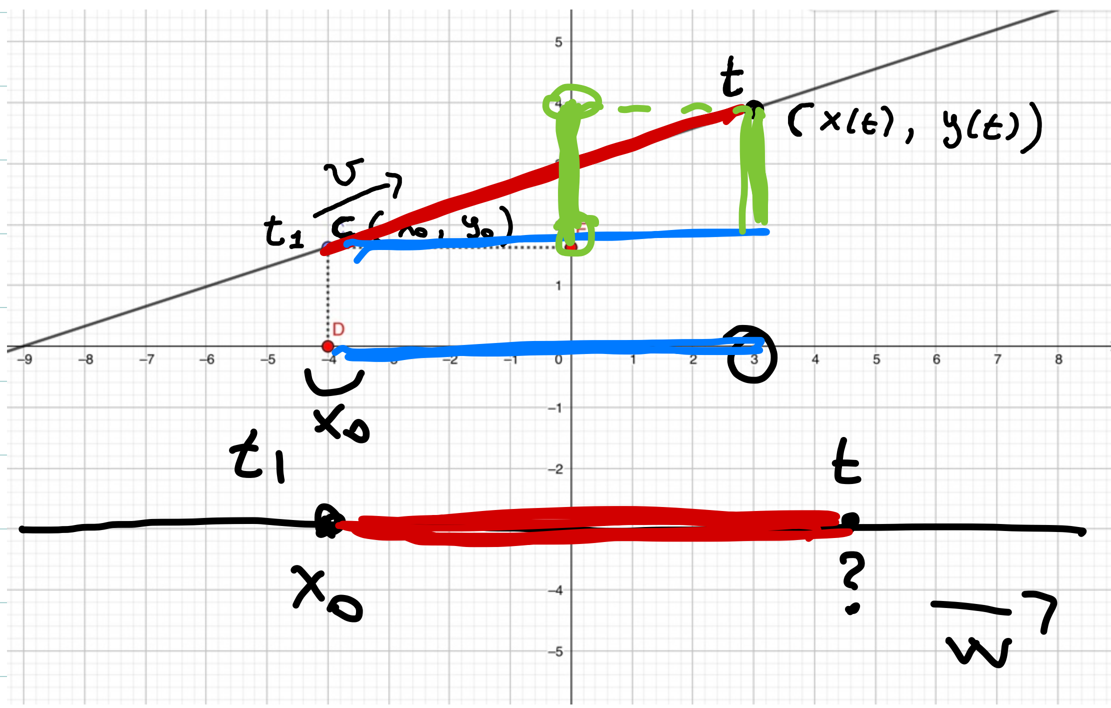
Lesson 7

Finish Chapter 4, start Chapter 5

Parametric equations of motion

Functions,domain range

Uniform linear motion



$$x(t) = x_0 + ??$$

$$y(t) = y_0 + ??$$

$$d = w(t - t_1)$$

$$d = v(t - t_1)$$

$$x(t) = x_0 + w(t - t_1)$$

$$x(t) = x_0 + v_x (t - t_1)$$

$$y(t) = y_0 + v_y (t - t_1)$$

Parametric equations. Uniform rectilinear motion.

Suppose an object is at (x_1, y_1) at time t_1 and it moves along a straight line at constant speed v.
The parametric equations of motion of the object are :

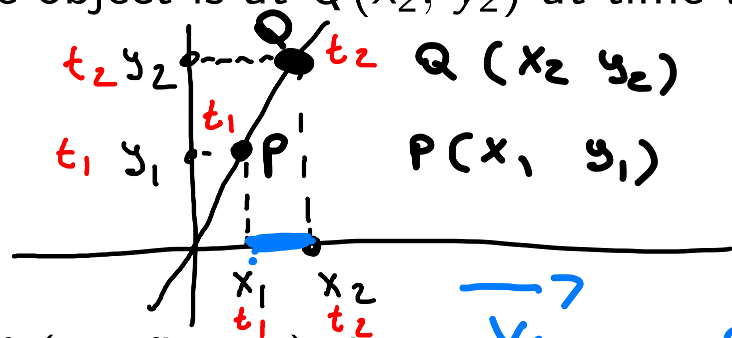
$$x(t)\cancel{x} = x_1 + v_x(t - t_1), \quad y(t)\cancel{y} = y_1 + v_y(t - t_1)$$

for t and t_1 greater than the time when the object started moving, where v_x is the horizontal component of the velocity and v_y is the vertical component of the velocity.

You can calculate v_x and v_y in different ways, depending on what the problem gives you :

- If you also know the object is at $Q(x_2, y_2)$ at time t_2 then

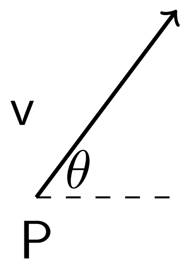
$$v_x = \frac{x_2 - x_1}{t_2 - t_1} \left(\frac{\Delta x}{\Delta t}\right)$$
$$v_y = \frac{y_2 - y_1}{t_2 - t_1} \left(\frac{\Delta y}{\Delta t}\right) \frac{d}{t}$$



d/t

- If you know v and θ (see figure) then

skip for now

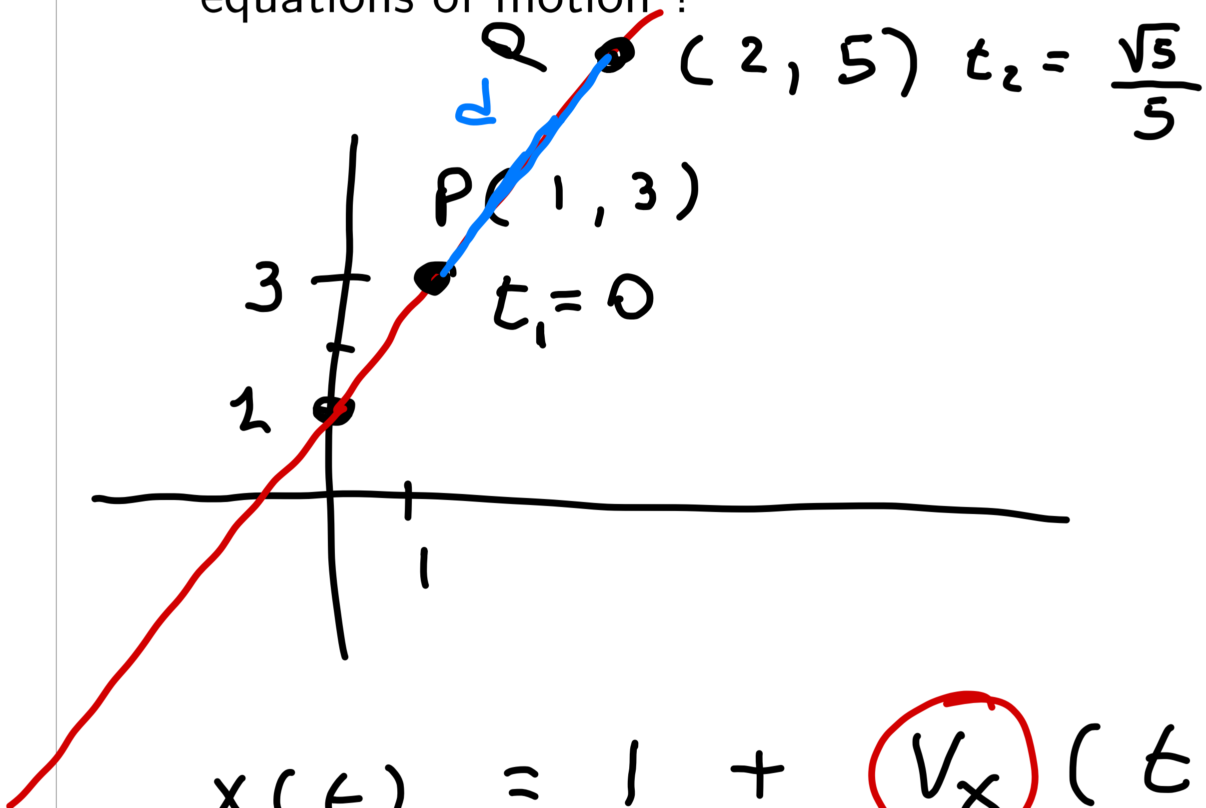


$$v_x = v\cos(\theta)$$
$$v_y = v\sin(\theta).$$

Note: in many problems time t_1 is just the initial time so $t_1 = 0$ in which case you have

$$x = x_1 + v_x t, \qquad y = y_1 + v_y t$$

Alice is running at a speed of 5mph starting at $P(1,3)$ along the line $y = 2x + 1$ in the NE direction. What are Alice' s parametric equations of motion ?



① Pick a point Q on line

Ex: $x = 2$

$y = 2 \cdot 2 + 1 = 5$

② Find out time time t_2 when Alice reaches Q: $t = \frac{d}{v}$

$d = d(P,Q) = \sqrt{(2-1)^2 + (5-3)^2} = \sqrt{5}$

$t = \frac{\sqrt{5}}{5}$

$$x(t) = 1 + V_x (t - 0)$$

$$y(t) = 3 + V_y (t - 0)$$

$$v_x = \frac{2-1}{\frac{\sqrt{5}}{5} - 0} = \frac{1}{\frac{\sqrt{5}}{5}} = \frac{5}{\sqrt{5}} = \frac{\sqrt{5}\cdot\cancel{\sqrt{5}}}{\cancel{\sqrt{5}}} = \sqrt{5}$$

$$v_y = \frac{5-3}{\frac{\sqrt{5}}{5} - 0} = \frac{2}{\frac{\sqrt{5}}{5}} = 2\cdot\frac{5}{\sqrt{5}} = 2\cdot\sqrt{5}$$

$$x(t) = 1 + \sqrt{5}\,t$$
$$y(t) = 3 + 2\sqrt{5}\,t$$

for any t when Alice is moving

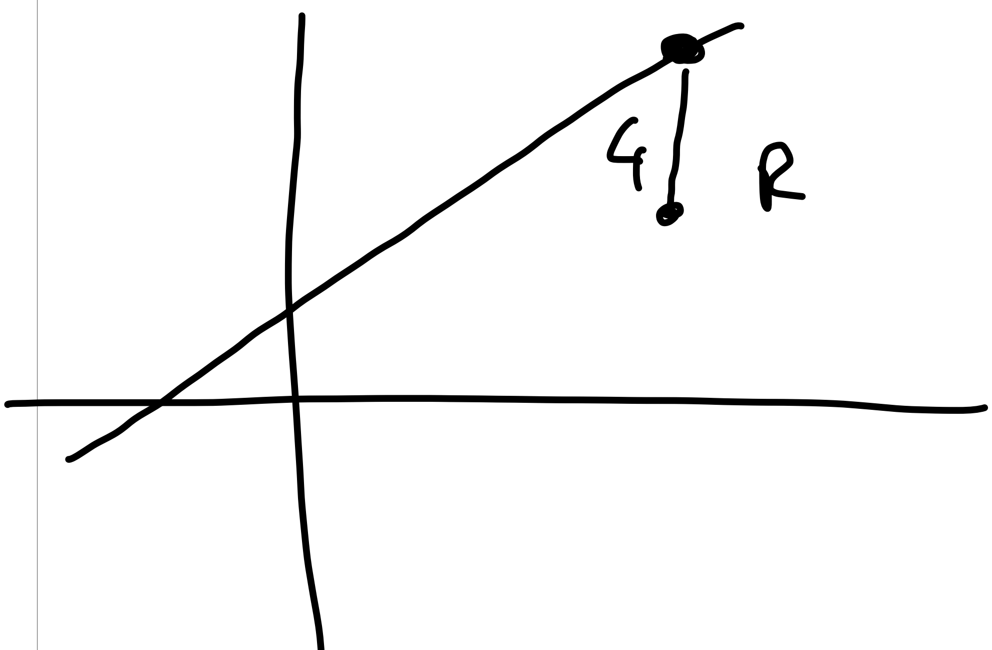
$t \geq 0$

For this problem not a general rule

When is Alice 's 4 mi away from the point $R(2,2)$?

At time t Alice is at $(1+\sqrt{5}t,\ 3+2\sqrt{5}t)$

s t ?



When the distance between Alice 's position $(1+\sqrt{5}t,\ 3+2\sqrt{5}t)$

and $R(2, 2)$ is 4.

$$\sqrt{(1+\sqrt{5}t-2)^2 + (3+2\sqrt{5}t-2)^2} = 4$$

solve for t

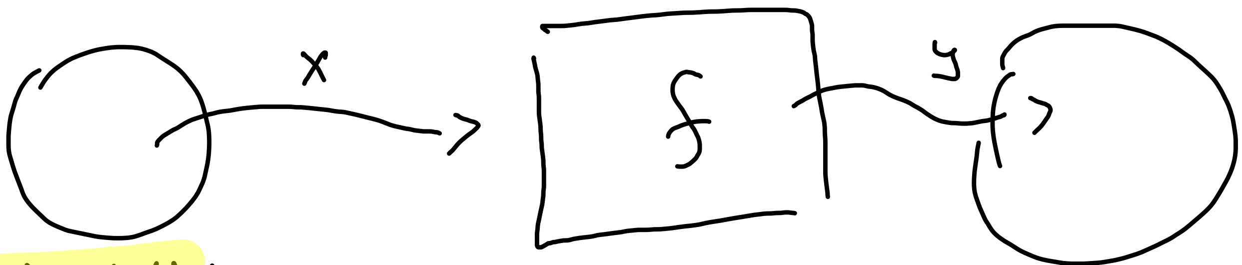
1) Square both sides
2) Simplify, expand squares, bring everything on the left
3) Use quadratic formula on

$$at^2 + bt + c = 0$$

$$(\sqrt{5}t - 1)^2 = 5t^2 - 2\cdot\sqrt{5}t + 1$$

solutions $t = -\cancel{0.8}4$ $\boxed{0.66}$

What is a function?



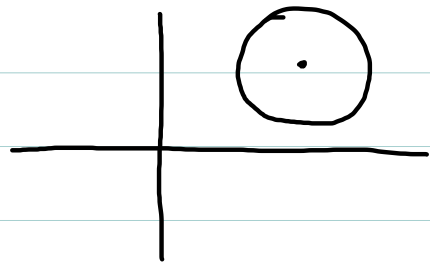
DOMAIN:

||

all possible inputs

Range:

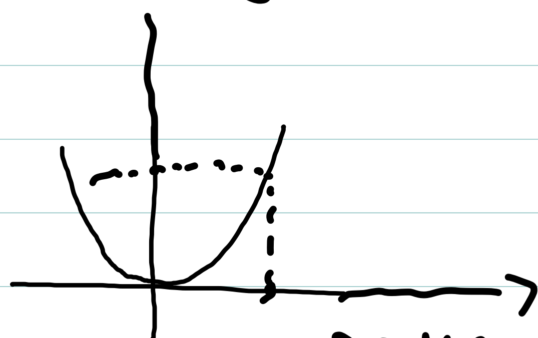
all possible outputs



$$(x-x_0)^2 + (y-y_0)^2 = r^2$$

Not a function

Ranges



DOMAIN

$$y = x^2$$

Function

Interval notation

$(2, 3)$ means all x with $2 < x < 3$
$[2, 3]$ means all x with $2 \leq x \leq 3$
$[2, 3)$ means all x with $2 \leq x < 3$
$(-\infty, +\infty)$ means all x in R

$(2, +\infty)$ $\quad 2 < x$