Lesson 11

Chapter 7. Min/max problems

Parabola through three points

Find the equation of the parabola through (1,2),(-1,1) and (2,3)

Start with standard form:

$$y = ax^2 + bx + c$$

plug in all three points to get three equations.

$$2 = a \cdot 1^2 + b \cdot 1 + c$$
$$1 = a(-1)^2 + b(-1) + c$$
$$3 = a \cdot 3^2 + b \cdot 3 + c$$

Solve a system

$$\begin{cases} 2 = a + b + c \\ 1 = a - b + c \\ 3 = 4a + 2b + c \end{cases}$$

$$\begin{cases} a = 2 - b - c \\ 1 = (2 - b - c) - b + c \\ 3 = 4(2 - b - c) + 2b + c \end{cases}$$

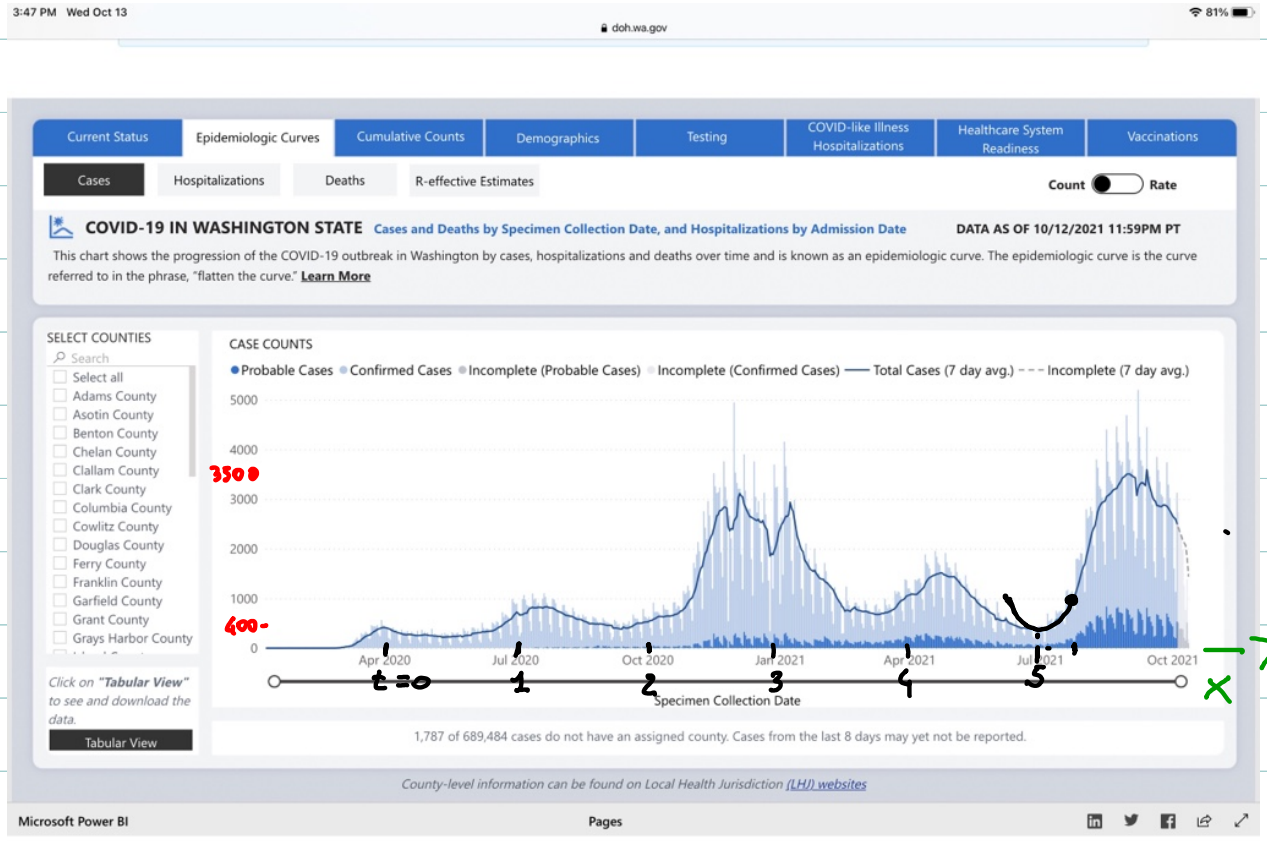
$$\begin{cases} a = 2 - b - c \\ 1 = 2 - 2b \\ 3 = 8 - 2b - 3c \end{cases}$$

$$\begin{cases} a = 2 - b - c \\ b = \frac{1}{2} \\ 3 = 8 - 2\left(\frac{1}{2}\right) - 3c \end{cases}$$

$$\cdot \begin{cases} a = 2 - b - c \\ b = \frac{1}{2} \\ \frac{3 - 8 + 1}{3} = c \end{cases}$$

$$\begin{cases} b = \frac{1}{2} \\ c = \frac{4}{3} \\ a = 2 - \frac{1}{2} - \frac{4}{3} = \frac{1}{6} \end{cases}$$

$$y = \frac{1}{6}x^2 + \frac{1}{2}x + \frac{4}{3}$$



Summary Data Tables

Find equation of parabola with vertex at (5, 400)

and through (5.25, 1000)

A min/max problem is a modeling problem where you need to minimize/maximize a quantity q.
In this class $q = q(x) = ax^2 + bx + c$
The min / max usually is at the vertex of the parabola

To solve a min/max problem

- Choose your variables and find a formula for q. q=q(x).
- In 120 q(x) should involve a quadratic function. Usually you find max/min by finding the vertex.
- Pay attention whether the problem is asking for an x value (h) or a q value (k) or both.

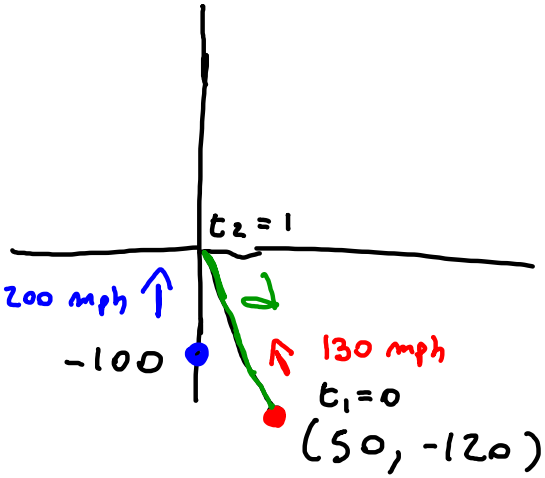
Issues–tricks

- The quantity is a distance given by a formula : $q = \sqrt{\cdots x \cdots}$.
- q depends on more than one variable. q=q(x,y).
- Min/max not at vertex.

Rosalie is organizing a circus performance to raise money for a charity. She is trying to decide how much to charge for tickets. From past experience she knows that the number of tickets sold is a linear function of the price. If she charges 5 dollars per ticket , she can sell 1000 tickets, if she charges 7 dollars she can only sell 900 tickets. How much should she charge per ticket to make the most money ?

You have 720 m of fencing with which to build 3 enclosures. Two are identical squares and one is a rectangle that is twice as long as it is wide. What should the dimensions of the squares be, in order to to minimize the combined area of all three enclosures ? What should the dimensions of the squares be, in order to maximize the combined area of all three enclosures ?

An American Airlines plane is flying North at a speed of 200 mph. At time $t = 0$ it is located 100 mi South of a control tower. A United Airlines plane is flying in a straight line towards the control tower with a speed of 130 mi/hour. At time $t = 0$ it is located 50 mi East and 100 mi South of the control tower. When are the planes closest ? How close do they get ?



$d = \sqrt{50^2 + 120^2} = 130$

$t_2 = \frac{130}{130} = 1$

AA $\quad x(t) = 0$

$y(t) = -100 + 200t$

UA : $x(t) = 50 + \frac{0-50}{1-0}t$

$y(t) = -120 + \frac{0-(-120)}{1-0}t$

AA $(0, -100 + 200t)$ $\quad$ UA $= (50 - 50t, -120 + 120t)$

$d(t) = \sqrt{(50-50t)^2 + (-100+200t-(-120+120t))^2}$ $\quad$ Minimize $d(t)$

SIMPLIFY

$$d(t) = \sqrt{8900t^2 - 1800t + 2900}$$