Lesson 4

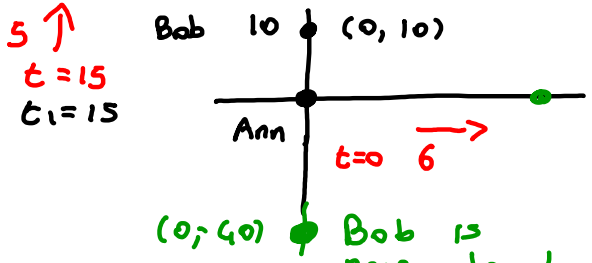
Read Chapter 3

lines and circles

Tricky version

$t = 0$

Bob is standing 10 feet North of Ann, when Ann starts moving East at 6 feet/sec. 15 seconds later Bob , starts moving North at 5 feet/sec. When is the distance between Ann and Bob 50 feet ?



Ann

$x_A(t) = 6t$

$y_A(t) = 0$

valid for $t \geq 0$

$(6t, 0)$

For $t = 5$ $(30, 0)$

Bob

$x_B(t) = 0$

$y_B(t) = 10 + 5(t - 15)$

valid for $t \geq 15$

$(0, 10 + 5(t-15))$

For $t = 5$ $(0, -40)$.

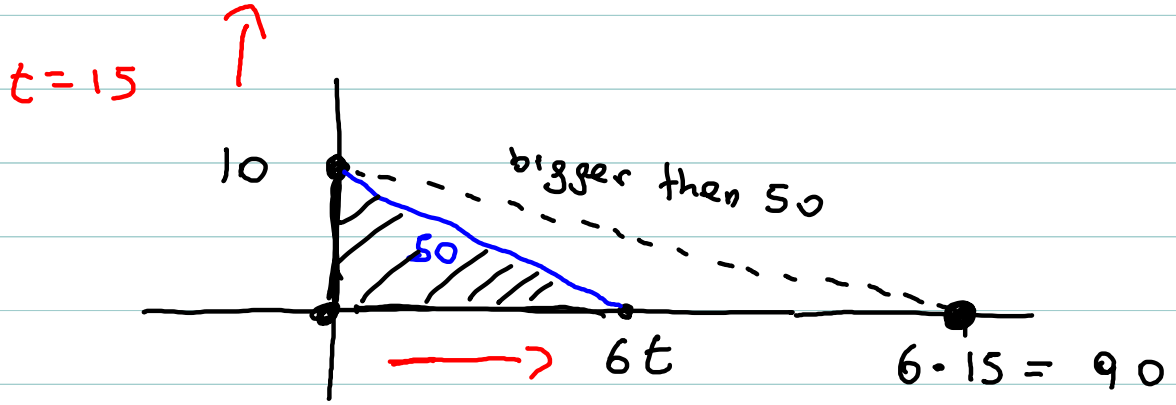
We want $d((6t, 0), (0, 10 + 5(t-15))) = 50$

$$50 = \sqrt{(6t - 0)^2 + (0 - (10 + 5(t-15)))^2}$$

do the algebra .... $t = 5, 5.66$

Can I accept these solutions ? No DISCARD BOTH

There is no time $t \geq 0$ when the distance between Ann and Bob is 50 ft?



Ann

$x_A(t) = 6t$

$y_A(t) = 0$

for $t \geq 0$

$(6t, 0)$

$50 = \sqrt{10^2 + (6t)^2}$

Bob

$x_B(t) = 0$

$y_B(t) = 10$

for $0 \leq t \leq 15$

$(0, 10)$

Ann at $(6t, 0)$ Bob at $(0, 10)$

$$50 = \sqrt{(6t)^2 + 10^2}$$

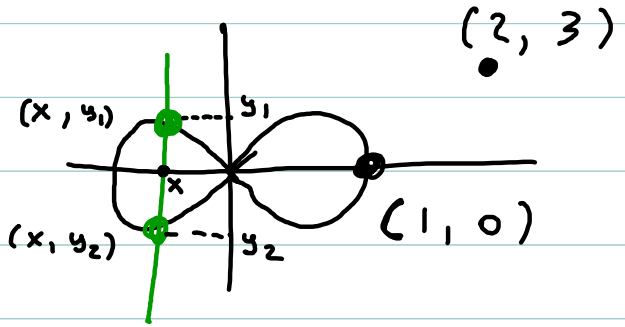
$$2500 = 36t^2 + 100$$

$$2400 = 36t^2$$

$$\pm\sqrt{\frac{2400}{36}} = t$$ discard negative solution

$$\sqrt{\frac{2400}{36}} \approx \boxed{8.16 \text{ sec}}$$

Chapter 3:



Equation:

$(x^2+y^2)^2 = (x^2-y^2)$

$(1^2+0^2)^2 = (1^2-0^2)$ ✓

$(2^2+3^2)^2 = (2^2-3^2)$ ✗

Function

Graph of a function

it passes the vertical line test

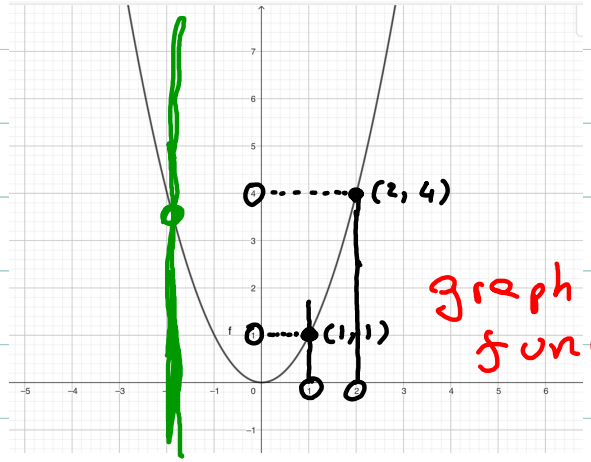
$y =$ expression in x

just one

Example of function

$f(x) = x^2$

$y = x^2$



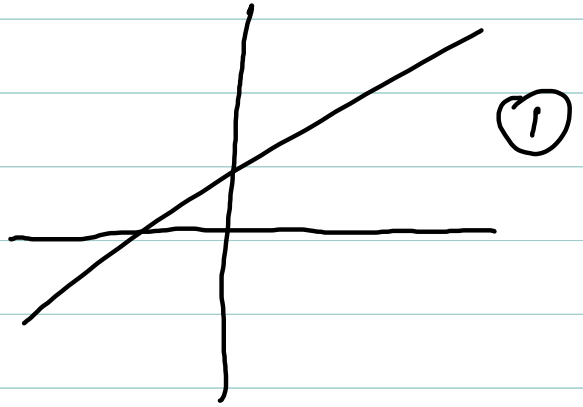
graph of function

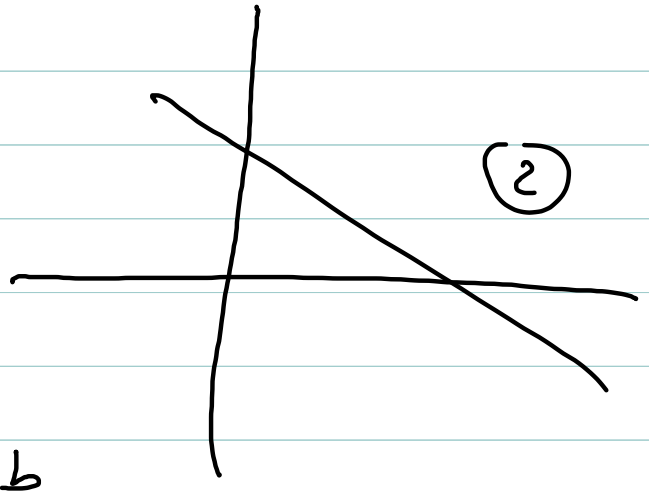
function

$y = 1^2 = 1$

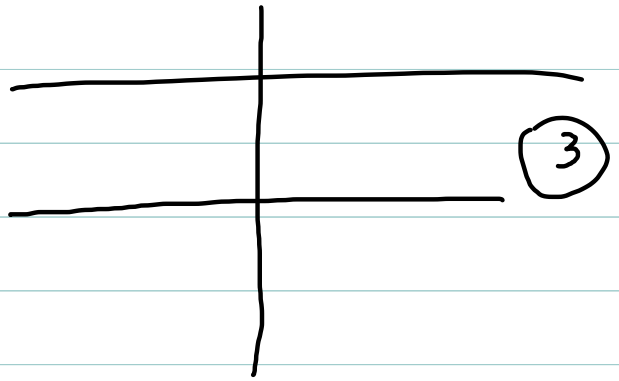
$y = 2^2 = 4$

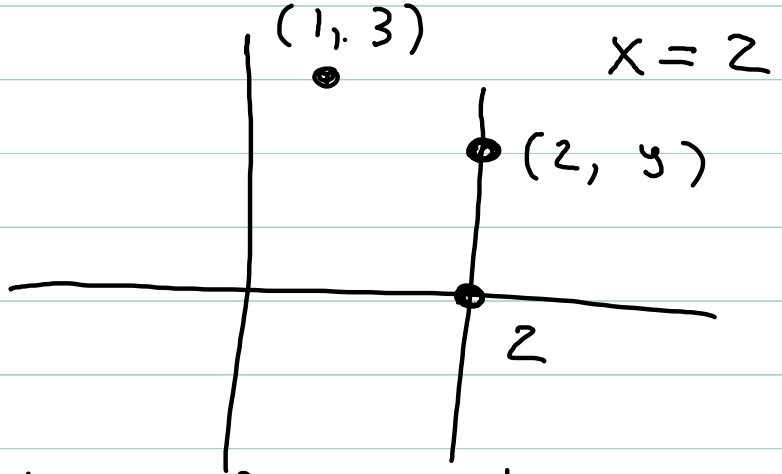
Is every line the graph of a function?





$y = mx + b$





Any line but a vertical line is the graph of a function

Lines equations

$ax+by+c=0$ for any line as long as we don't have both $a=0$ and $b=0$

$y = mx + b$ slope, y intercept

$y = y_1 + m(x - x_1)$ point slope

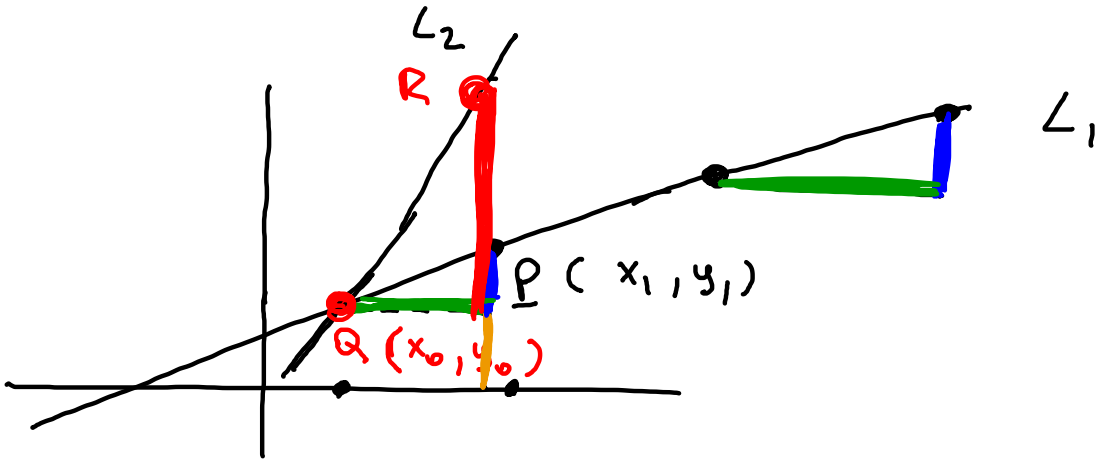
} for a line that is non vertical

slope of a line $= m$

$y = mx + b$

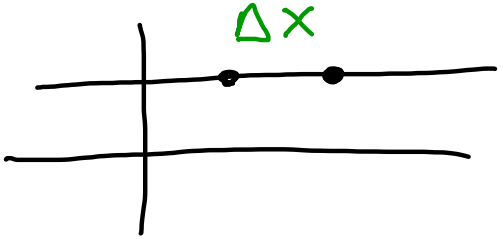
Ex $y = 2x + 1$

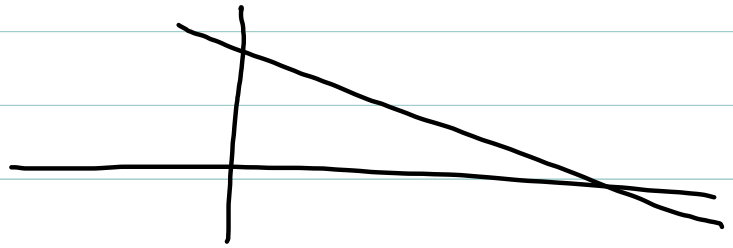
$y = y_1 + m(b - x_1)$



$$m_1 = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{y_0 - y_1}{x_0 - x_1} = \frac{\text{rise}}{\text{run}}$$

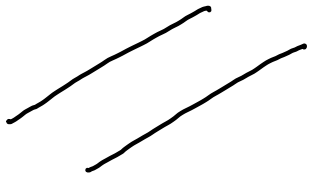
$m_2 = \dfrac{\Delta y}{\Delta x}$ $\qquad m_2 > m_1$



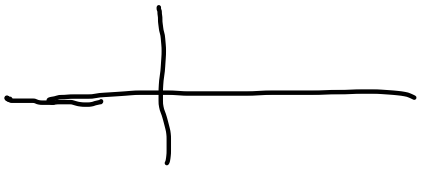


slope is negetive

Useful facts about lines



1. Two lines $L_1 : y = m_1x + b_1$ and $L_2 : y = m_2x + b_2$ are parallel iff $m_1 = m_2$.
2. Two lines $L_1 : y = m_1x + b_1$ and $L_2 : y = m_2x + b_2$ are perpendicular iff $m_1 = -\frac{1}{m_2}$



3. The slope of the line through the points (x_0, y_0) and (x_1, y_1) is $m = \frac{y_1 - y_0}{x_1 - x_0}$
4. The equation of a line through $P(x_0, y_0)$ with slope m is $y = y_0 + m(x - x_0)$
5. The equation of the line through point P$=(x_0, y_0)$ and Q$=(x_1, y_1)$ is $y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$ if $x_1 - x_2 \neq 0$ and $y = x_1$ if $x_1 = x_0$.