

Lesson 17

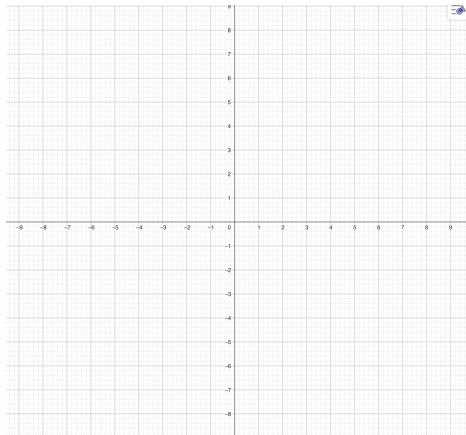
Read Chapter 13

Graphical tools

Motivation

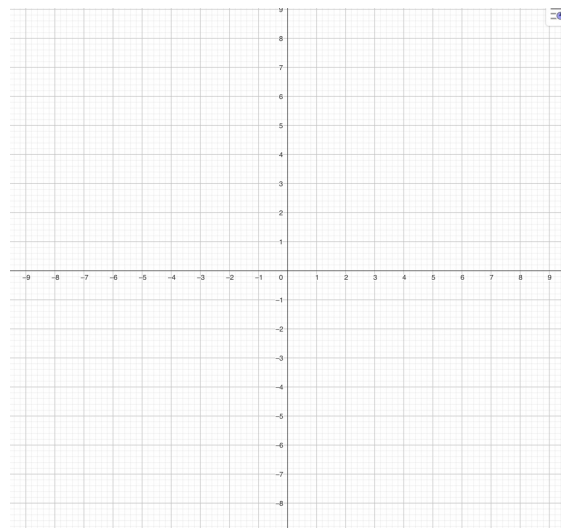
In an old problem we considered graphing $y = 1 - |x|$.

①



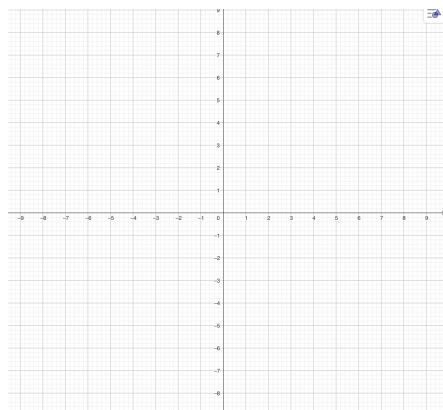
$$y = |x|$$

②



$$y = -|x|$$
$$J_{\text{new}} = -J_{\text{old}}$$

③



$$y = 1 - |x|$$
$$J_{\text{new}} = 1 + J_{\text{old}}$$

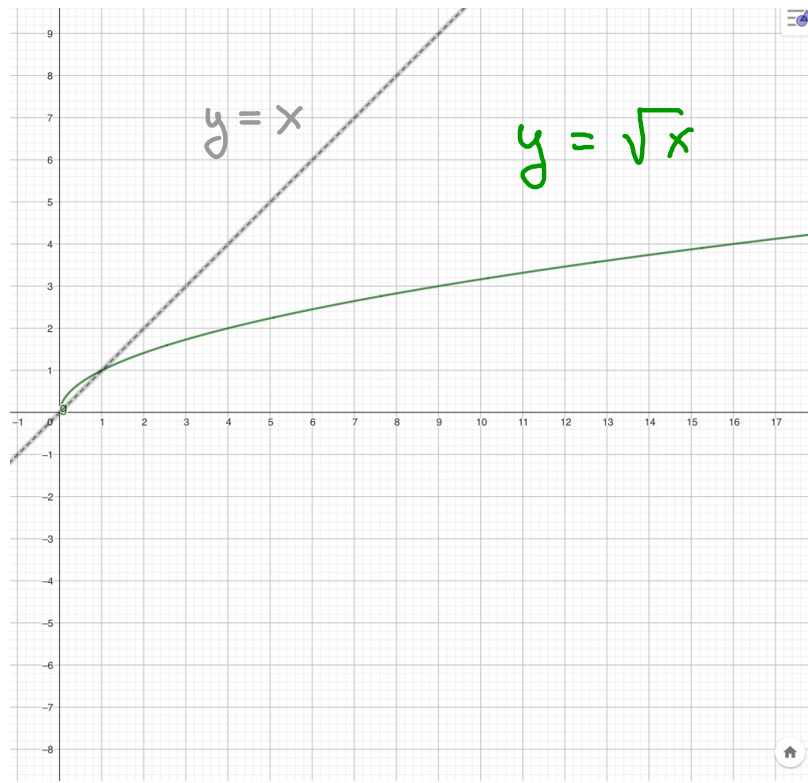
Goals

Draw the graph of $af(bx+c)+d$ from the graph of $f(x)$

Find the formula for the function whose graph is obtained from the graph of $f(x)$ by performing a series of graphical operations (shifts, reflections and scalings)

Which graphs should you know to start with?

Linear functions, quadratic functions, exponential functions, $\ln x$,
 \sqrt{x} , $|x|$



Domain $[0, +\infty)$
Range $[0, +\infty)$

Vertical translation

Given $f(x) = x^2$, what do the graph of

$$y = f(x) + 5 \text{ and}$$

$$y = f(x) - 5$$

look like ?

Vertical Reflections

Given $f(x) = |x|$

what does the graph of $y = -|x|$ look like ?

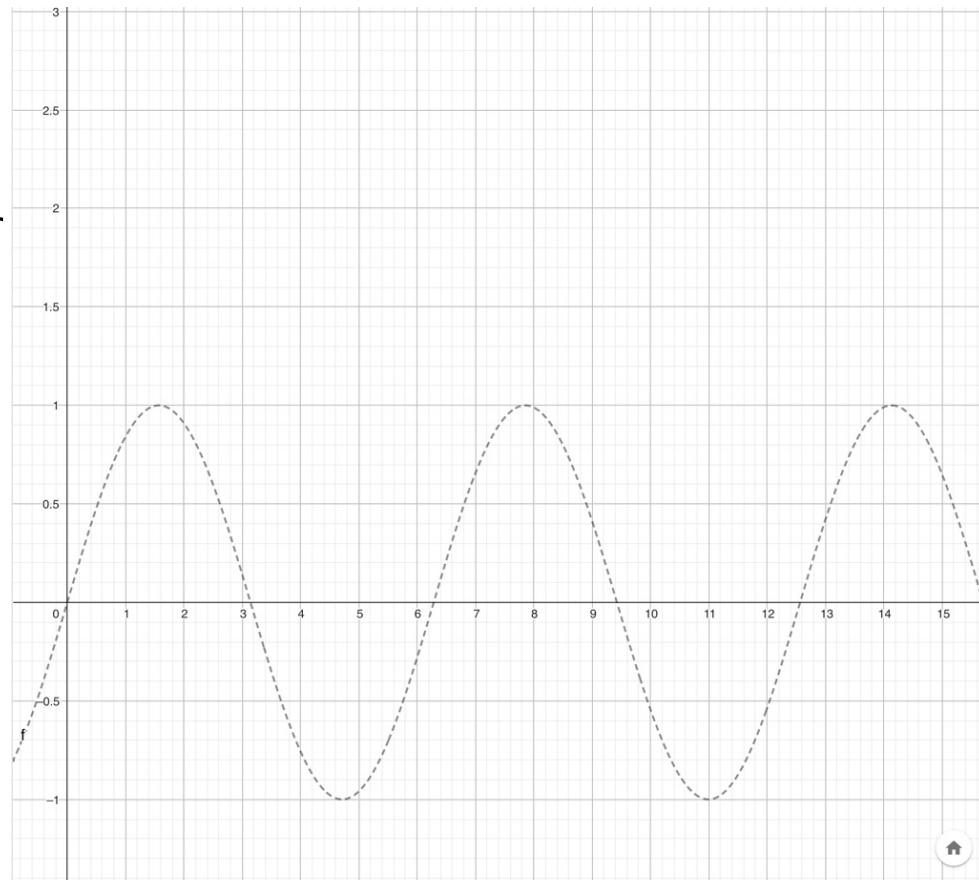
Vertical scaling (expansion or compression)

Given $f(x) = \sin x$ what do the graphs of

$y = 2f(x)$ and of

$y = \frac{1}{2}f(x)$

look like ?



$$y = -2f(x) + 3$$

Vertical translation up 3 units

reflection across x axis

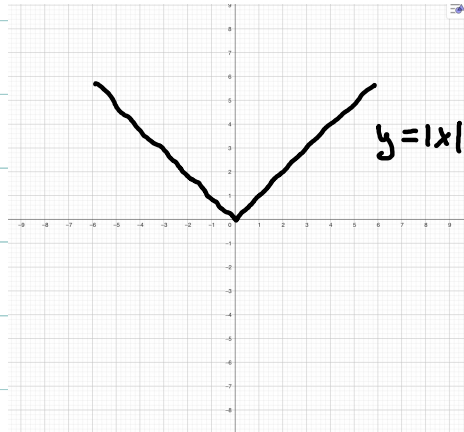
vertical scaling
of a factor of 2

In which order?

$$y = 1 - |x|$$

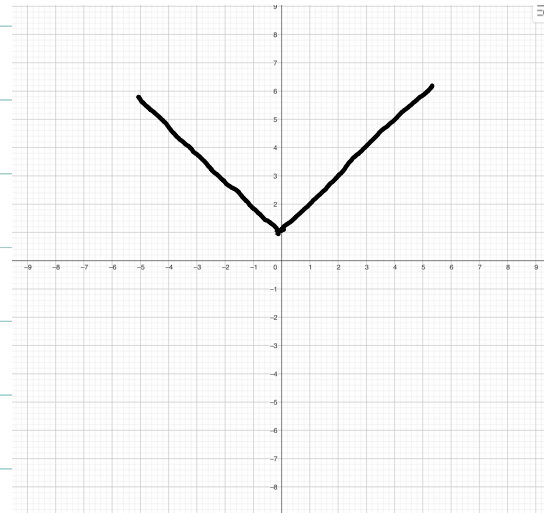
WRONG ORDER

①

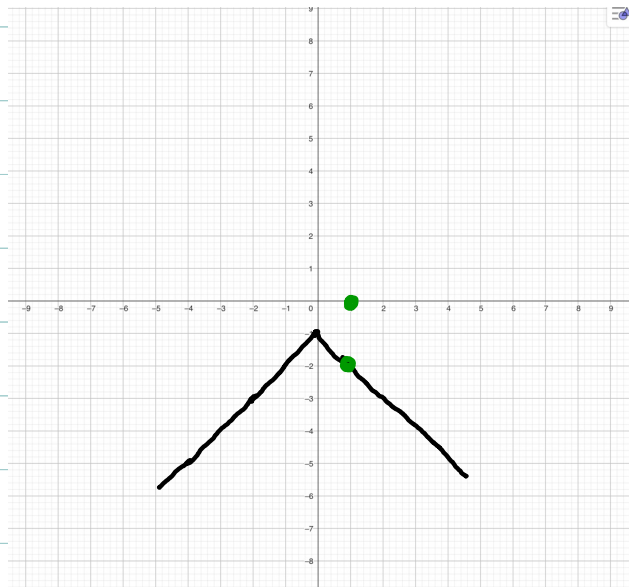


②

Shift up
1 unit



③



reflect across x axis

WRONG GRAPH !

when $x = 1$ y should be 0

Horizontal translation

Given $f(x) = x^2$ what do the graph of

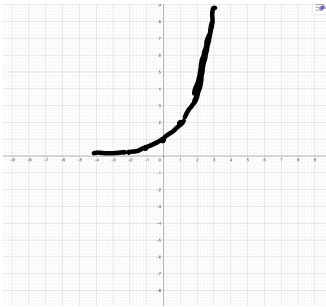
$$f(x + 5) =$$

and $f(x - 5) =$

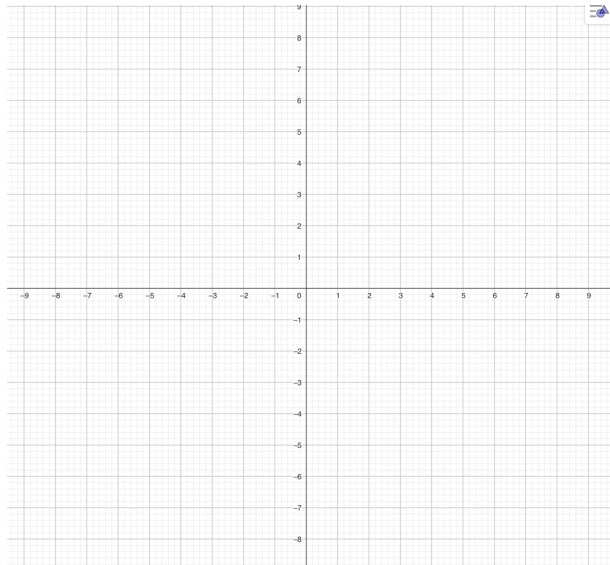
look like ?

Horizontal Reflections

Given $f(x) = 2^x$ what does the graph of



$f(-x) =$ looks like ?



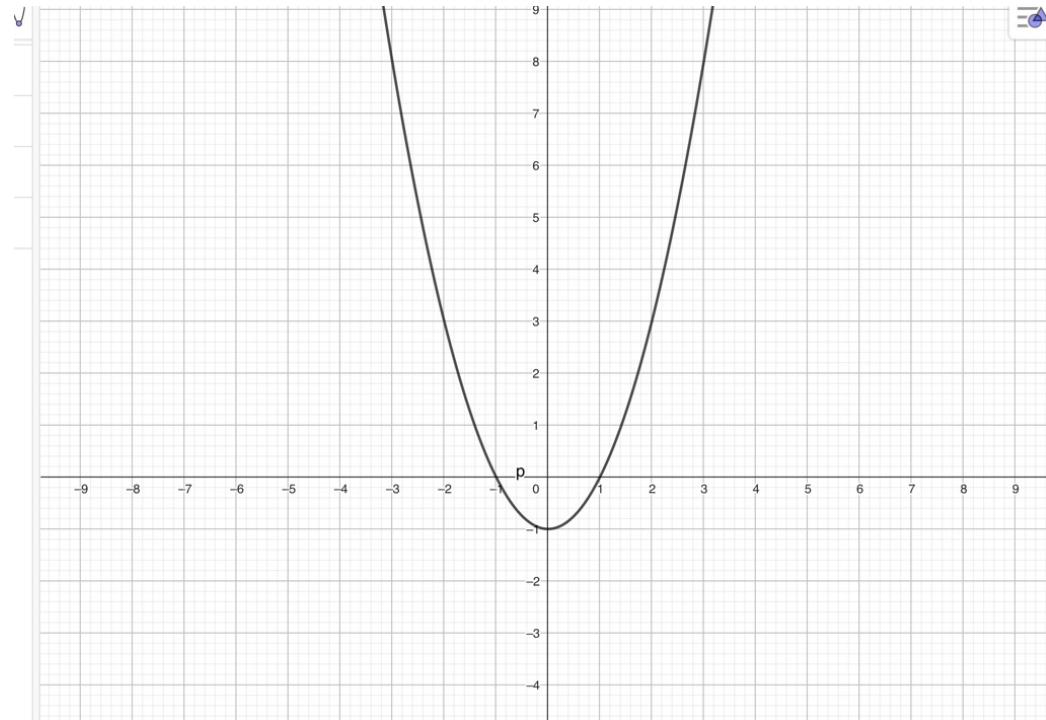
Horizontal scaling (expansion or compression)

Given $f(x) = x^2 - 1$ what do the graphs of

$$p(x) = f\left(\frac{x}{2}\right) =$$

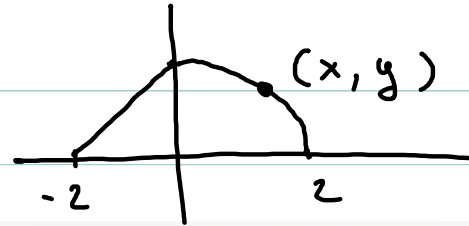
.

$$\text{and of } q(x) = f\left(\frac{x}{1/2}\right)$$



look like ?

original function $y = f(x)$



Shifting (Assume $c > 0$)

Symbolic Change	New Equation	Graphical Consequence	Picture
Replace x with $(x - c)$.	$y = f(x - c)$	A shift to the right c units.	
Replace x with $(x + c)$.	$y = f(x + c)$	A shift to the left c units.	
Replace $f(x)$ with $(f(x) + c)$.	$y = f(x) + c$	A shift up c units.	
Replace $f(x)$ with $(f(x) - c)$.	$y = f(x) - c$	A shift down c units.	

Horizontal

Vertical

(x, y) moves to

$(x + c, y)$

$(x - c, y)$

$(x, y + c)$

$(x, y - c)$

Table 13.2: Shifting $y = f(x)$.

Reflection

Symbolic Change	New Equation	Graphical Consequence	Picture
Replace x with $-x$.	$y = f(-x)$	A reflection across the y -axis.	
Replace $f(x)$ with $-f(x)$.	$y = -f(x)$	A reflection across the x -axis.	

(x, y) moves to

$(-x, y)$

$(x, -y)$

Table 13.1: Reflecting $y = f(x)$.

Dilation

Symbolic Change	New Equation	Graphical Consequence	Picture
If $c > 1$, replace x with $(\frac{x}{c})$.	$y = f(\frac{x}{c})$	A horizontal expansion.	
If $0 < c < 1$, replace x with $(\frac{x}{c})$.	$y = f(\frac{x}{c})$	A horizontal compression.	
If $c > 1$, replace $f(x)$ with $(cf(x))$.	$y = cf(x)$	A vertical expansion.	
If $0 < c < 1$, replace $f(x)$ with $(cf(x))$.	$y = cf(x)$	A vertical compression.	

horizontal

vertical

(x, y) moves to

(cx, y)

(x, cy)

(x, cy)

How to graph $a f(bx + c) + d$

1. Graph $y = f(x)$

Horizontally :

2. Shift $|c|$ units, left if c is positive, right if c is negative .

3. Scale horizontally of a factor $\frac{1}{|b|}$ (compression if $|b| > 1$, expansion if $|b| < 1$)

4. Reflect across y axis if b is negative . Skip this step if b is positive.

Vertically:

5. Scale by a factor of $|a|$ (compression if $|a| < 1$, expansion if $|a| > 1$)

6. Reflect across x axis if a is negative . Skip this step if a is positive.

7. Shift $|d|$ units, up if c is positive, down if c is negative .

Note: the order is important.

How to graph $a f(b(x + c)) + d$.

1. Graph $y = f(x)$

Horizontally:

2. Scale horizontally of a factor $\frac{1}{|b|}$. (compression if $|b| > 1$, expansion if $|b| < 1$)

3. Reflect across y axis if b is negative. Skip this step if b is positive.

4. Shift $|c|$ units, left if c is positive, right if c is negative.

Vertically:

5. Scale by a factor of $|a|$ (compression if $|a| < 1$, expansion if $|a| > 1$)

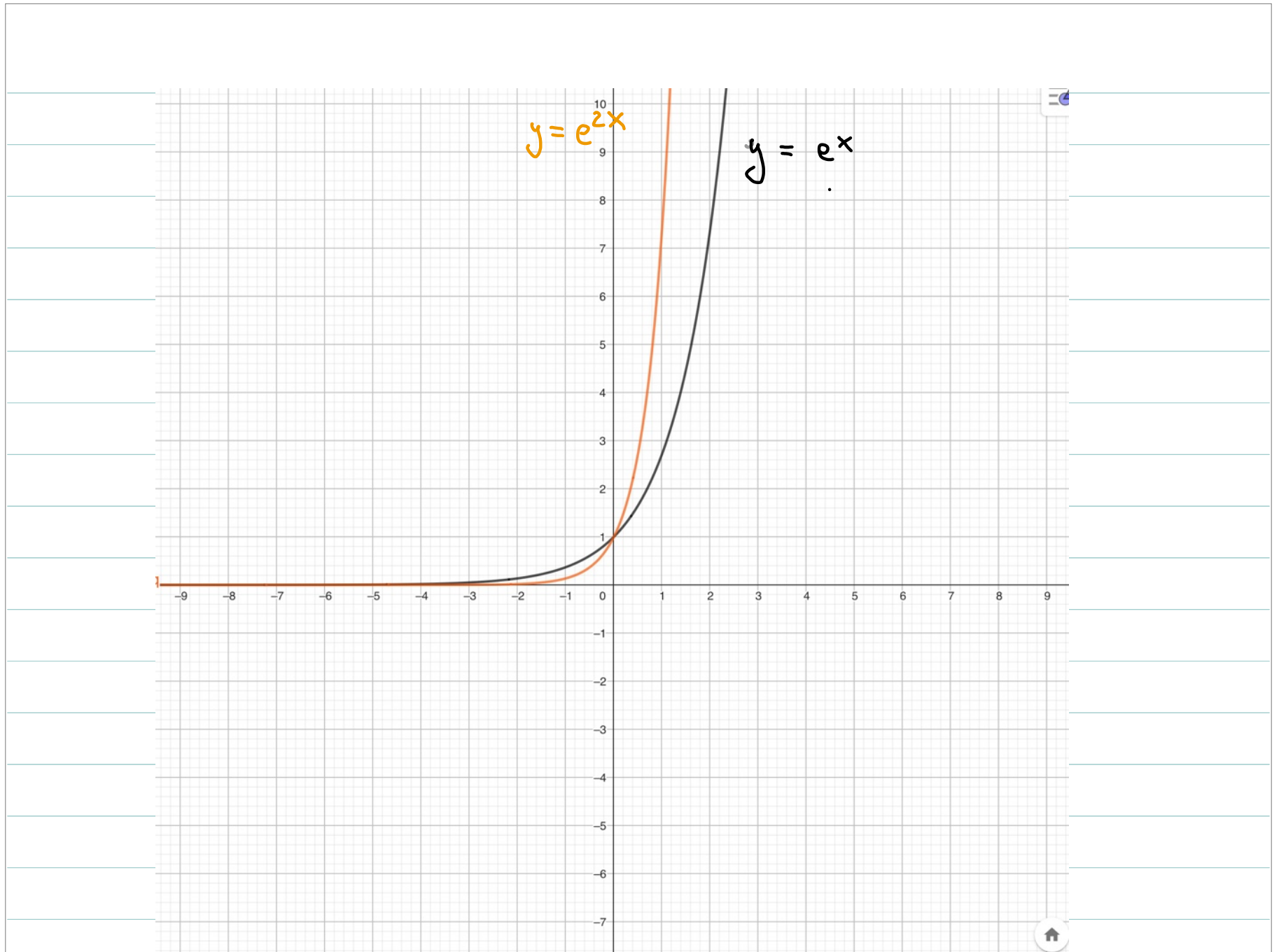
6. Reflect across x axis if a is negative. Skip this step if a is positive.

7. Shift $|d|$ units, up if d is positive, down if d is negative.

Note: the order is important.

Sketch the graph of $g(x) = 3|2x - 5| + 1$

Sketch the graph of $g(x) = 3e^{2(x-5)} + 1$





f is even if $f(x) = f(-x)$.

EX: $\cos x$ is even

f is odd if $f(x) = -f(-x)$.

EX: $\sin x$ is odd