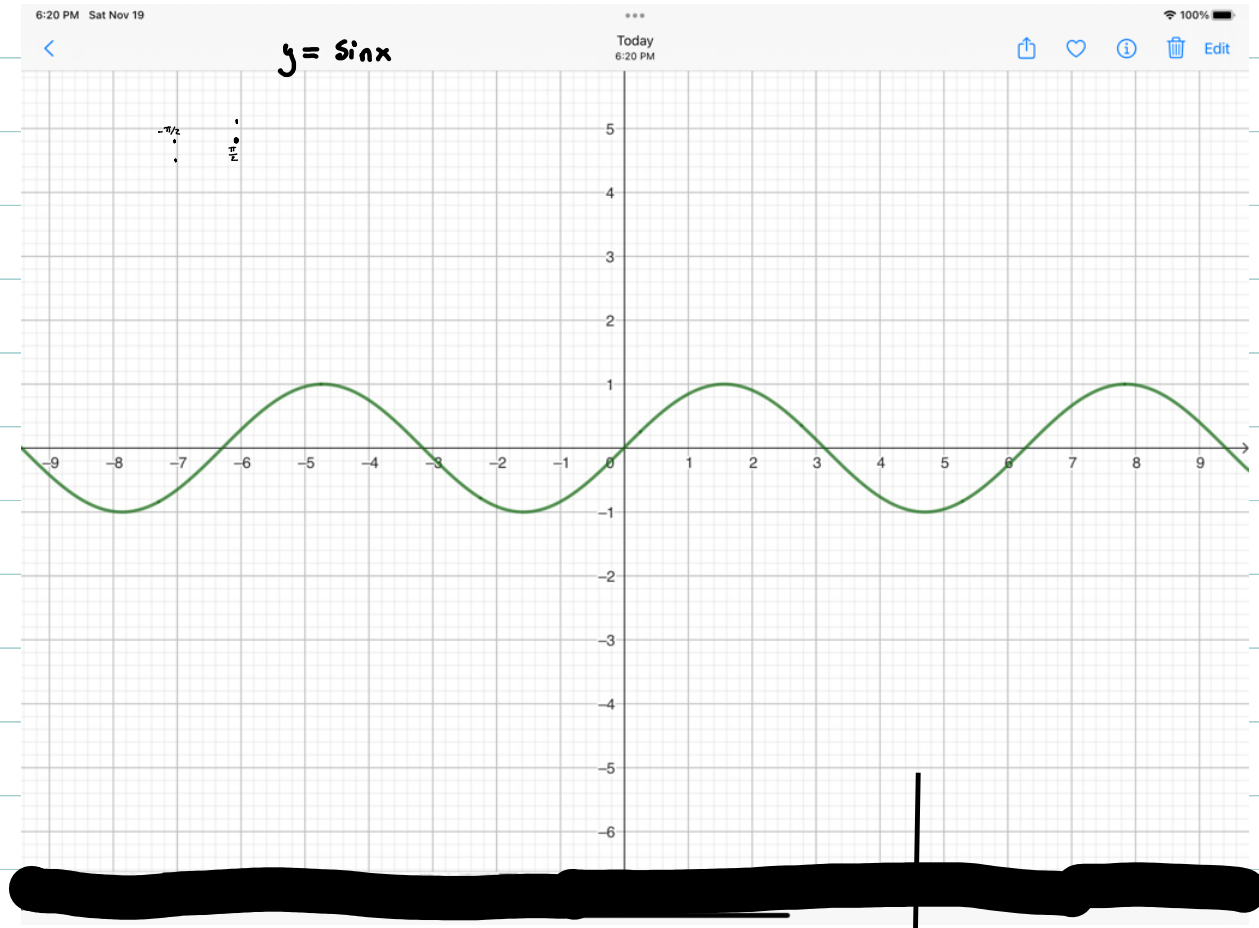
Lesson 25

Read Chapter 19 and 20

Inverse trigonometric functions

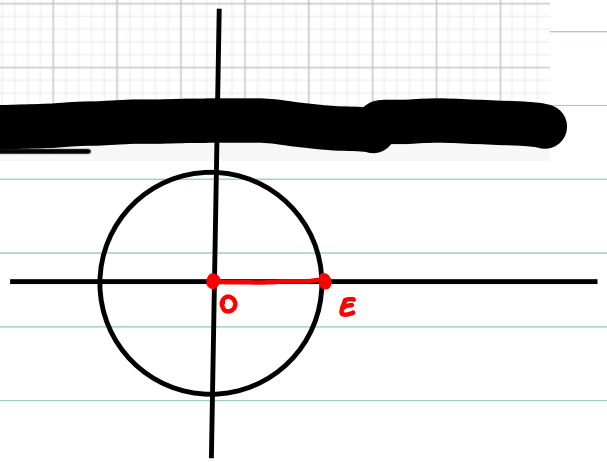
Sinusoidal functions

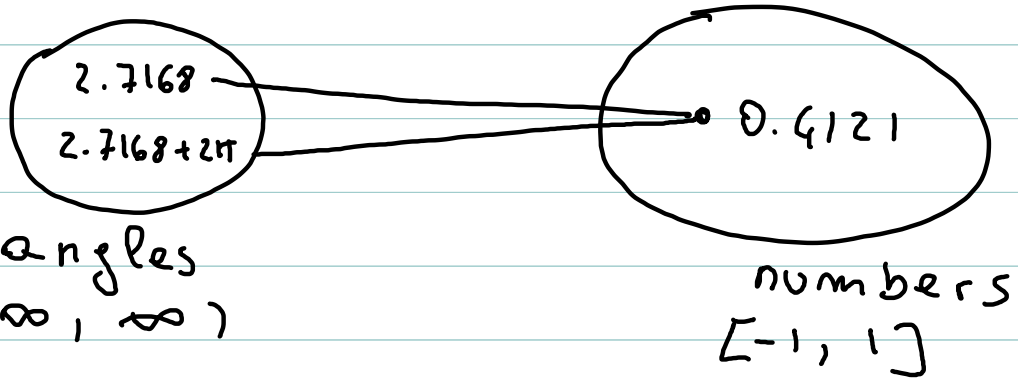


$\sin(9) = 0.4121$

$9 - 2\pi = 2.7168$

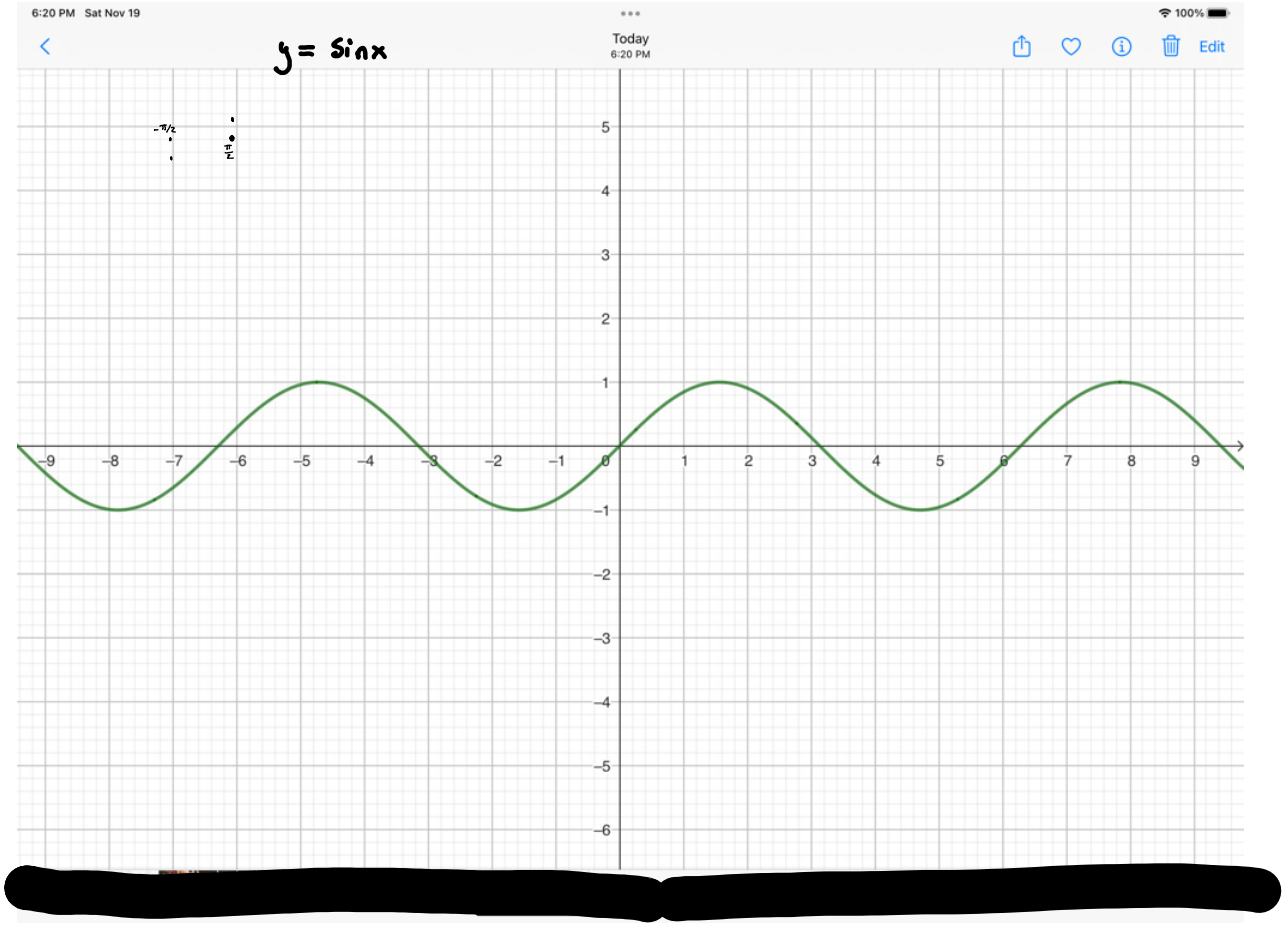
$9 = 2\pi + 2.7168$



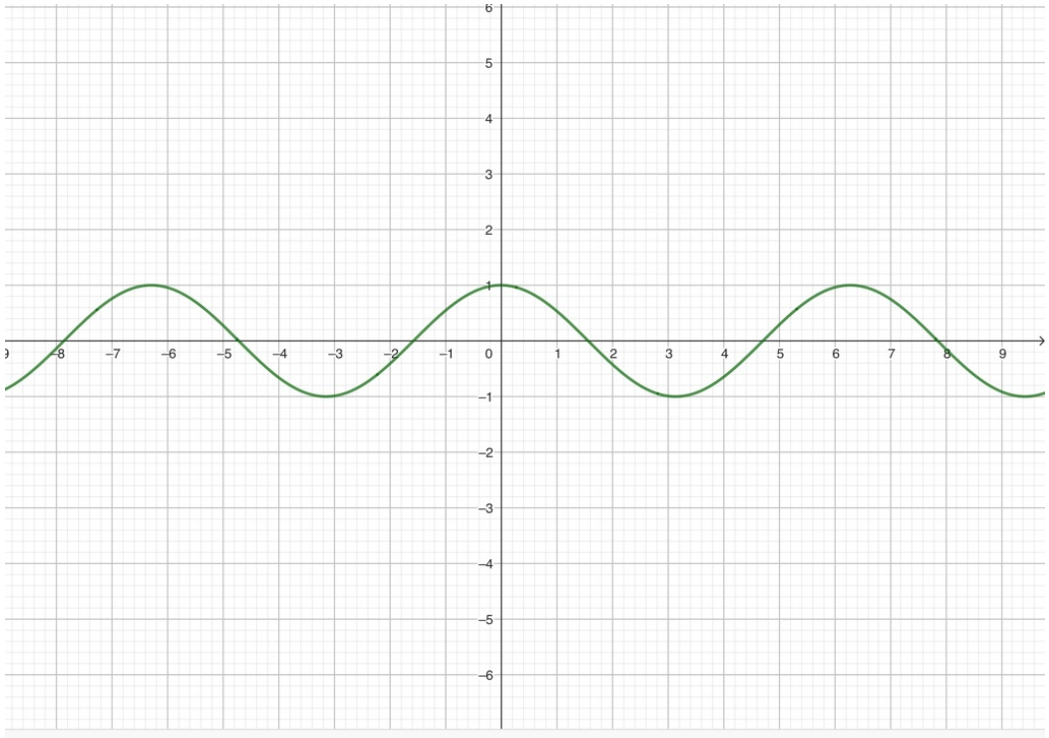


NOT invertible

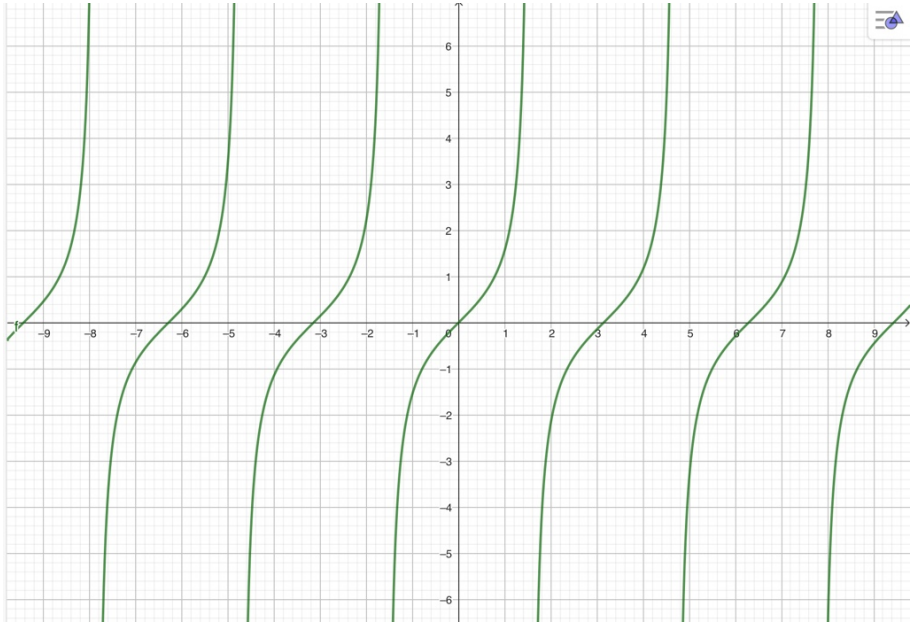
arcsin(x) is the inverse of $\sin x$ restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

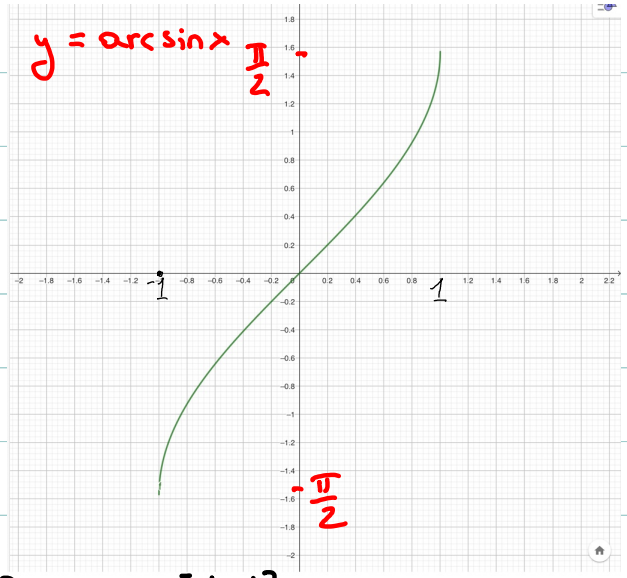


$\arccos(x)$ is the inverse of $\cos x$ restricted to $[0, \pi]$

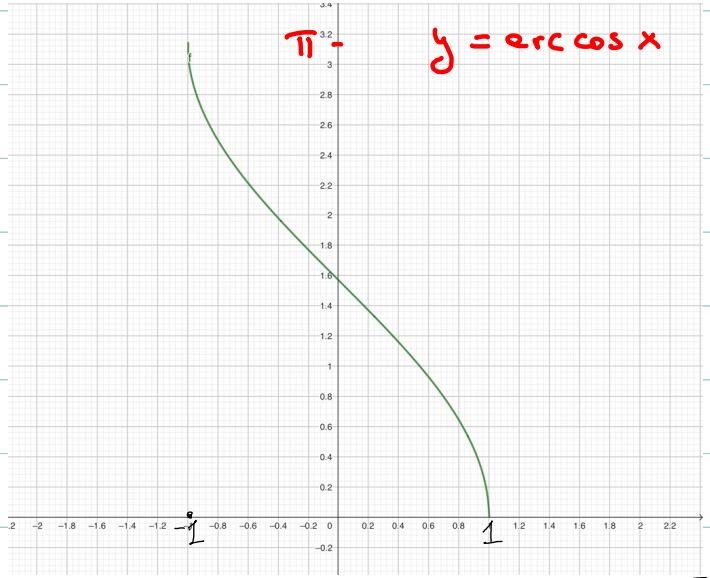


arctan(x) is the inverse of $\tan x$ restricted to $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

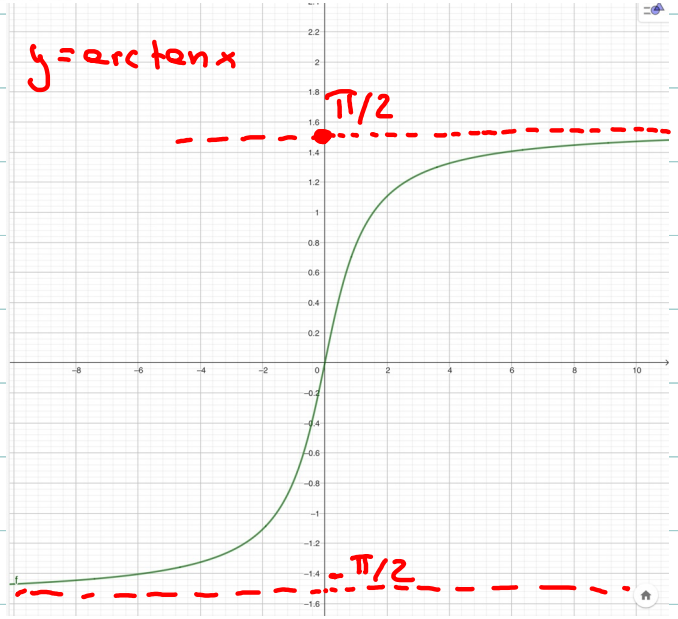




Domain $[-1, 1]$
Range $[-\frac{\pi}{2}, \frac{\pi}{2}]$

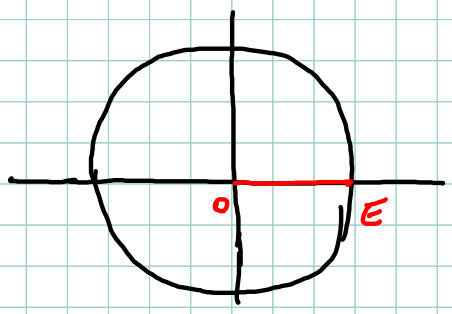


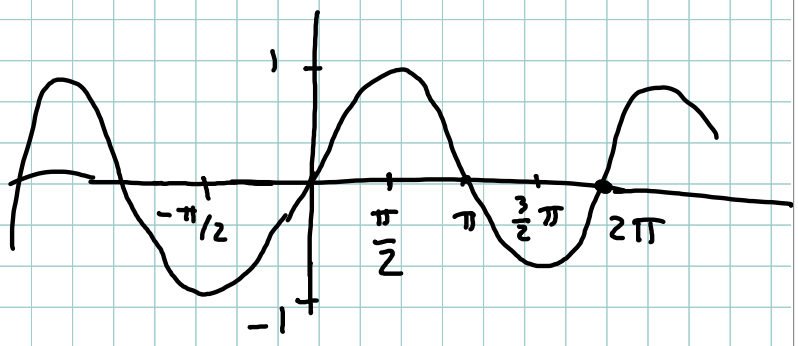
Domain $[-1, 1]$
Range $[0, \pi]$

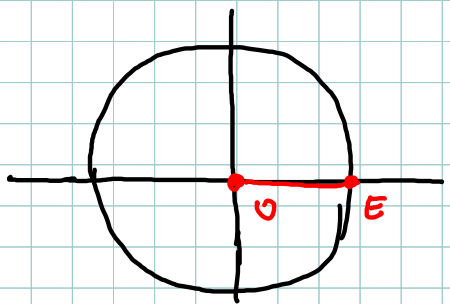


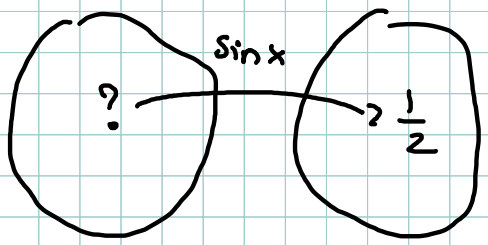
Domain $(-\infty, +\infty)$
Range $(-\frac{\pi}{2}, \frac{\pi}{2})$

Solve $\sin(x) = \frac{1}{2}$





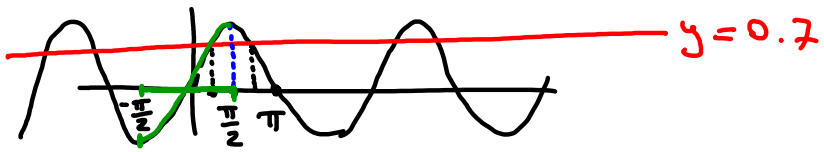


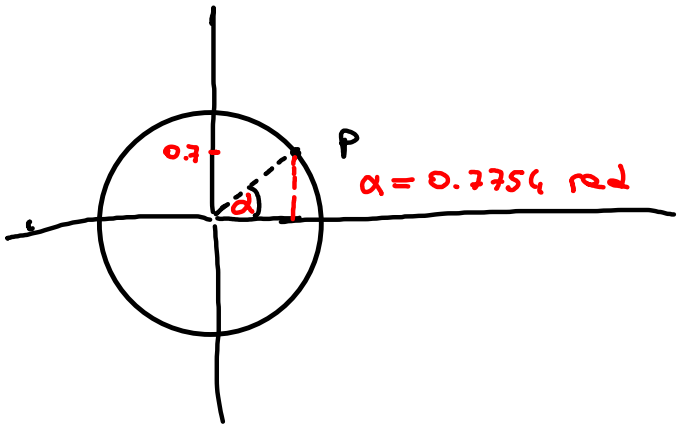


angles $\left[-\frac{\pi}{2} \quad \frac{\pi}{2}\right]$

numbers $[-1 \quad 1]$

Solve $\sin(x) = 0.7$





1) $\sin^{-1}(0.7) = 0.7754$ CALCULATOR MODE IS RAD

($\frac{\pi}{4} \approx 0.7854$) Principal

2) $\pi - 0.7754 \simeq 2.3662$ rad symmetry

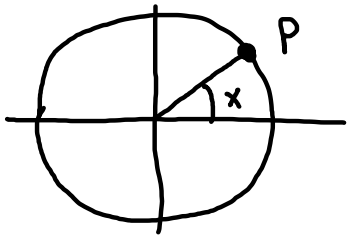
ALL sol $0.7754 + 2k\pi$

$2.3662 + 2k\pi$

How to solve $\sin(x) = c$

- $x_1 = \arcsin(c)$ This is the principal solution. $-\frac{\pi}{2} \leq x_1 \leq \frac{\pi}{2}$
- All values $x_1 + 2\pi k$, $k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.
- $x_2 = \pi - x_1$. This is the symmetric solution. $\frac{\pi}{2} \leq x_2 \leq \frac{3\pi}{2}$
- All values $x_2 + 2\pi k$ $k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.

How to solve $\cos(x) = c$



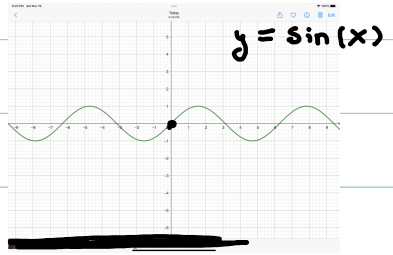
- $x_1 = \arccos(c)$ This is the principal solution. $0 \leq x_1 \leq \pi$
- All values $x_1 + 2\pi k, k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.
- $x_2 = -x_1$. This is the symmetry solution. $-\pi \leq x_2 \leq 0$
- All values $x_2 + 2\pi k \;\; k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.

How to solve arctan (x)=c

- $x_1 = \arctan(c)$ This is the principal solution. $-\frac{\pi}{2} < x_1 < \frac{\pi}{2}$
- All values $x_1 + \pi k$ $k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.

Sinusoidal functions

$$f(x) = A \sin\left(\frac{2\pi}{B}(x - C)\right) + D$$



1) h scale of $\frac{B}{2\pi}$

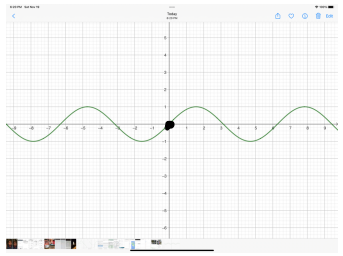
2) Shift right C units (if C is positive)
   left C units (if C is negative)

3) V scale of A

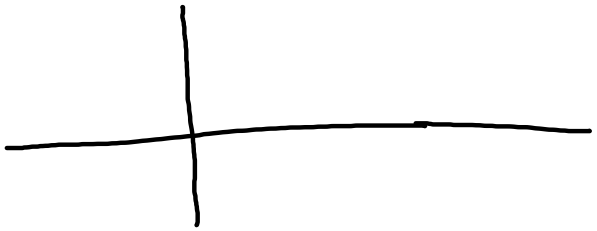
4) Shift up D units (if D is positive)
   down D units (if D is negative)

Graph $f(x) = A\sin(\frac{2\pi}{B}(x - C)) + D$

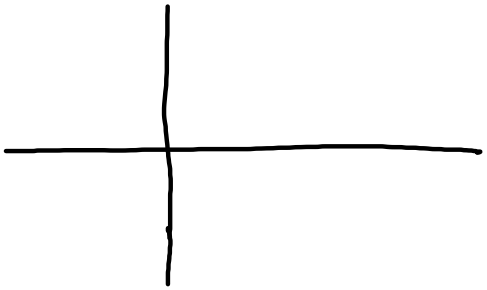
A, B > 0



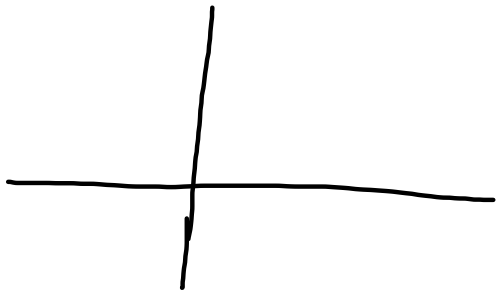
1



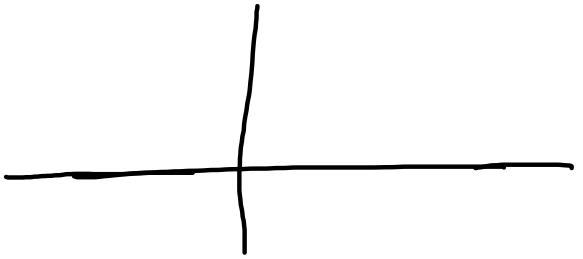
2



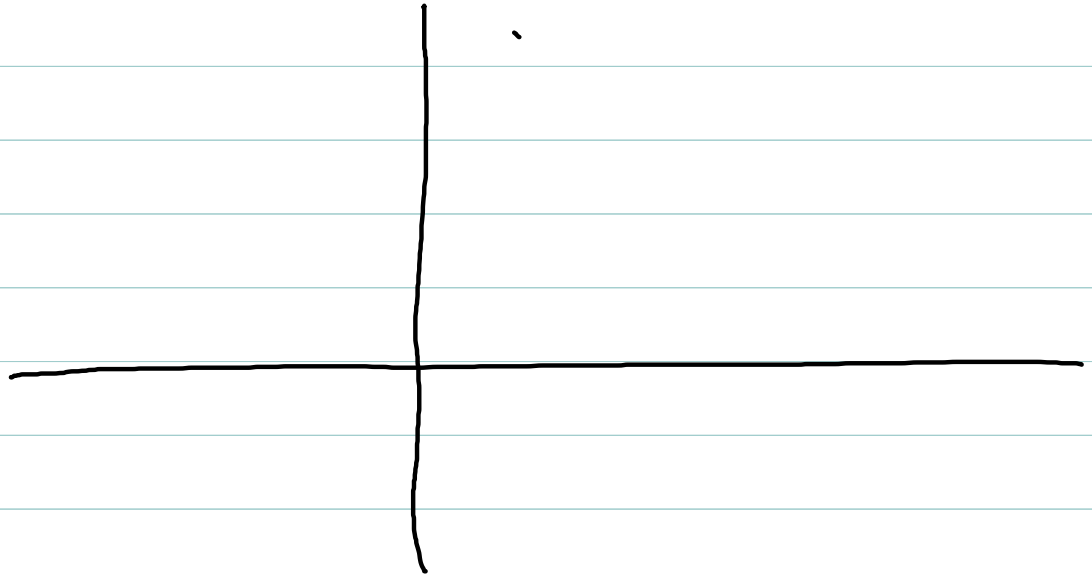
3



4



Graph $f(t) = \underbrace{3}_{A} \sin \underbrace{\frac{2\pi}{5}}_{B}\left(t - \underbrace{\frac{7}{4}}_{C}\right) + \underbrace{2}_{D}$



1) Draw $y = D$

2) Draw points (C, D) $\left(C + \frac{B}{4}, D\right)$ $\left(C + \frac{B}{2}, D\right)$ $\left(C + \frac{3}{4}B, D\right)$ $(C + B)$

3) Draw points $\left(C + \frac{B}{4}, D + A\right)$, $\left(C + \frac{3}{4}B, D - A\right)$

4) Draw basic S shape and repeat