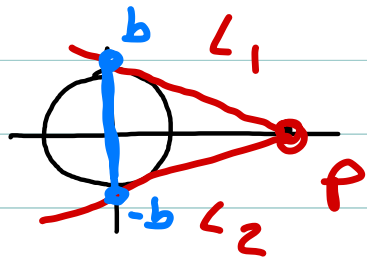# Lesson 6

Read Chapter 4   (no uniform   motion yet)

Linear modeling

Lines and Circles word problems

Dave's silo problem

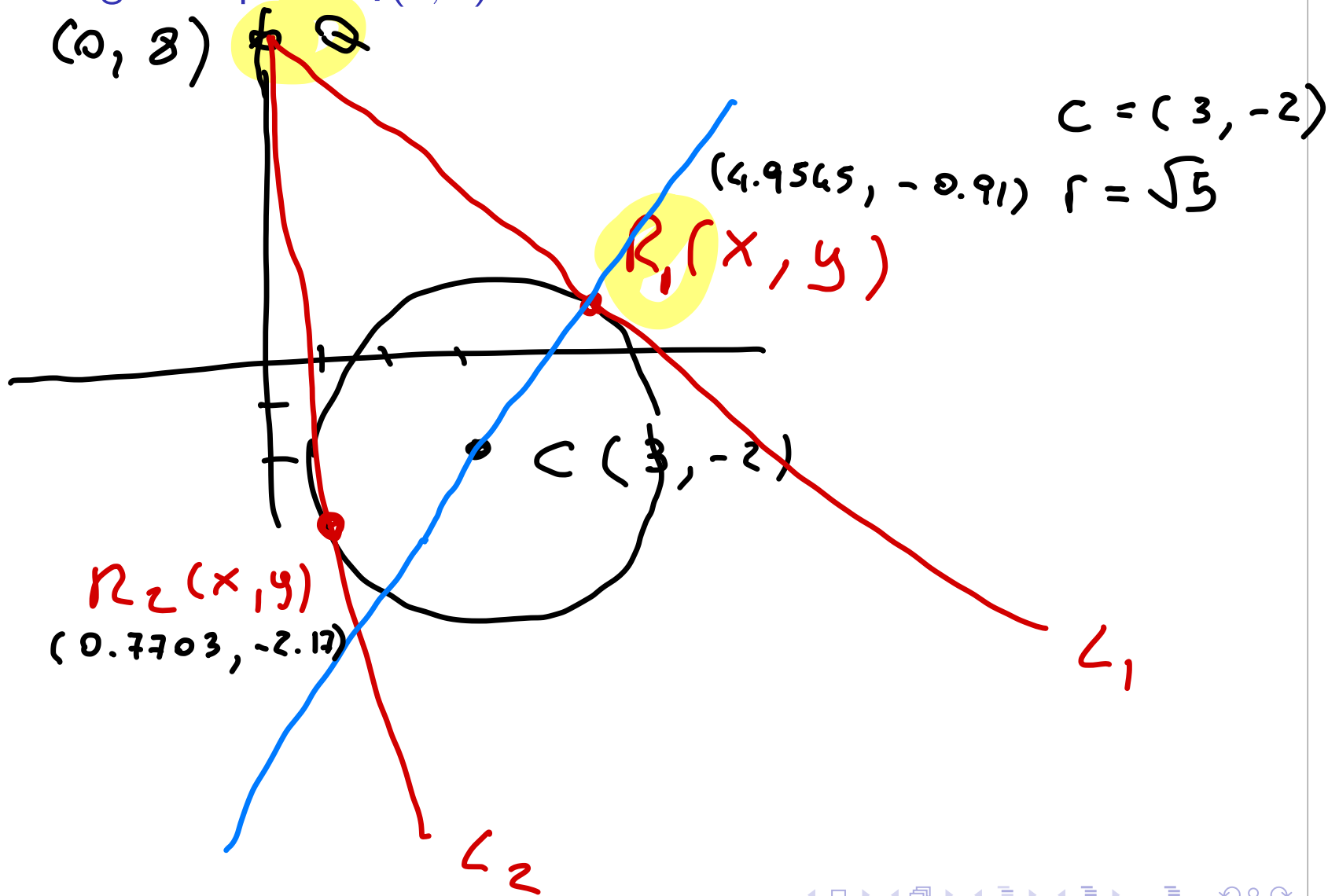Find equations

of Lines tangent to the circle

through P

Find y intercept

Answer should $(-b, b)$ cannot see

Find the tangent to the circle $(x-3)^2 + (y+2)^2 = 5$
through the point $Q(0,8)$

$(0,8)$ Q

$C = (3,-2)$

$(4.9545, -0.91)$ $r = \sqrt{5}$

$R_1(x,y)$

$C(3,-2)$

$R_2(x,y)$

$(0.7703, -2.17)$

L_1

L_2

Want to find R_1 (x, y). I need two equations

$$\begin{cases} \text{Equation of circle} \\ \\ \text{slope of } L_1 \text{ way 1} \quad = \quad \text{Slope of } L_1 \text{ way 2} \\ \quad L_1 \perp \text{ line } C R_1 \end{cases}$$

slope of L_1 way 1 $=$ Slope of L_1 way 2

$L_1 \perp$ line $C R_1$

use $Q(0, 8)$ $R(x, y)$

$$\frac{8-y}{0-x} = m$$

$$m = - \frac{1}{\text{slope of line } C R_1}$$

$$m = - \frac{1}{\dfrac{-2-y}{3-x}} = \frac{1}{\left(\dfrac{2+y}{3-x}\right)} = \frac{3-x}{2+y}$$

$C(3, -2)$

$R_1 (x, y)$

$$\begin{cases} (x-3)^2 + (y+2)^2 = 5 \quad \text{②} \\ \dfrac{3-x}{2+y} = \dfrac{8-y}{-x} \quad \text{②} \end{cases}$$

$= m =$

Expand squares

cross multiply

$$\begin{cases} x^2 - 6x + 9 + y^2 + 4y + 4 = 5 \\ -3x + x^2 = 16 + 8y - 2y - y^2 \end{cases}$$

Simplify

Simplify

$$\begin{cases} x^2 + y^2 = 6x - 4y - 8 \\ x^2 + y^2 = 16 + 6y + 3x \end{cases}$$

Keep

$$\begin{cases} x^2 + y^2 = 6x - 4y - 8 \\ 6x - 4y - 8 = 16 + 6y + 3x \end{cases}$$

⌐ solve for y (or x)

$$\begin{cases} x^2 + y^2 = 6x - 4y - 8 \\ 3x - 24 = 10y \end{cases}$$

$$\begin{cases} x^2 + \left(\dfrac{3x - 24}{10}\right)^2 = 6x - 4\left(\dfrac{3x - 24}{10}\right) - 8 \\ y = \dfrac{3x - 24}{10} \end{cases}$$

$$\begin{cases} x^2 + \dfrac{9x^2 - 2 \cdot 3 \cdot 24x + 24^2}{100} = 6x - \dfrac{12x - 96}{10} - 8 \\ y = \dfrac{3x - 24}{10} \end{cases}$$

$$\underbrace{109}_{a} x^2 \underbrace{- 624}_{b} x \underbrace{+ 416}_{c} = 0 \quad \Big\langle \begin{array}{l} x \approx 4.95 \\ x = 0.77 \end{array}$$

quadratic formula

$$y = \frac{3x - 24}{10}$$

For $x = 4.9545$ $y \approx -0.91$ R_1

For $x = 0.7703$ $y \approx -2.17$ R_2

tangent 1   line through $Q(0,8)$ $R_1 (4.95, -0.91)$

$$y = 8 + \frac{8 - (-0.91)}{0 - 4.95} x ; \quad \boxed{y = 8 - 1.8 x}$$

tangent 2 : line through $Q(0,8)$ $R_2 (0.77, -2.17)$

$$y = 8 + \frac{8 + 2.17}{-0.77} x ; \quad \boxed{y = 8 - 13.21 x}$$

Find the equation of the line tangent to to the circle $(x - 3)^2 + (y + 2)^2 = 5$ and parallel to the line $4x - 2y + 10 = 0$

Video with solution in Canves

quantity $\qquad y(x) = \underline{mx + b}$

$= \text{slope}$

Clue words: <u>LINEAR</u> , <u>CONSTANT RATE</u>
Goal: find the equation of a line and use it to answer questions in the problem

Ex: ch 4 #2 average sale price of home in Seattle and port Towsend
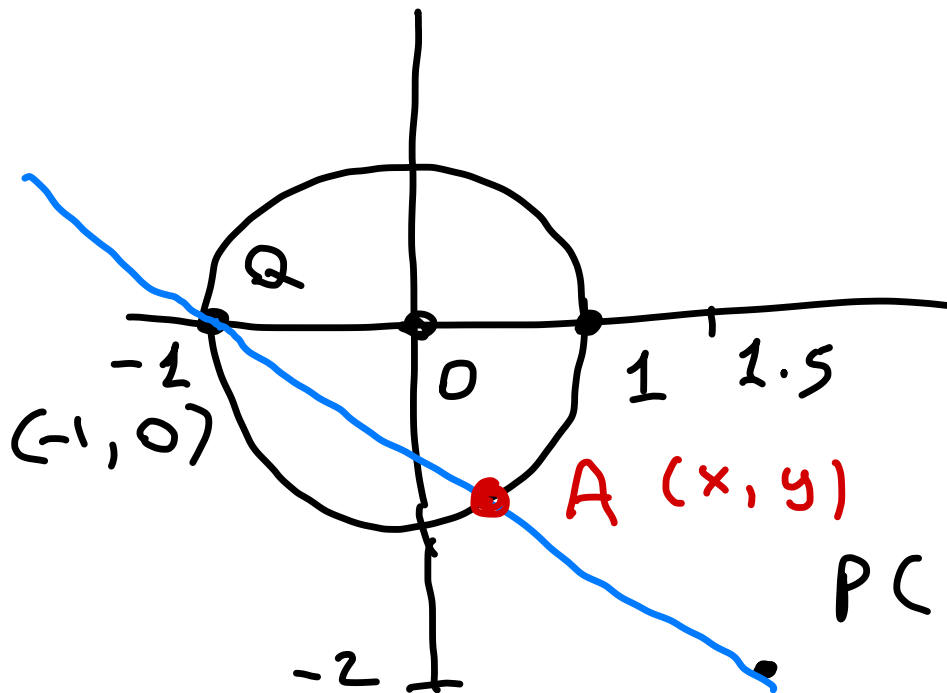
$\textcircled{y} = y_0^s + m_s(x - x_0^s)$ $\qquad\qquad$ For seattle
$\qquad\qquad\qquad\qquad\qquad \searrow$ year i.e 1970

$\vee$ sale price

$\textcircled{y} = y_0^p + m_p(x - x_0^p)$

Similar to ch 4 #3

A crop dusting airplane flying a constant speed of 120mph is first spotted 2 miles South and 1.5 miles East of the center of circular irrigated field. The irrigated field has radius 1 mile. The plane flies in a straight line to a point 1 mile West of the center of the irrigated field. Impose a coordinate system with the origin at the center of the field.
Find the location A where the crop duster enters the airspace above the field

To find x, y
Solve
{ Circle
  blue line P_1
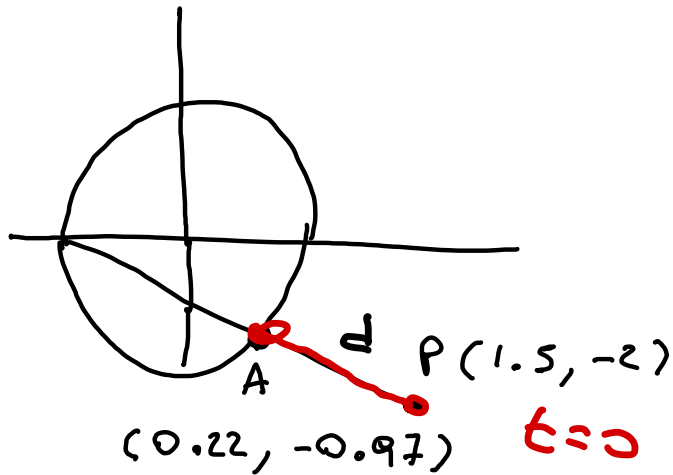
Q

−1

(−1,0)

O   1   1.5

A (x, y)

P (1.5, −2)
t = 0

$$\begin{cases} x^2 + y^2 = 1 \\ y = 0 + \dfrac{-2 - 0}{1.5 - (-1)} \; (x - (-1)) \end{cases}$$

do the algebra...
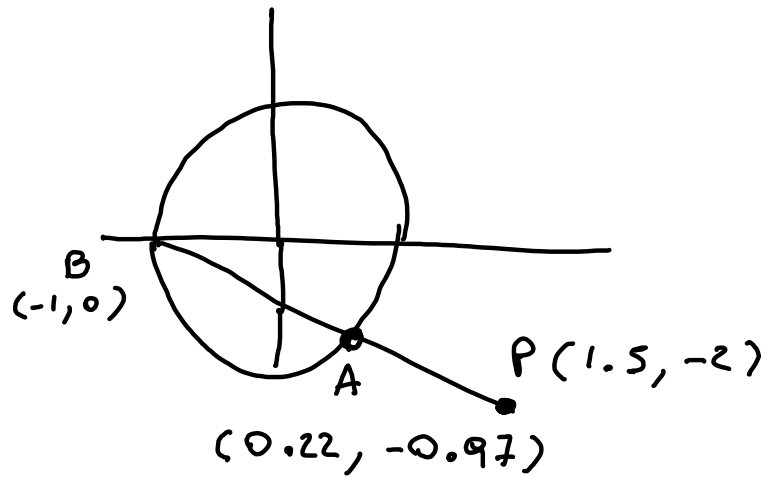
When does the plane first enter the airspace above the field ?
(Assume time t=0 corresponds to when the plane is first spotted)

$P(1.5, -2)$

$(0.22, -0.97)$ $t=0$

$$E = \frac{d(P, A)}{120}$$

How much time does the plane spend flying over the irrigated field?

B
(-1, 0)

A

P (1.5, -2)

(0.22, -0.97)

How close does the plane get to the center of the field?

B
(-1,0)

A
(0.22, -0.97)

P(1.5, -2)