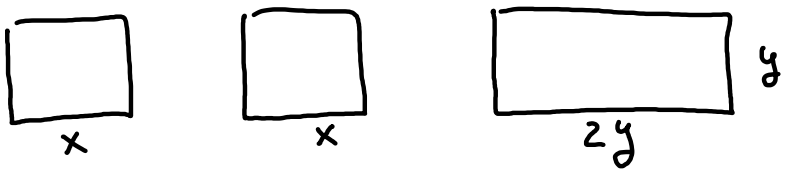
Lesson 12

Read Chapter 8

Composition

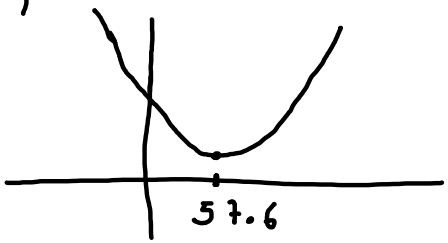
You have 720 m of fencing with which to build 3 enclosures. Two are identical squares and one is a rectangle that is twice as long as it is wide. What should the dimensions of the squares be, in order to to minimize the combined area of all three enclosures ? What should the dimensions of the squares be, in order to maximize the combined area of all three enclosures ?



From last time:

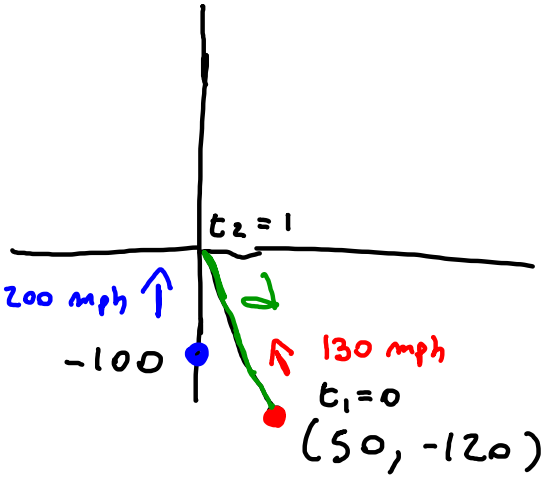
$A(x) = \frac{50}{9}x^2 - 640x + 28800$

$720 = 8x + 6y$



How about maximizing area?

An American Airlines plane is flying North at a speed of 200 mph. At time $t = 0$ it is located 100 mi South of a control tower. A United Airlines plane is flying in a straight line towards the control tower with a speed of 130 mi/hour. At time $t = 0$ it is located 50 mi East and 100 mi South of the control tower. When are the planes closest ? How close do they get ?



$d = \sqrt{50^2 + 120^2} = 130$

$t_2 = \frac{130}{130} = 1$

AA $\quad x(t) = 0$

$y(t) = -100 + 200t$

UA : $x(t) = 50 + \frac{0-50}{1-0}t$

$y(t) = -120 + \frac{0-(-120)}{1-0}t$

AA $(0, -100 + 200t)$ $\quad$ UA $= (50 - 50t, -120 + 120t)$

$d(t) = \sqrt{(50-50t)^2 + (-100+200t-(-120+120t))^2}$ $\quad$ Minimize $d(t)$

SIMPLIFY

$$d(t) = \sqrt{8900t^2 - 1800t + 2900}$$

1) For which value of t is $d(t)$ minimum?
2) What is minimum distance?

1) Trick Look at $d^2(t) = 8900t^2 - 1800t + 2900$

min for $t = \frac{1800}{2 \cdot 8900} \approx 0.1$

why? For positive values squaring does not change order

Ex:

t	$f(t)$	$f^2(t)$
1	5	25
2	2	4
3	1	1
4	7	49

2) Compute $d(0.1) = \sqrt{8900(0.1)^2 - 1800 \cdot 0.1 + 2900} \approx 53$ miles

What is a function ?

g(f(x)) in pictures

Example $f(x) = x^2 + 1, \quad g(x) = 2x + 3$

$g(f(x)) =$

$f(g(x)) =$

Suppose $f(x)$ is the profit made by selling x barrels of apples and $g(x)$ is the number of barrels of apples produced by x trees.
Explain in words the meaning of $f(g(x))$

Example $f(x) = \begin{cases} x+1 & \text{if } x \leq 0 \\ 2x^2+x+1 & \text{if } x > 0 \end{cases}$ $g(x) = 2x+3$

$g(f(x)) =$

$f(g(x)) =$

Write the following functions as composition of two functions:

e^{x^3}

$\sqrt{x^3+1}$

$f(x) = |1 - |x||$ find a multipart formula for f (no $|\ |$ there) and draw the graph of f.