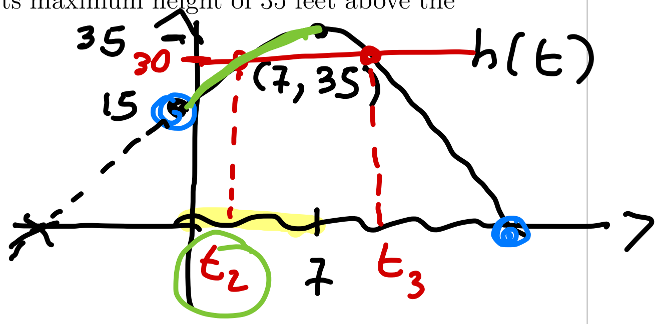
3. Jack is standing on a cliff 15 feet above the sea. He kicks a ball into the sea. The height of the ball above the sea $t$ seconds after being kicked, is described by a quadratic function h(t). You know that h(0) = 15 and that the ball reaches its maximum height of 35 feet above the sea 7 seconds after being kicked.




(a) Find a formula for h(t).

$$h(t) = a(t-7)^2 + 35$$
$$15 = a(0-7)^2 + 35$$
$$-20 = 49a \qquad a = -\frac{20}{49}$$
$$h(t) = -\frac{20}{49}(t-7)^2 + 35$$

(b) Find the time $t_1$ when the ball hits the water.

$$h(t) = 0$$
$$-\frac{20}{49}(t-7)^2 + 35 = 0 \rightarrow \frac{49 \cdot 35}{20} = \cancel{\frac{20}{49}}(t-7)^2$$
$$7 \pm \sqrt{\frac{49 \cdot 35}{20}} = t \cancel{-7} \qquad t_1 = 7 + \sqrt{\frac{49 \cdot 35}{20}} \text{ sec}$$
$$t_1 = 7 + \frac{7}{2}\sqrt{7}$$

(c) At what time, when the ball is going up, is it 30 feet above the sea?

$$h(t) = 30 \quad \text{solve for } t$$
$$-\frac{20}{49}(t-7)^2 + 35 = 30 \ ; \ \frac{49 \cdot 5}{20} = \cancel{\frac{20}{49}}(t-7)^2$$
$$7 \pm \sqrt{\frac{49 \cdot 5}{20}} = t \cancel{-7} \qquad t_2 = 7 - \sqrt{\frac{49}{4}} = 7 - \frac{7}{2}$$

(d) Find a formula for the function $t = g(y)$, that gives you the time when the ball is going up and it is at an height $y$ above the sea level. Give the domain and range for $g$.

Find the inverse of $y = h(t)$ on $[0\ 7]$

$g = h^{-1}$ on $[0\ 7]$

Domain of $g = h^{-1}$ = range of $h$ is $[15, 35]$

Range of $g = h^{-1}$ = domain of $h$ $[0, 7]$

$0 \le t \le 7$

To find a formula for g

set $y = h(t)$ solve for t

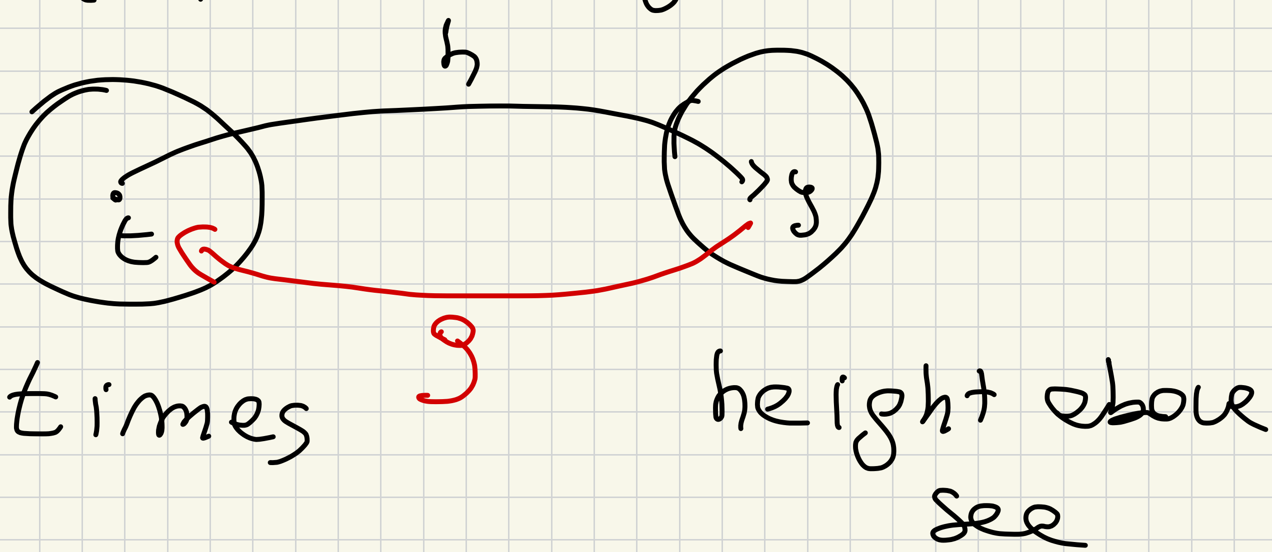
$$y = -\frac{20}{49}(t-7)^2 + 35$$

$$\cancel{\frac{20}{49}}(t-7)^2 = (35-y)\,\frac{49}{20}$$

$$t\cancel{-7} = 7 \pm \sqrt{(35-y)\,\frac{49}{20}}$$

$$g(y) = 7 - \sqrt{(35-y)\,\frac{49}{20}}$$

$$h(t) = y$$



$h(t) = at^2 + bt + c$ stendord

$a(t-h)^2 + k$ vertex form

6 Mrs. Peacock is standing in the study, where a candlestick is positioned on the floor.

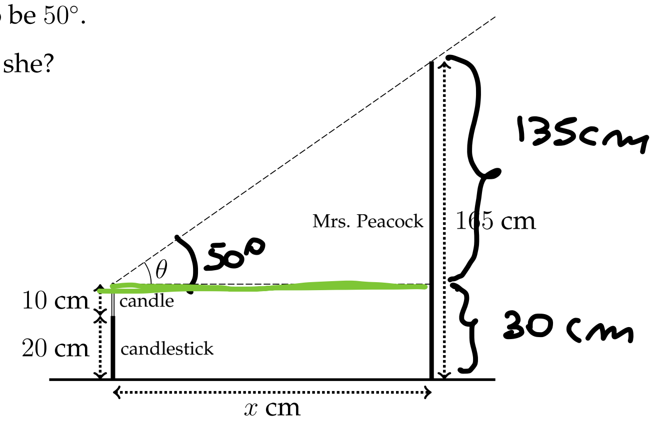
Mrs. Peacock is $165$ centimeters tall. The candlestick is $20$ centimeters tall, and it's holding a candle which is $10$ centimeters tall.

Let $\theta$ be the angle of elevation of Mrs. Peacock's head relative to the top of the candle, as shown in the picture below.

(a) **[5 points]** Mrs. Peacock measures $\theta$ to be $50°$.

How far away from the candlestick is she?

*(In other words, what's $x$?)*




$$\tan(30°) = \frac{135}{x}$$

$$x = \frac{135}{\tan(50°)} = 113.278$$

$t = 0$

(b) **[5 points]** The candle burns at a constant ~~speed~~ rate. After $1$ minute, the angle $\theta$ is $51°$.

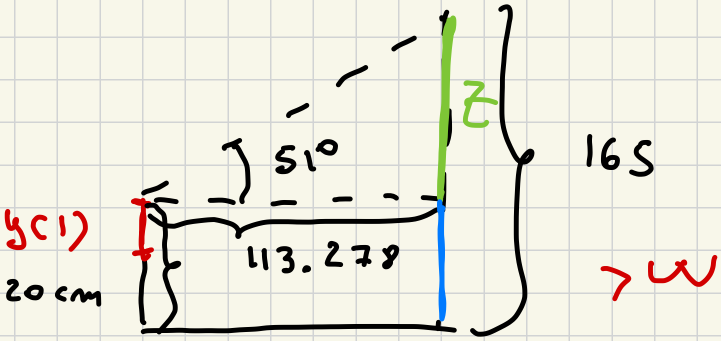
When will the candle burn all the way down?

$y(t)$ = length of candle at time $t$

Want time when $y(t) = 0$

$y(t) = b + mt$ $\quad (0, 10) \quad (1, ?)$

Figure out $y(1)$




$t = 1$

$$\frac{z}{113.278} = \tan(51°) \qquad z = 113.278 \tan(51°) \approx 139.8867$$

$$w = 165 - 139.8867 \approx 25.1133$$

$$y(1) = 25.1133 - 20 = 5.1133$$

$$y(t) = 10 + mt$$

$$(0, 10) \qquad (1, 5.1133)$$

$$m = \frac{10 - 5.1133}{0 - 1} = -4.8867$$

$$y(t) = 10 - 4.8867\, t$$

want

$$0 = 10 - 4.8867\, t$$

$$4.8867\, t = 10$$

$$t = \frac{10}{4.8867} \approx 2.05 \text{ min}$$