

Lesson 27

Sinusoidal functions problems

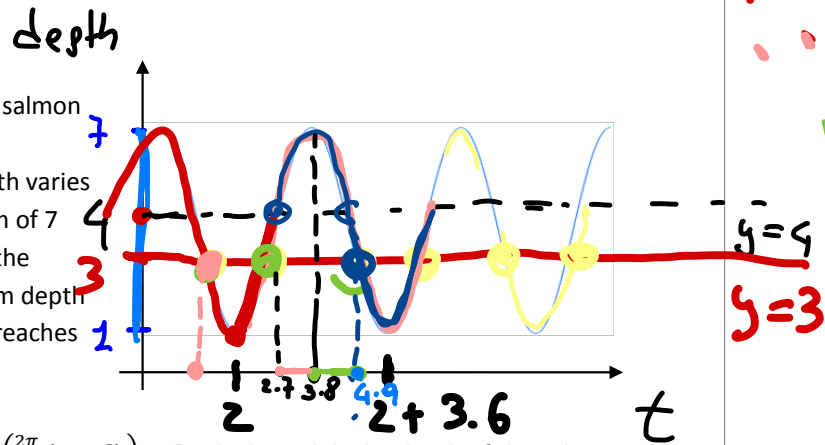
15

20.75

21.5

Spring 2012

Problem 7 (16 pts) The **depth** of a swimming salmon below the water surface can be modeled by a sinusoidal function of time. The salmon's depth varies between a minimum of 1 foot and a maximum of 7 feet below the surface of the water. It takes the salmon 1.8 minutes to move from its minimum depth to its successive maximum depth, and it first reaches the minimum depth at $t = 2$ minutes.



- a) Find the sinusoidal function $d(t) = A \sin\left(\frac{2\pi}{B}(t - C)\right) + D$ which models the depth of the salmon after t minutes.

$$A = \frac{7-1}{2} = 3 \quad D = \frac{7+1}{2} = 4 \quad B = 1.8 \times 2 = 3.6$$

$$C = 2 + \frac{3.6}{4} = 2.9 \quad \text{or} \quad x_{\max} - \frac{B}{4} = 3.8 - \frac{3.6}{4}$$

- b) Compute all the times during the first 5 minutes when the depth of the fish is exactly 3 feet.

$$3 \sin\left(\frac{2\pi}{3.6}(t - 2.9)\right) + 4 = 3$$

$$\cancel{3} \sin\left(\frac{2\pi}{3.6}(t - 2.9)\right) = \frac{-1}{3}$$

$$\frac{2\pi}{3.6}(t - 2.9) = \sin^{-1}\left(-\frac{1}{3}\right) \cdot \frac{3.6}{2\pi}$$

$$t = 2.9 + \frac{3.6}{2\pi} \sin^{-1}\left(-\frac{1}{3}\right) = 2.7 \quad \text{principal}$$

$$S = \underbrace{3.8}_{x_{\max}} + \underbrace{3.8}_{x_{\max}} - \underbrace{2.7}_{\text{principal}} = 4.9$$

$$4.9 - 3.6 = 1.3$$

S - period

ALL solutions

$$2.7 + k \cdot 3.6$$

$$k = 0, \pm 1, \pm 2, \dots$$

$$4.9 + k \cdot 3.6$$

Between $0 \leq t \leq 5$ I

have

$$2.7, 4.9, 4.9 - 3.6 = 1.3$$

Win 2013

3. The temperature at your house in the desert is a sinusoidal function of time with a 24 hour period. The maximum daily temperature is 45 degrees Celsius and occurs at 5:00 PM. The minimum daily temperature is 11 degrees Celsius. " 17

(a) Let t be hours after midnight last night. Find the function $f(t)$ that gives the temperature at time t .

$$B = 24 \quad A = \frac{45 - 11}{2} = 17, \quad D = \frac{45 + 11}{2} = 28$$

$$C = x_{\max} - \frac{B}{4} = 17 - \frac{24}{4} = 11$$

(b) For how much of each day is the temperature above 35 degrees Celsius?

$$17 \sin\left(\frac{2\pi}{24}(t - 11)\right) + 28$$

(c) Starting from midnight last night, how long will it be until the temperature has been above 35 degrees Celsius for 22 hours?

$$17 \sin\left(\frac{2\pi}{24}(t-11)\right) + 28 = 35$$

$$17 \sin\left(\frac{2\pi}{24}(t-11)\right) = 7$$

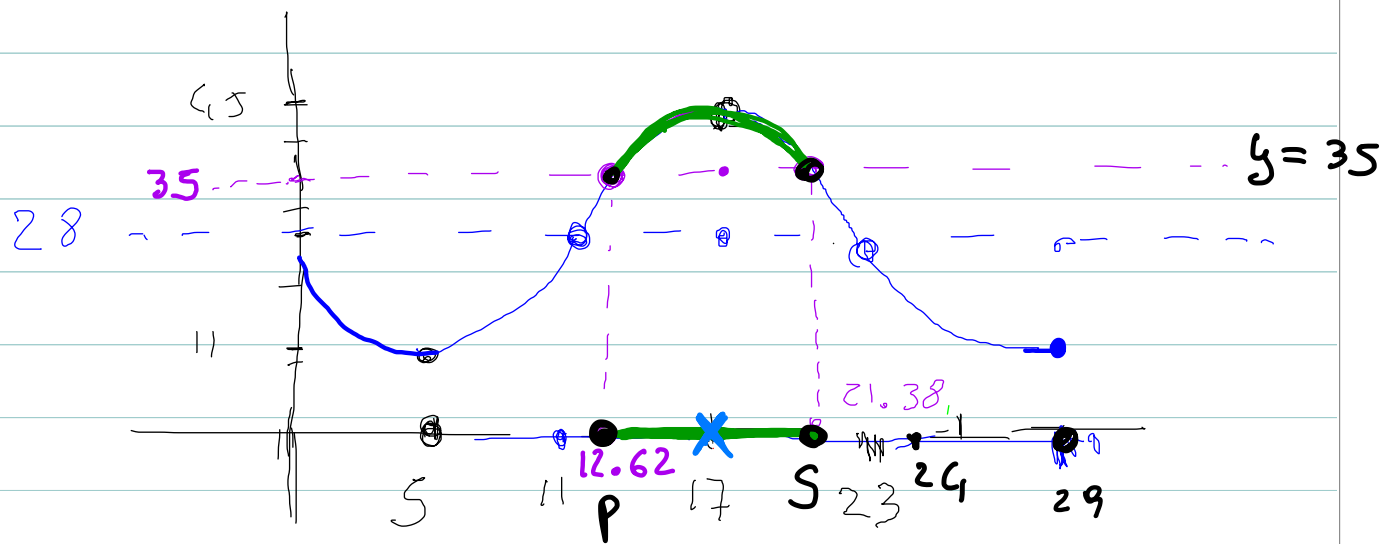
$$\sin\left(\frac{2\pi}{24}(t-11)\right) = \frac{7}{17}$$

$$\frac{2\pi}{24}(t-11) = \overset{\sin^{-1}}{\arcsin}\left(\frac{7}{17}\right)$$

$$t-11 = \frac{24}{2\pi} \arcsin\left(\frac{7}{17}\right)$$

$$t = 11 + \frac{24}{2\pi} \arcsin\left(\frac{7}{17}\right)$$

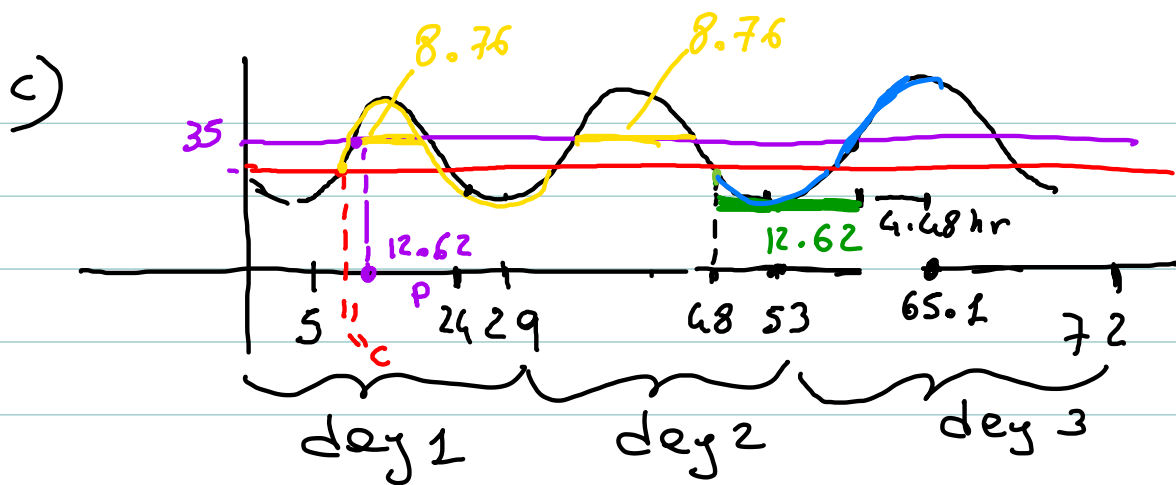
$$t_1 \approx 12.62 \quad \text{principle}$$



$$\text{Symmetry sol} = 17 + (17 - 12.62) = 21.38$$

$$\text{Above } 35 \text{ deg for } 21.38 - 12.62 = \boxed{8.76} \text{ hrs}$$

Need above 35 for 22 hours



$$22 - 8.76 - 8.76 = 4.48 \text{ hrs}$$

I still need 4.48 hrs above 35° in day 3

$$48 + \underbrace{12.62}_{\text{time needed}} + 4.48 = 65.1$$

time needed
for temperature
to rise above
35° in day 3