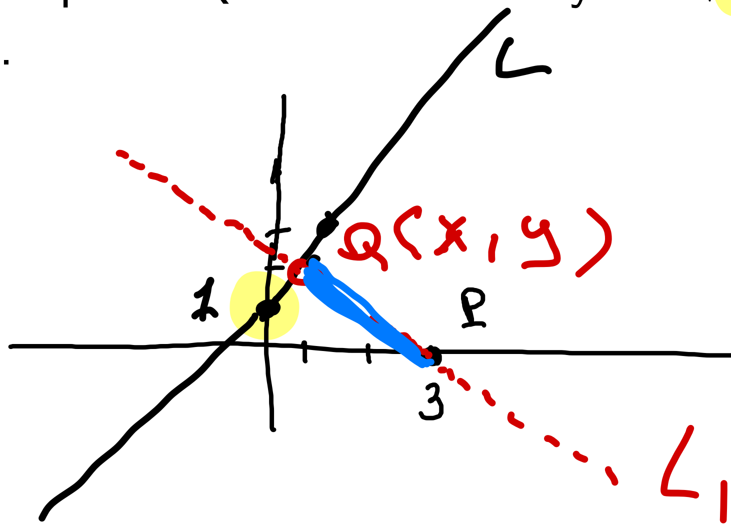
Lesson 5

Read Chapter 3

circles

Find the point Q on the line L: y=2x+1 that is closest to the point P(3,0).



$y = mx + b$

y intercept

When $x = 0$ $y = 1$

When $x = 1$ $y = 3$

1) Find equation of L_1, line $\perp L$ through P

2) Q is the intersection of L and L_1

$$y = y_0 + m(x - x_0)$$

$$y = 0 - \frac{1}{2}(x - 3)$$

$L_1 \perp L$

L_1 has slope $m = -\frac{1}{2}$

L has slope 2

To find (x, y) coordinates of Q

solve

$$\begin{cases} y = 2x + 1 \\ y = -\frac{1}{2}(x-3) \end{cases}$$

$$2x + 1 = -\frac{1}{2}(x - 3)$$

solve for x

$$x = 1/5$$

$y = 2 \cdot \frac{1}{5} + 1 = \frac{7}{5}$

$Q\left(\frac{1}{5}, \frac{7}{5}\right)$

What is the distance of P from L ?

Calculate $d(P, Q)$

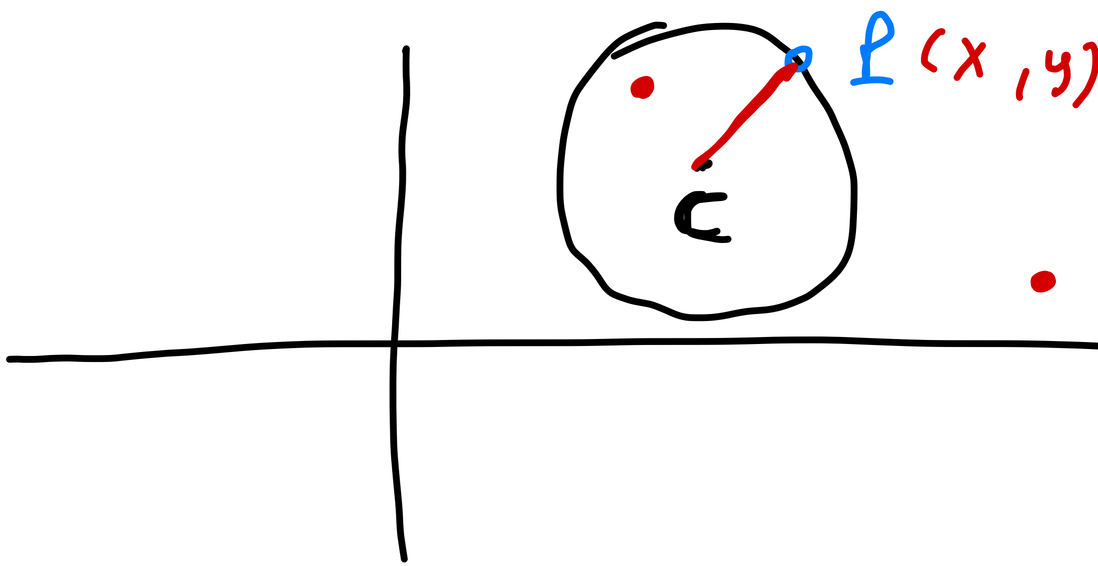
Ch 4 3 d) golf course

Equation of a circle

$$d(P, C) = \sqrt{(x - x_0)^2 + (y - y_0)^2} = r$$

The circle has center (x_0, y_0) and radius r.
A point $P(x_1, y_1)$

- is on the circle if : $(x_1 - x_0)^2 + (y_1 - y_0)^2 = r^2$ standard form
- is inside the circle if:$(x_1 - x_0)^2 + (y_1 - y_0)^2 < r^2$
- is out side the circle if $(x_1 - x_0)^2 + (y_1 - y_0)^2 > r^2$



$C(x_0, y_0)$

$d(P, C) = r$

Find the center and radius of the circle

$$\frac{1}{3}\left(3x^2 + 18x + 3y^2 - 6y + 6\right) = 0 \cdot \frac{1}{3}$$

$$x^2 + 6x + y^2 - 2y + 2 = 0$$

$$(x-x_0)^2 + (y-y_0)^2 = r^2$$

$$x^2 - 2x_0 x + y^2 - 2y_0 y + x_0^2 + y_0^2 - r^2 = 0$$

$$-2x_0 = 6 \qquad -2y_0 = -2$$

$$x_0 = -\frac{1}{2} \cdot 6 = -3 \qquad y_0 = -\frac{1}{2}(-2) = 1$$

$$C(-3,\ 1)$$

$$x_0^2 + y_0^2 - r^2 = 2$$

$$(-3)^2 + (1)^2 - 2 = r^2$$

$$8 = r^2$$

$$\sqrt{8} = r$$

$$1 \cdot x^2 + ax + 1 \cdot y^2 + by + c = 0$$

is the equation of a circle with center at $x_0 = -\frac{a}{2}$, $y_0 = -\frac{b}{2}$ and radius $r = \sqrt{\underbrace{\frac{a^2}{4}}_{x_0^2} + \underbrace{\frac{b^2}{4}}_{y_0^2} - c}$

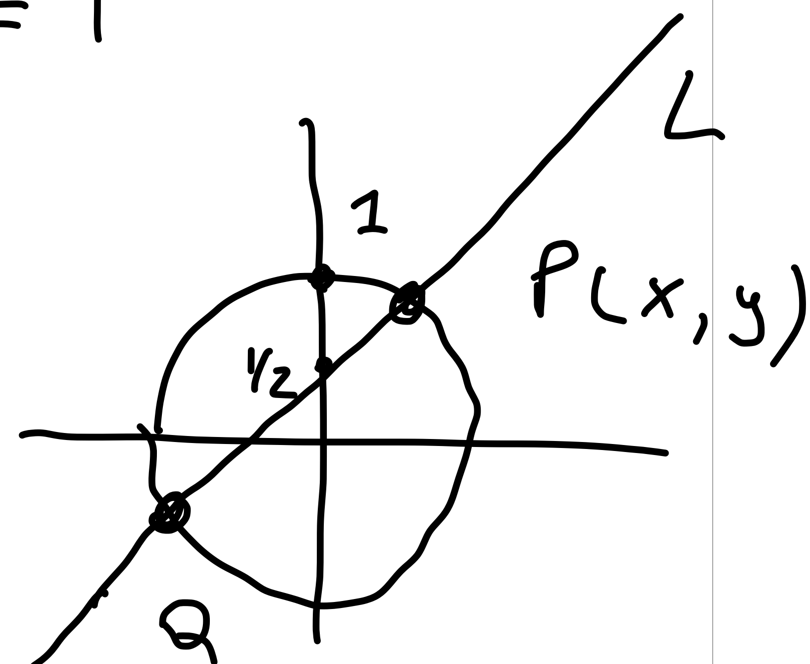
Intersection of a line and a circle

Find the intersection of the unit circle and the line $y = x + \frac{1}{2}$

Center $(0,0)$ $\quad r = 1$

$$(x-0)^2 + (y-0)^2 = 1^2$$

$$\begin{cases} x^2 + y^2 = 1 \\ y = x + \frac{1}{2} \end{cases}$$



$$\begin{cases} x^2 + y^2 = 1 \\ y = x + \frac{1}{2} \end{cases}$$

$$\begin{cases} y = x + \frac{1}{2} \\ x^2 + \left(x + \frac{1}{2} \right)^2 = 1 \end{cases}$$

$$x^2 + x^2 + x + \frac{1}{4} - 1 = 0$$

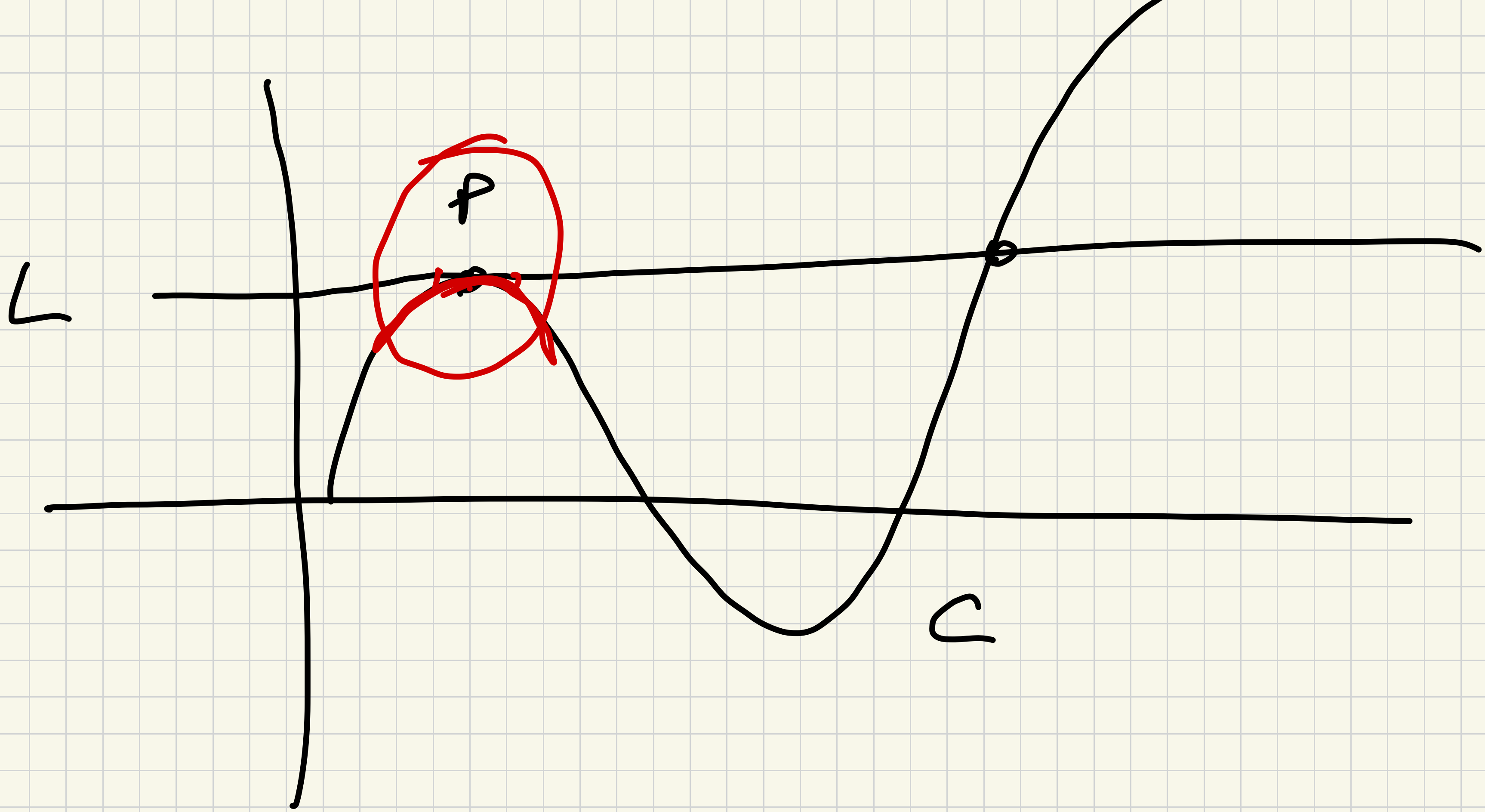
$$2x^2 + x - \frac{3}{4} = 0 \quad ; \quad x = \frac{-1 \pm \sqrt{1 + 6}}{4}$$

If $x = \frac{-1+\sqrt{7}}{4}$ $y = \frac{-1+\sqrt{7}}{4} + \frac{1}{2}$

$P = \left(\frac{-1+\sqrt{7}}{4}, \frac{1}{4} + \frac{\sqrt{7}}{4} \right)$

if $x = \frac{-1-\sqrt{7}}{4}$, $y = \frac{-1-\sqrt{7}}{4} + \frac{1}{2}$

$Q \left(\frac{-1-\sqrt{7}}{4}, \frac{1}{4} - \frac{\sqrt{7}}{4} \right)$



Tangent intersects curve in two points.

Tangent to a circle

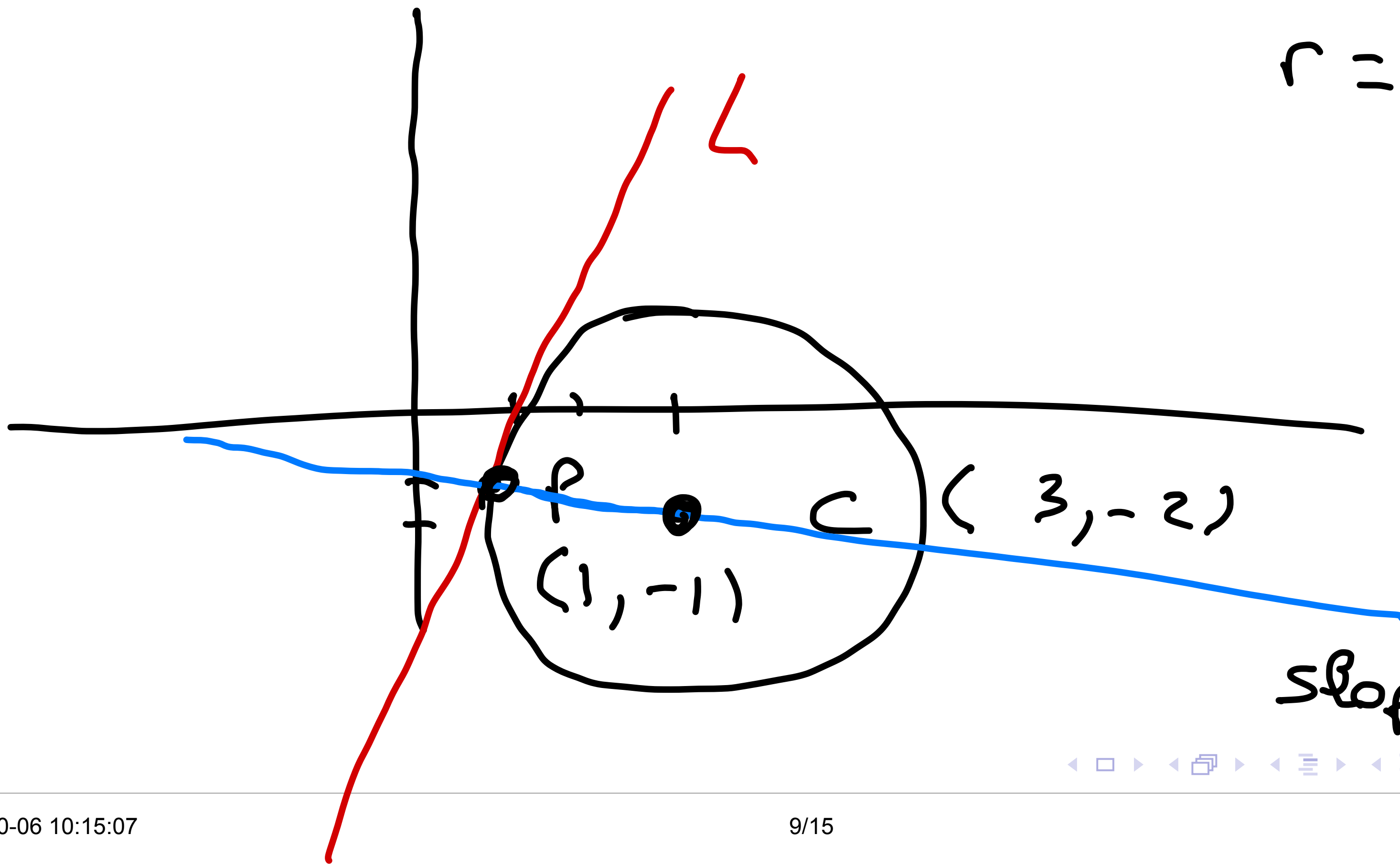
Fact: if a line L is tangent to a circle at P, then the line is perpendicular to the radius CP.

$$(y-(-2))^2 = r^2$$

Find the tangent to to the circle $(x-3)^2 + (y+2)^2 = 5$ at the point $P(1,-1)$

$C(3,-2)$

$r = \sqrt{5}$



slope $-\frac{1}{2}$

$L \perp$ line PC

P is on L

$$y = y_0 + m(x - x_0)$$

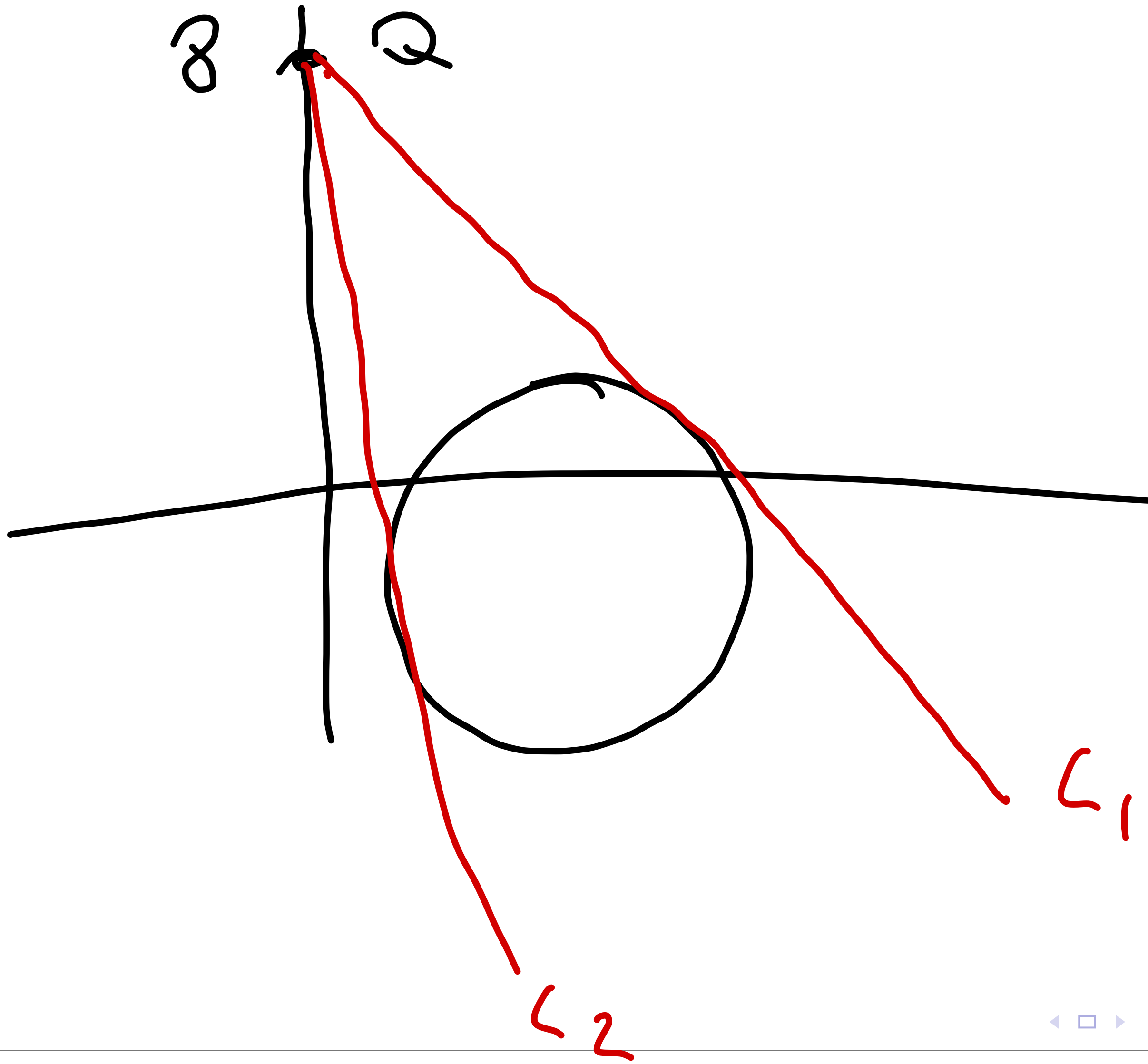
$$\boxed{y = -1 + 2(x - 1)}$$

slope of line PC $\quad \dfrac{\Delta y}{\Delta x} = \dfrac{-1-(-2)}{1-3}$

$$= -\frac{1}{2}$$

$$m = -\frac{1}{-\frac{1}{2}} = 2$$

Find the tangent to to the circle $(x-3)^2+(y+2)^2=5$ through the point $Q(0,8)$



NEXT TIME