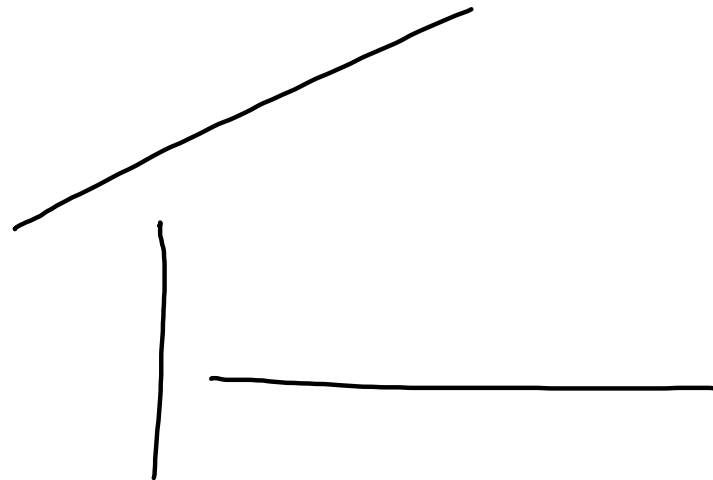


Lesson 3

Still on Chapter 2

Motion problems



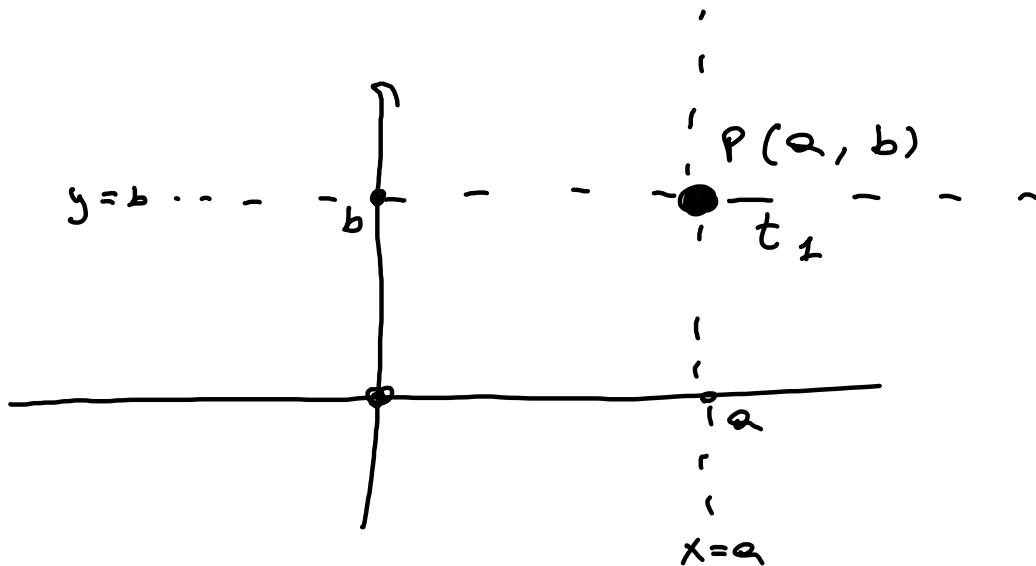
Recall

a, b, t_1, v constant numbers
 t variable

If an object is moving with constant velocity v and at time t_1 it is at (a, b) then the position of the object at time t is:

▶ $x(t) = a + v(t - t_1), \quad y(t) = b$ object is moving along horizontal line $y = b$.

▶ $x(t) = a, \quad y(t) = b + v(t - t_1)$ if object is moving along vertical line $x = a$.



$$y(t) - y(t_1) = v(t - t_1)$$

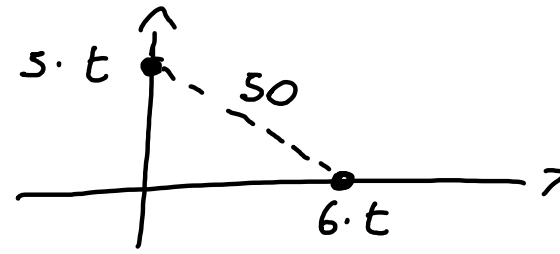
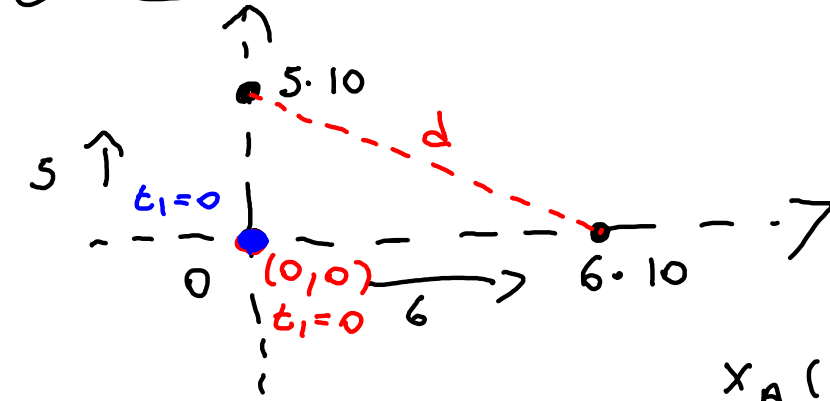
$d = v \underline{\underline{t}}$
Length of time

Example

Ann and Bob start moving at the same time from the same location. Ann moves East at 6 feet/sec. Bob moves North at 5 feet/sec.

What is the distance between Ann and Bob 10 sec later ?

When is the distance between Ann and Bob 50 feet ?



$$d = \sqrt{60^2 + 50^2} \text{ ft.}$$

$$x_A(t) = 0 + 6(t-0)$$

$$x_B(t) = 0$$

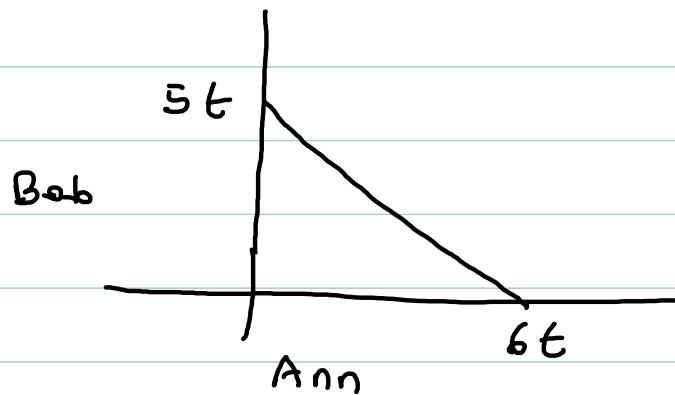
$$y_B(t) = 0$$

$$y_B(t) = 0 + 5(t-0)$$

$$(6t, 0)$$

$$(0, 5t)$$

$$50 = \sqrt{(6t)^2 + (5t)^2}$$



When is distance
between Ann and Bob
equal to 50 feet?

Ann $(6t, 0)$

Bob $(0, 5t)$

$$50 = \sqrt{(6t)^2 + (5t)^2}$$

$$50 = \sqrt{36t^2 + 25t^2} = \sqrt{61t^2}$$

$$2500 = 61t^2$$

$$\frac{2500}{61} = t^2$$

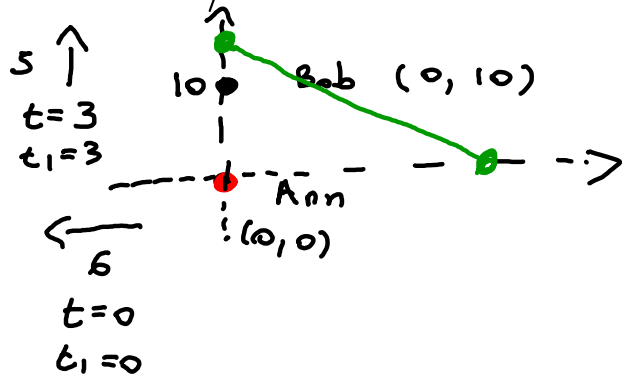
$$\pm \sqrt{\frac{2500}{61}} = t, \text{ discard negative } \boxed{t = \sqrt{\frac{2500}{61}} \text{ sec}}$$

Algebra question

Is $\sqrt{x^2} = x$?

What if Ann and Bob start at different times from different locations ?

Bob is standing 10 feet North of Ann, when Ann starts moving ~~WEST~~ ^{WEST} at 6 feet/sec. 3 seconds later Bob, starts moving North at 5 feet/sec. When is the distance between Ann and Bob 50 feet ?



$$x_A(t) = 0 - 6(t-0)$$

$$y_A(t) = 0$$

$$(-6t, 0)$$

$$t \geq 0$$

$$x_B(t) = 0$$

$$y_B(t) = 10 + 5(t-3)$$

$$(0, 10 + 5(t-3))$$

$$t \geq 3$$

$$50 = \sqrt{(-6t-0)^2 + (10+5(t-3))^2} \quad \text{solve for } t$$

$$50 = \sqrt{36t^2 + (10 + 5(t-3))^2}$$

① square both sides

$$2500 = 36t^2 + (10 + 5(t-3))^2$$

② simplify inside \rightarrow

$$2500 = 36t^2 + (5t - 5)^2$$

③ Expand square: recall $(a+b)^2 = a^2 + 2ab + b^2$
 $(a-b)^2 = a^2 - 2ab + b^2$

$$2500 = 36t^2 + 25t^2 - 50t + 25$$

④ move 2500 to the right and simplify

$$0 = \underbrace{61}_{a}t^2 - \underbrace{50}_{b}t - \underbrace{2475}_{c}$$

⑤ Use quadratic formula

recall the solutions of $ax^2 + bx + c = 0$

are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ if $b^2 - 4ac \geq 0$

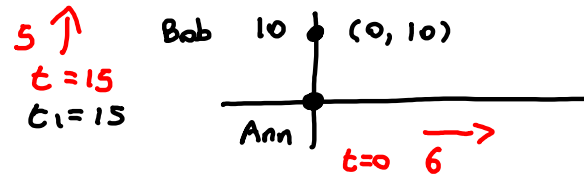
$$t = \frac{50 \pm \sqrt{50^2 + 4 \cdot 61 \cdot 2475}}{2 \cdot 61} \approx -5.97, 6.79$$

For our problem, the negative solution would be a time before Ann and Bob start moving so it does not make sense so

$$t = 6.79 \text{ sec}$$

Tricky version

Bob is standing 10 feet North of Ann, when Ann starts moving East at 6 feet/sec. 15 seconds later Bob, starts moving North at 5 feet/sec. When is the distance between Ann and Bob 50 feet ?



Ann

$$\begin{aligned}x_A(t) &= 6t \\ y_A(t) &= 0\end{aligned}$$

valid for $t \geq 0$
 $(6t, 0)$

Bob

$$\begin{aligned}x_B(t) &= 0 \\ y_B(t) &= 10 + 5(t-15)\end{aligned}$$

valid for $t \geq 15$
 $(0, 10 + 5(t-15))$

We want $d((6t, 0), (0, 10 + 5(t-15))) = 50$

$$50 = \sqrt{(6t-0)^2 + (0-(10+5(t-15)))^2}$$

do the algebra $t = 5, 5.66$ DISCARD BOTH

Can I accept these solutions? NO

Are there any solutions? We will finish on wed