Lesson 9

Finish Chapter 6

$|ax + b|$

Midterm problems

Fall 2023 Math 120 A midterm 1

NAME (First,Last) : .....

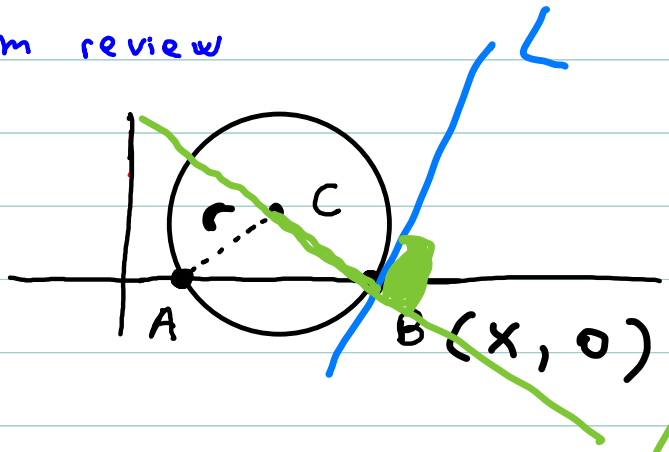
UW email: .....

Student ID .....

Section .....

- Please write your name as it appears in Canvas.
- IMPORTANT: Write your NAME (first, last) on top of every odd page of this exam.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough. Your work needs to be neat and legible.
- The only calculator allowed is the TI 30X IIS. You are allowed an 8x11 sheet of notes, written both sides. Do not turn in your sheet of notes.
- IMPORTANT : you are allowed to use scratch paper, do not turn in any scratch paper.
- Unless stated otherwise, you **MUST** show work for credit.
- If you run out of space, continue your work on the back of the last page and indicate clearly on the problem page that you have done so.
- Unless the problem gives you different instructions, you can give exact answers or round off your answers to 2 decimal places.
- Box your final answer, when appropriate.
- Your exam should have 3 pages, printed double sided, with only the last half page left blank. Please check you have a complete exam.
- Raise your hand if you have a question.

Midterm review



A has coordinates (1, 0)

C has coordinates (3, 2)

Find the equation of the tangent to the circle at B

1) Find x

a) $r = d(A, C) = \sqrt{(3-1)^2 + (2-0)^2}$

$= \sqrt{8}$

b) $(x-3)^2 + (y-2)^2 = 8$

c) $(x-3)^2 + (0-2)^2 = 8$

solve for x

$(x-3)^2 = 8-4 = 4$

$(x-3) = \pm\sqrt{4}$

$(x-3) = \pm 2$

$x = 3 \pm 2 = 5, 1$

B (5, 0)

Find equation of L

1) B is on L

2) slope of $L = m = -\frac{1}{\text{slope } CB}$

C (3, 2) B (5, 0)

slope of $CB = \frac{2-0}{3-5} = -1$

$m = -\frac{1}{-1} = 1$

$$y = y_0 + m(x - x_0)$$

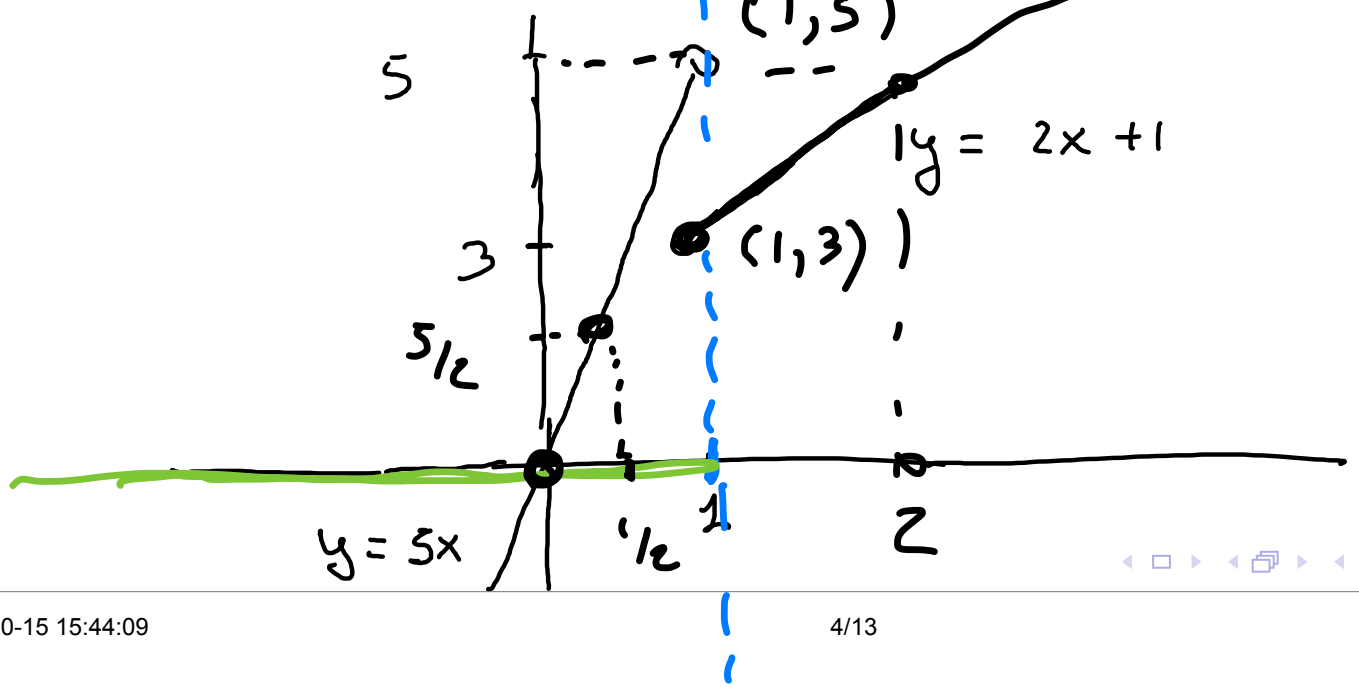
$$y = 0 + 1 \cdot (x - 5)$$

$$\boxed{y = x - 5}$$

Example of multipart function (ch 6 not in midterm)

$k(1) = 3$

$$k(x) = \begin{cases} 2x+1 & \text{if } x \geq 1 \\ 5x & \text{if } x < 1 \end{cases}$$

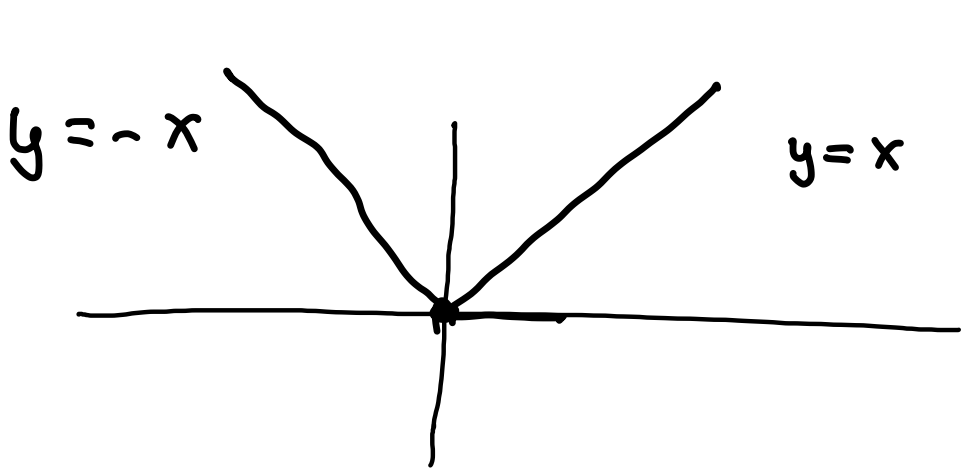


Example

$|2| = 2$

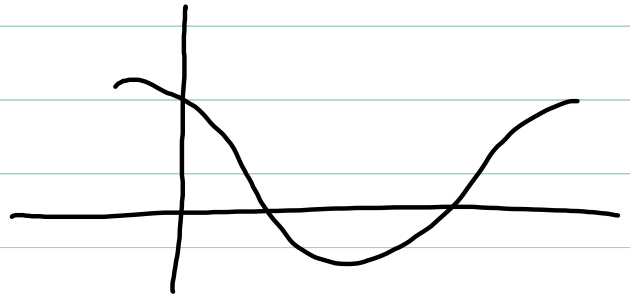
$|-2| = -(-2)$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

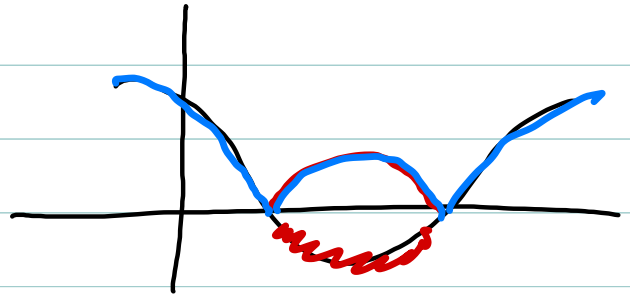


How to graph $y = |f(x)|$

Ex:



$y = f(x)$



$y = |f(x)|$

Flip parts of graph of $y = f(x)$ that are below x axis above

x axis

How to solve an equation involving $|f(x)|$

.... $|f(x)|$ ... = ...

splits into two parts

1) ... $f(x)$ ... = ...

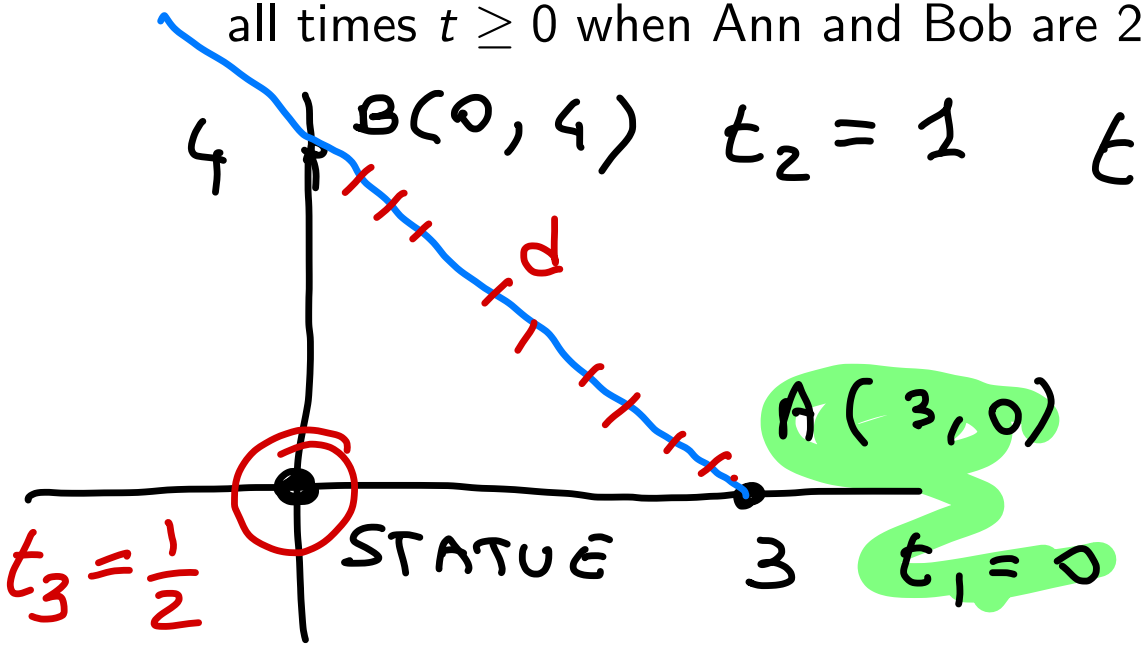
keep only solutions x such that $f(x) \geq 0$

2) ... $-f(x)$ ... = ...

keep only solutions x s.t $f(x) \leq 0$

Ann is located 3 mi east of a statue. At time t=0 she starts walking in a straight line, at a speed of 5mph, to a point located 4 mi North of the statue.Assume Ann keeps walking forever.

1. Find the parametric equations of motion for Ann.
2. Assume Bob stands still by the statue for the first 30 min, then he moves North at 6mph (forever) with a speed of 6 mph. Find all times $t \geq 0$ when Ann and Bob are 2.8 miles apart.



$t_2 = 1$ $\quad t_2 - t_1 = \frac{d}{r}$

$t_3 = \frac{1}{2}$ $\quad t_1 = 0$

$d = \sqrt{(3-0)^2 + (4-0)^2}$

$= 5$

$x_A(t) = 3 + v_x(t-0)$ $\quad t_2 - 0 = \frac{5}{5} = 1$

$y_A(t) = 0 + v_y(t-0)$

$$v_x = \frac{0-3}{1-0} = -3 \qquad \frac{3-0}{0-1} = -3$$

$$v_y = \frac{4-0}{1-0} = 4$$

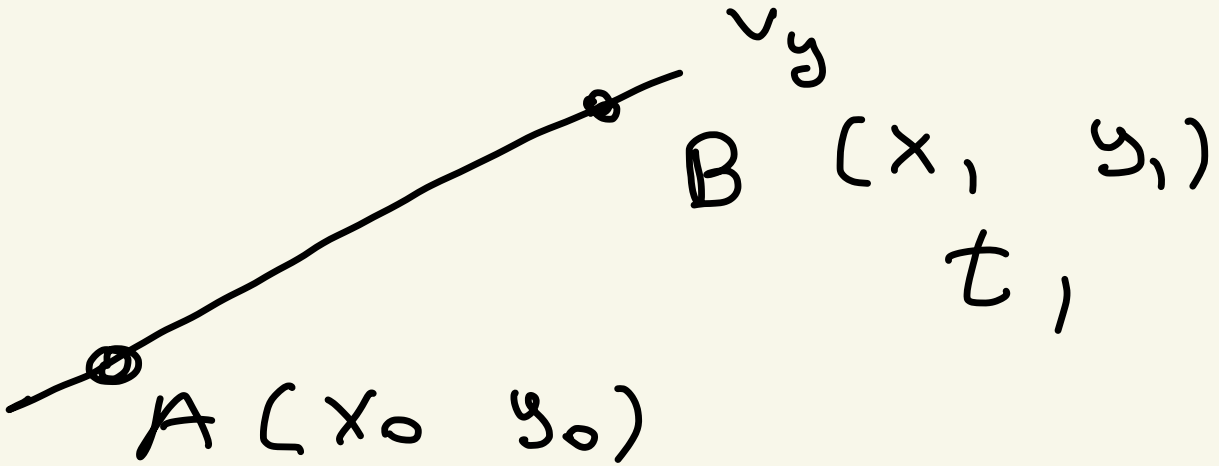
$$x_A(t) = 3 - 3t$$

$$y_A(t) = 4t$$

for $t \geq 0$

$$x(t) = x_0 + \underbrace{\frac{x_1 - x_0}{t_1 - t_0}}_{v_x}(t - t_0)$$

$$y(t) = y_0 + \underbrace{\frac{y_1 - y_0}{t_1 - t_0}}(t - t_0)$$



$$x_B(t) = 0$$

$$y_B(t) = 0 + 6\left(t - \frac{1}{2}\right)$$

for $t \geq \frac{1}{2}$

Ann at $(3-3t,\ 4t)$ Bob at $(0,\ 6(t-\frac{1}{2}))$

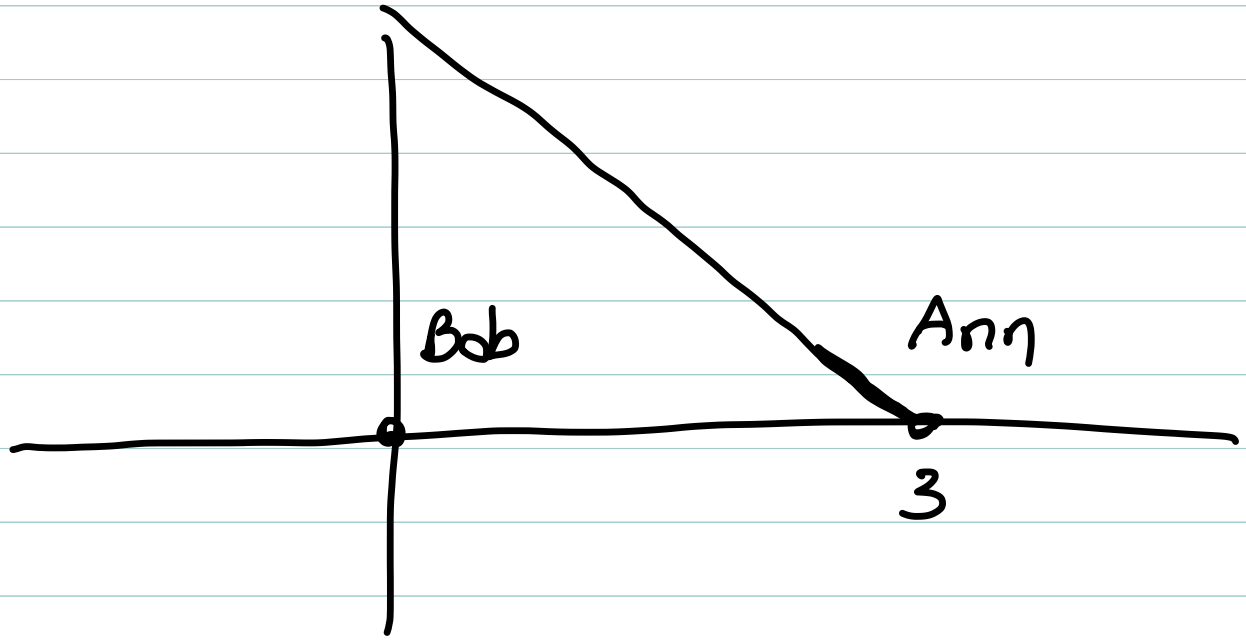
$d(A,B) = \sqrt{(3-3t)^2+(4t-6(t-\frac{1}{2}))^2} = 2.8$

For $t \geq 0.5$

$(3-3t)^2 + (-2t+3)^2 = 2.8^2$

$9 - 18t + 9t^2 + 4t^2 - 12t + 9 - 7.84 = 0$

$13t^2 - 30t + 10.16 = 0$ $\quad t = \dfrac{30 \pm \sqrt{30^2 - 4 \cdot 13 \cdot 10.16}}{2 \cdot 13} = 1.9, 0.41$



Is there a time $0 \le t \le 0.5$ that works? Ann $(3-3t,\ 4t)$ Bob $(0,0)$

$\sqrt{(3-3t)^2+(4t)^2} = 2.8$ For $0 \le t \le 0.5$

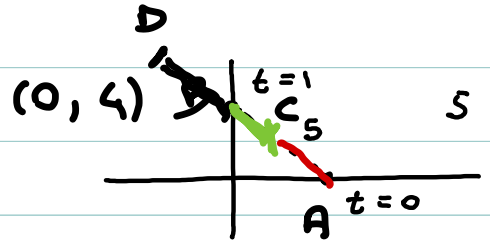
$(3-3t)^2 + (4t)^2 = 2.8^2$

$9 - 18t + 9t^2 + 16t^2 - 7.84 = 0$

$25t^2 - 18t + 1.16 = 0$

$t = \dfrac{18 \pm \sqrt{18^2 - 4 \cdot 25 \cdot 1.16}}{2 \cdot 25} = 0.07, 0.5$

Is there a time when Ann is 2.8 m from ~~the statue~~ B?



In order to be 2.8 miles from B

Ann has to travel 5 − 2.8 = 2.2 mi

or 5 + 2.8 = 7.8 mi

so she reaches C at $\frac{2.2}{5}$

D at $\frac{7.8}{5}$

Or, but it is longer

A $(3-3t, 4t)$ B $(0, 4)$

$d(A, B) = 2.8$

$$\sqrt{(3-3t-0)^2 + (4t-4)^2} = 2.8$$

$$(3-3t)^2 + (4t-4)^2 = 2.8^2$$

$$9 - 18t + 9t^2 + 16t^2 - 32t + 16 - 2.8^2 = 0$$

$$25t^2 - 50t + 17.16 = 0$$

$$t = \frac{50 \pm \sqrt{2500 - 1716}}{50}$$

$$\frac{50 \pm 28}{50} = \begin{cases} \frac{7.8}{5} \\ \frac{2.2}{5} \end{cases}$$