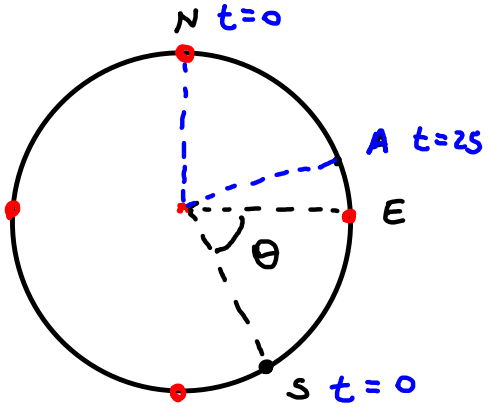
Lesson 21

Read Chapter 17

Trigonometric functions. Triangle definition.

Problems involving two triangles

Two objects move around a circle . They start at the same time. Object 1 moves in the counterclockwise direction, with angular speed of $\frac{\pi}{50}$ rad/sec; from where it starts it takes it 20 seconds to reach the easternmost part of the track. Object 2 moves in the clockwise direction, starting from the northernmost part of the track 's with a speed of 4 feet / sec . The two objects pass each other after 25 sec. What is object 1's starting position ? (Give your answer as an angle). What is the radius of the track ?

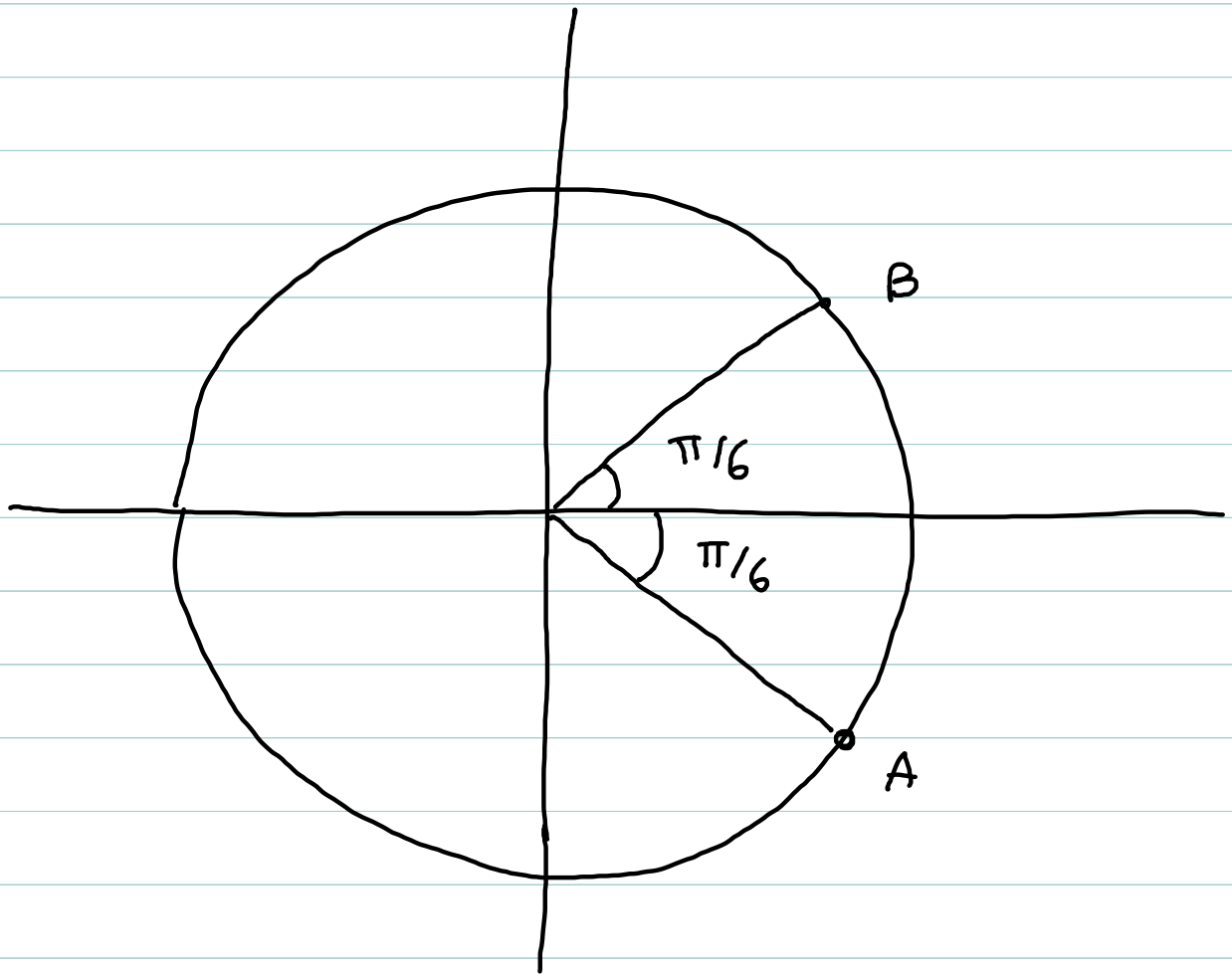


$T = \frac{2\pi}{\omega} = 100$ sec

$\frac{T}{4} = 25$

$\theta = \omega t$

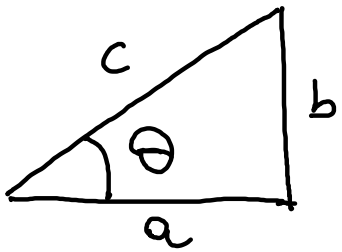
$\theta = \frac{\pi}{50} \cdot 20 = \frac{2}{5}\pi$ rad



1) How do we distinguish the two angles?

2) What are the coordinates of A and B?

Trig for angles $0 < \theta < \frac{\pi}{2}$



$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

$\theta = \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}$

$a^2 + a^2 = 1 \; ; \; 2a^2 = 1 \; ; \; a^2 = \frac{1}{2} \; ; \; a = \frac{1}{\sqrt{2}}$

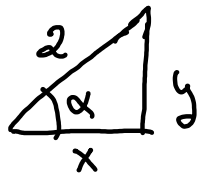
$\sin \frac{\pi}{4} =$ $\quad \cos \frac{\pi}{4} =$ $\quad \tan \frac{\pi}{4} =$

$h^2 + \left(\frac{1}{2}\right)^2 = 1 \; ; \; h^2 = 1 - \frac{1}{4} \; ; \; h^2 = \frac{3}{4} \; ; \; h = \frac{\sqrt{3}}{2}$

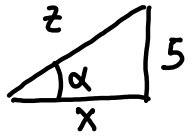
$\sin \frac{\pi}{3} =$ $\quad \cos \frac{\pi}{3} =$ $\quad \tan \frac{\pi}{3} =$

$\sin \frac{\pi}{6} =$ $\quad \cos \frac{\pi}{6} =$ $\quad \tan \frac{\pi}{6} =$

If $\alpha = 1.3$ rad, find x and y

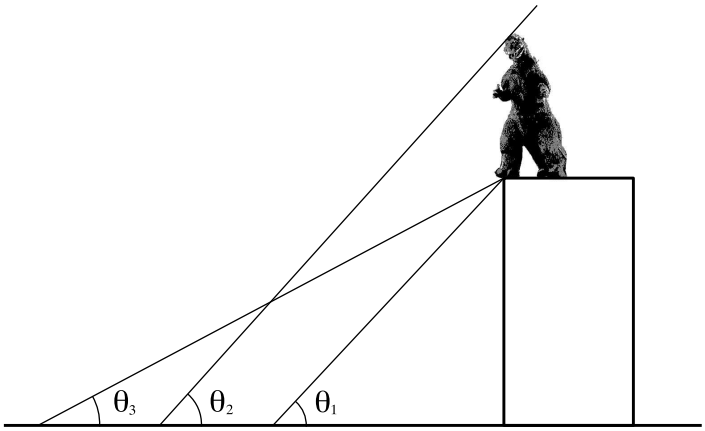


If $\alpha = 0.5$ rad, find x and z



3. Godzilla is attacking, but at the moment he is standing on top of a building downtown. You want to determine Godzilla's height, so you measure three angles. First, from a certain distance away from the building, you measure the angle the top of the building makes with the horizontal: $\theta_1 = 72°$. You then move 50 meters farther from the building and measure the angle Godzilla's head makes with the horizontal: $\theta_2 = 74°$. You then move 75 meters farther from the building and measure the angle the top of the building makes with the horizontal: $\theta_3 = 60°$.

   The figure may not be to scale.



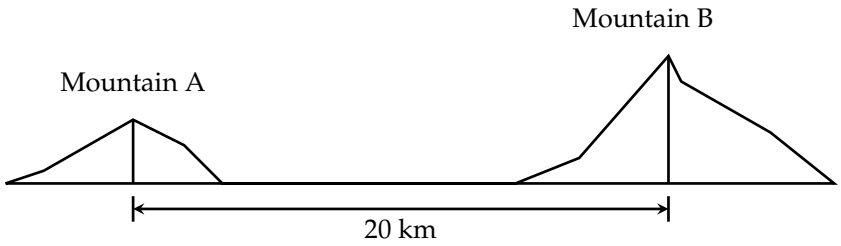
   How tall is Godzilla?

4. You are on a road connecting the bases of Mountain A and Mountain B.

   You look at Mountain A and measure the angle of elevation to the top of Mountain A to be $15°$.

   You then travel 2 km toward Mountain B.

   You measure Mountain B's angle of elevation from your new location to be $17°$.



   Mountain A and Mountain B are 20 km apart as shown in the figure, and Mountain B is exactly twice as tall as Mountain A.

   What is the height of Mountain A?

$$\begin{cases} \frac{x}{y} = \tan(15^\circ) \\ \frac{2x}{18-y} = \tan(17^\circ) \end{cases} \qquad \begin{cases} x = y\tan(15^\circ) \\ 2x = (18-y)\tan(17^\circ) \end{cases}$$

$$2y\tan(15^\circ) = 18\cdot\tan(17^\circ) - y\tan(17^\circ)$$

$$2y\tan(15^\circ) + y\tan(17^\circ) = 18\tan(17^\circ)$$

$$y(2\tan(15^\circ) + \tan(17^\circ)) = 18\tan(17^\circ)$$

$$y = \frac{18\tan(17^\circ)}{2\tan(15^\circ)+\tan(17^\circ)}$$

$$x = \frac{18\tan(17^\circ)}{2\tan(15^\circ)+\tan(17^\circ)} \cdot \tan(15^\circ) \simeq 1.75 \text{ km}$$