Lesson 26

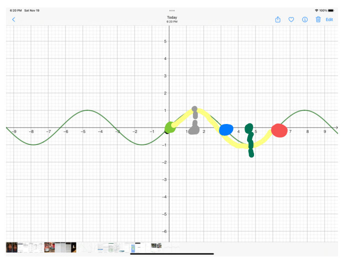
Read Chapter 19 and 20

Sinusoidal functions

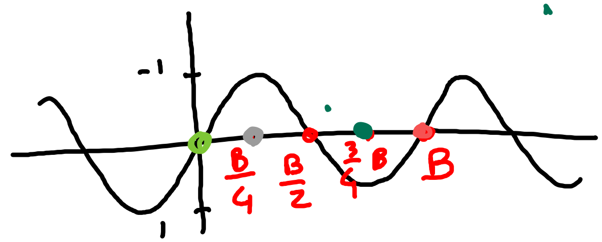
Sinusoidal equations

~~Sinusoidal modelling problems~~

Graph $f(x) = A\sin(\frac{2\pi}{B}(x - C)) + D$ $\quad$ $A, B > 0$

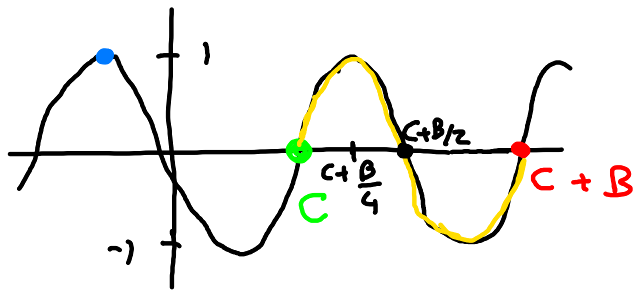


① h scaling of $\frac{B}{2\pi}$

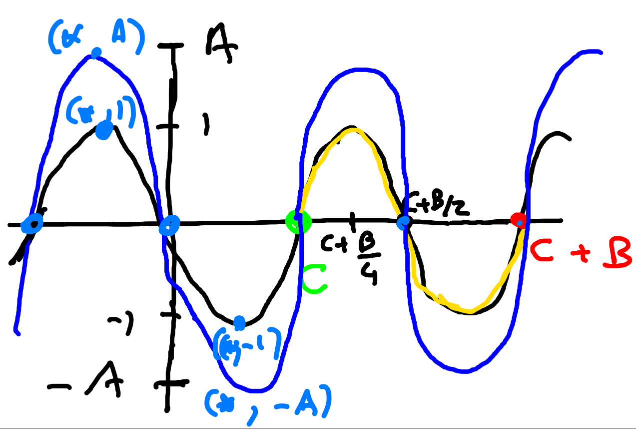


$(0,0)$ $(2\pi, 0)$ $(\pi, 0)$ $(\frac{\pi}{2}, 0)$ $(\frac{3}{2}\pi, 0)$

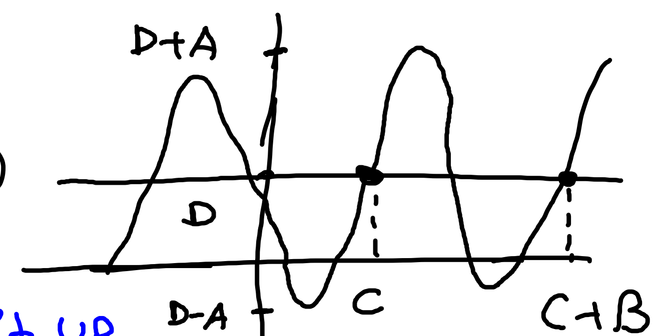
② shift right C units (if $C \geq 0$)



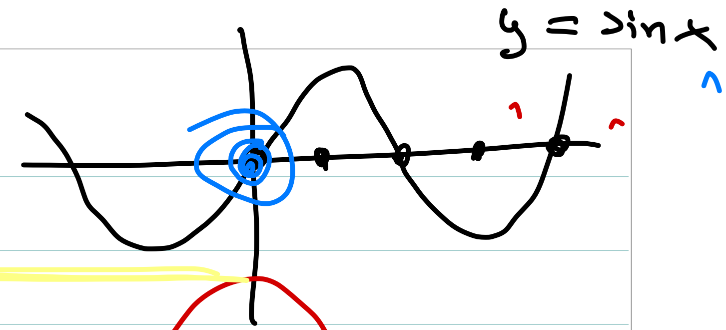
③ vertical scaling

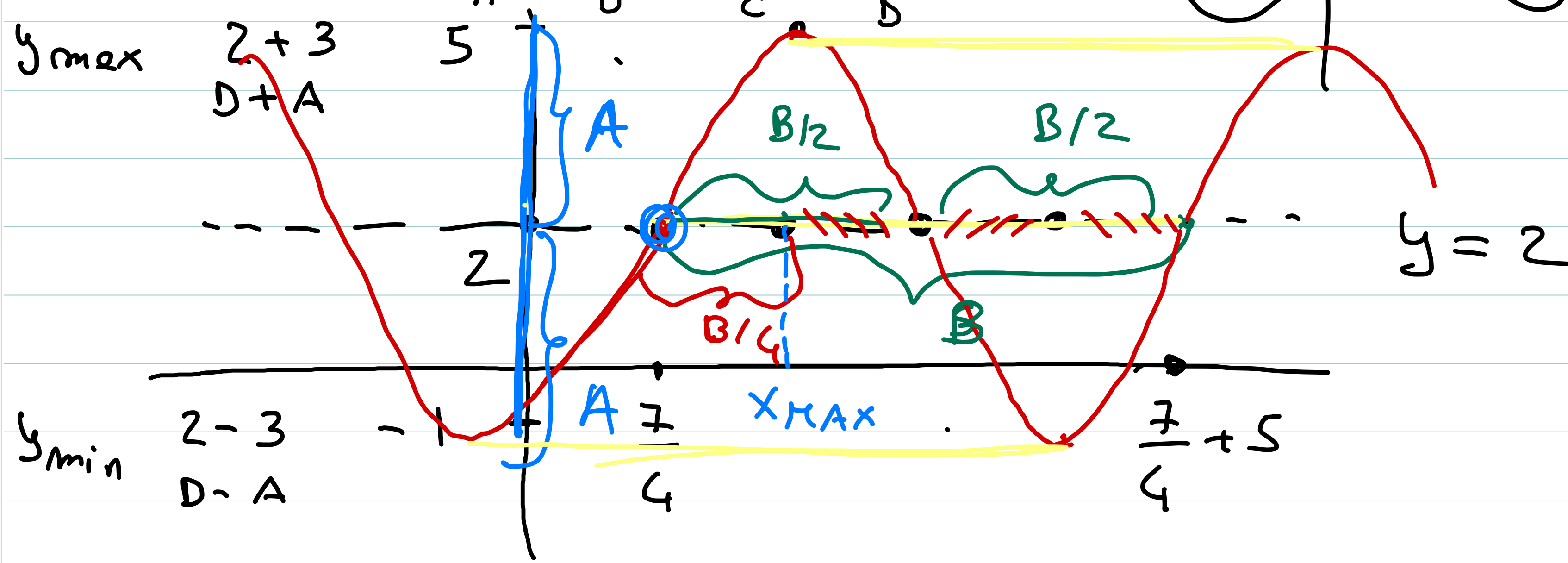


④ shift up D units ($D > 0$)



Graph $f(t) = \underbrace{3}_{A} \sin \underbrace{\frac{2\pi}{5}}_{B}\left(t - \underbrace{\frac{7}{4}}_{C}\right) + \underbrace{2}_{D}$





1) Draw $y = D$

2) Draw points (C, D) $\left(C + \frac{B}{4}, D\right)$ $\left(C + \frac{B}{2}, D\right)$ $\left(C + \frac{3}{4}B, D\right)$
$(C + B)$

3) Draw points $\left(C + \frac{B}{4}, D + A\right)$, $\left(C + \frac{3}{4}B, D - A\right)$

4) Draw basic S shape and repeat

Sinusoidal functions

$$f(x) = A\sin(\frac{2\pi}{B}(x - C)) + D \quad A, B > 0$$

A: amplitude. Half total height $= \frac{y_{max} - y_{min}}{2}$.

y coordinate of highest point

y coordinate of lowest point

B: period. Horizontal distance between two consecutive peaks or valleys, or double the horizontal distance between one peak and the next valley or one valley and the next peak.

= highest points

lowest points

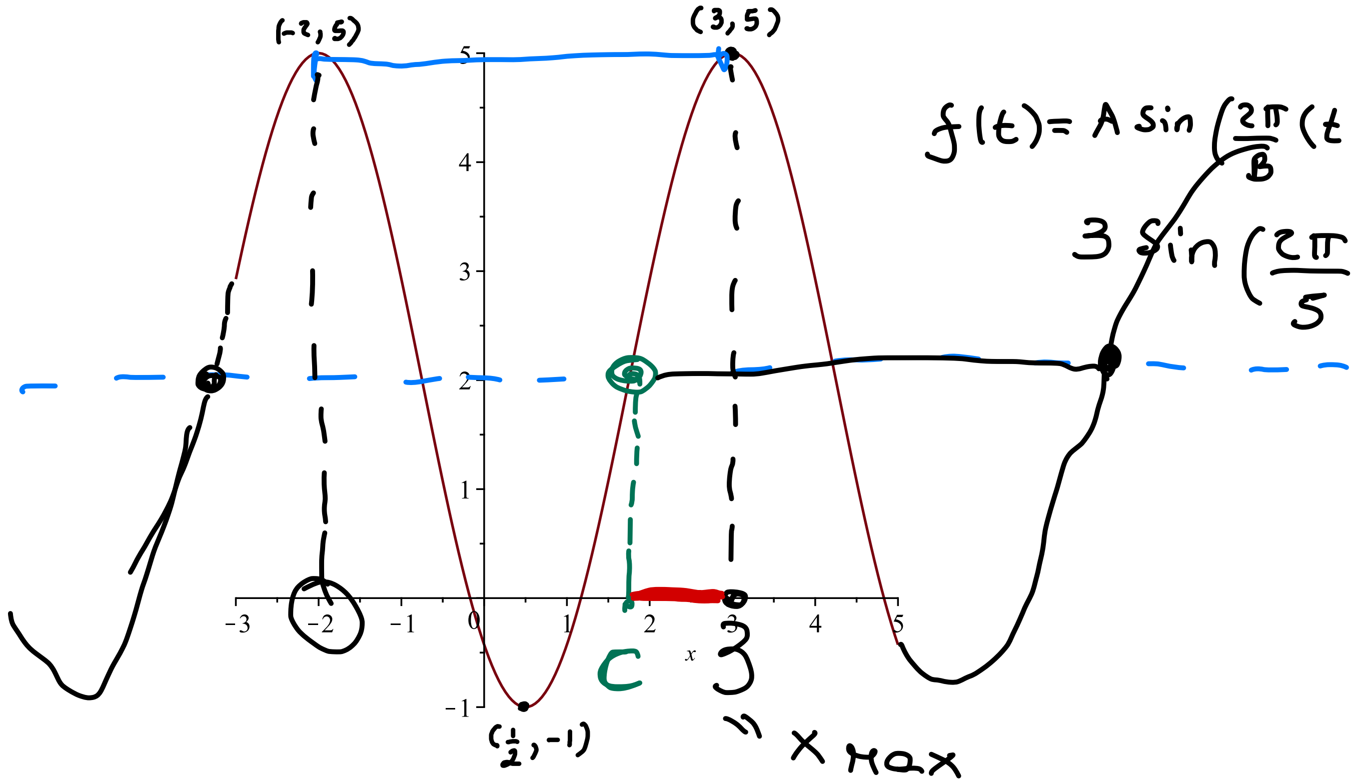
C: phase shift. x-coordinate of max - $\frac{B}{4}$ or x-coordinate of a point half way (vertically) between a valley and a peak.

to left

to right

D:mean. Half way vertical point $= \frac{y_{max} + y_{min}}{2}$.

Find a formula for the sinusoidal function below



$f(t) = A\sin\left(\frac{2\pi}{B}(t - C)\right) + D$

$3\sin\left(\frac{2\pi}{5}\left(t - \frac{7}{4}\right)\right) + 2$

$A = \frac{5 - (-1)}{2} = 3$, $D = \frac{5 + (-1)}{2} = 2$

$|-2|$

$B = 2 + 3 = 5$, $C = 3 - \frac{5}{4} = \frac{7}{4}$

$C_1 = -2 - \frac{5}{4} = -\frac{13}{4}$ $C_2 = \frac{7}{4} + 5$

solve $3\sin(\frac{2\pi}{5}(t-\frac{7}{4}))+2=4$

[rembember $\sin x = \frac{1}{2}$]

Find ALL solutions

1) $\cancel{3}\sin\left(\frac{2\pi}{5}\left(t-\frac{7}{4}\right)\right) = \frac{4-2}{3} = \frac{2}{3}$

$$\sin^{-1}\left(\sin\left(\frac{2\pi}{5}\left(t-\frac{7}{4}\right)\right)\right) = \sin^{-1}\left(\frac{2}{3}\right)$$

$$\frac{2\pi}{5}\left(t-\frac{7}{4}\right) = \sin^{-1}\left(\frac{2}{3}\right)$$

$$t-\frac{7}{4} = \frac{5}{2\pi}\sin^{-1}\left(\frac{2}{3}\right)$$

$$t = \frac{7}{4}+\frac{5}{2\pi}\sin^{-1}\left(\frac{2}{3}\right)$$

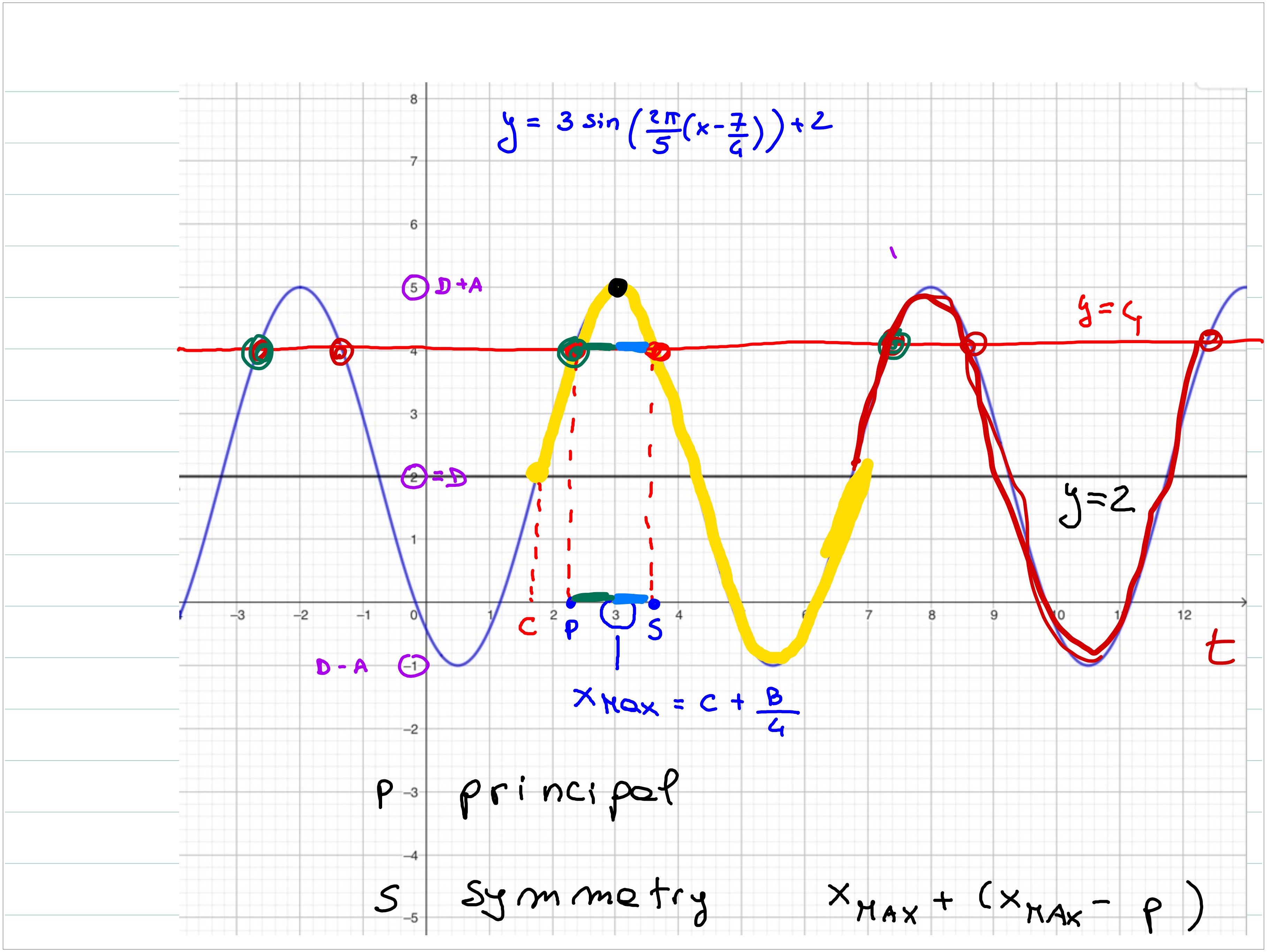
$t = p \approx 2.33$ (calc in red)

2) $\boxed{2.33 + \underbrace{5}_{B \text{ period}} k}$ principal $k = 0, \pm 1, \pm 2, \ldots$

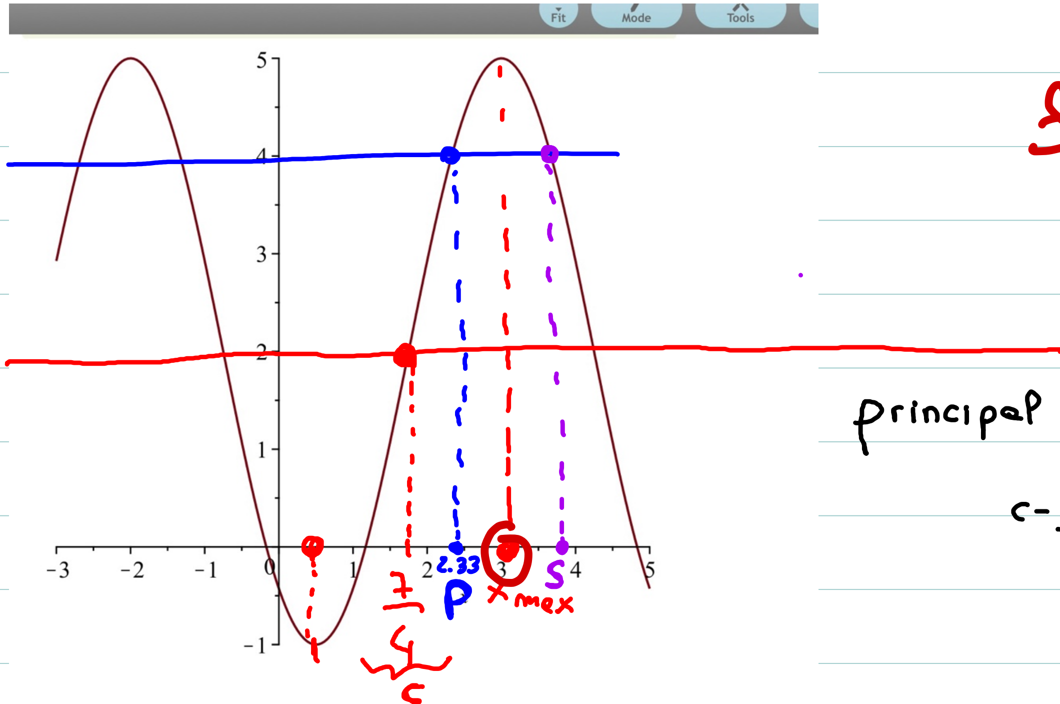
3) symmetry $s = \underbrace{3}_{x_{max}} + (3 - \underbrace{2.33}_{p}) = 3.67$

x_{max}: x coordinate of highest point immediately to the right of p

4) $\boxed{3.67 + 5k}$



$y = 3\sin\left(\frac{2\pi}{5}\left(x - \frac{7}{4}\right)\right) + 2$
5 D+A
2 = D
-1 D - A
y = 4
y = 2
t
C
P
3
S
$x_{Max} = C + \frac{B}{4}$
P principal
S symmetry
$x_{MAX} + (x_{MAX} - P)$



$$f(t) = A\sin\left(\frac{2\pi}{B}(x-c)\right)+D$$

principal solution always between

$c-\frac{B}{4}$ and $c+\frac{B}{4}$

symmetry solution $S = x_{MAX} + x_{MAX} - \text{principal}$

$c+\frac{B}{4} + c+\frac{B}{4} - \text{principal}$

$S = 2c + \frac{B}{2} - \text{principal}$

sin x

Other way to find symmetry solution

Solve $3\sin\left(\frac{2\pi}{5}\left(t-\frac{7}{4}\right)\right)+2=4$

$\sin\left(\underbrace{\frac{2\pi}{5}\left(t-\frac{7}{4}\right)}_{\theta}\right) = \frac{2}{3}$ $\qquad \frac{5}{2\pi}\sin^{-1}\left(\frac{2}{3}\right)+\frac{7}{4} = P$

$\sin(\theta) = \frac{2}{3}$ $\qquad$ principal $\sin^{-1}\left(\frac{2}{3}\right)$

symmetry is $\theta = \pi - \sin^{-1}\left(\frac{2}{3}\right)$

$\frac{2\pi}{5}\left(t-\frac{7}{4}\right) = \pi - \sin^{-1}\left(\frac{2}{3}\right)$ $\qquad$ solve for t

$t-\frac{7}{4} = \frac{5}{2\pi}\left(\pi - \sin^{-1}\left(\frac{2}{3}\right)\right)$

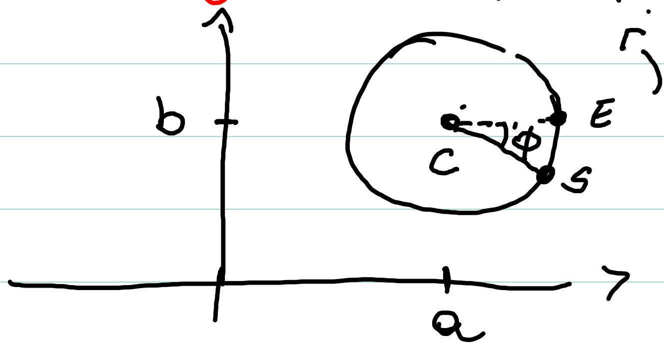
$t = \frac{7}{4} + \frac{5}{2} - \frac{5}{2\pi}\sin^{-1}\left(\frac{2}{3}\right)$

$\underbrace{\frac{7}{4}+\frac{7}{4}+\frac{5}{2}}_{2C \; + \; B/2} - \left(\frac{5}{2\pi} - \frac{7}{4}\sin^{-1}\frac{2}{3}\right)$

How to solve $A\sin(\frac{2\pi}{B}(x - C)) + D = V$

- Do some algebra first: $\sin(\frac{2\pi}{B}(x - C)) = \frac{V-D}{A}$
- Use arcsin : $\frac{2\pi}{B}(x - C) = \arcsin(\frac{V-D}{A})$
- Do some more algebra to solve for x:
  $x = C + \frac{B}{2\pi}\arcsin(\frac{V-D}{A})$.
- $x_1 = C + \frac{B}{2\pi}\arcsin(\frac{V-D}{A})$ is the principal solution.
  $C - \frac{B}{4} \leq x_1 \leq C + \frac{B}{4}$
- All values $x_1 + kB$,$k = 0, \pm1, \pm2, \pm3, \cdots$ are also solutions.
- The symmetry solution is $x_2 = x_{\text{max}} + (x_{\text{max}} - \text{principal})$, where x_{max} is the x coordinate of the first max to the right of C.
- All values $x_2 + kB$ $k = 0, \pm1, \pm2, \pm3, \cdots$ are also solutions.

WebAssign hw (ch 19 #3)



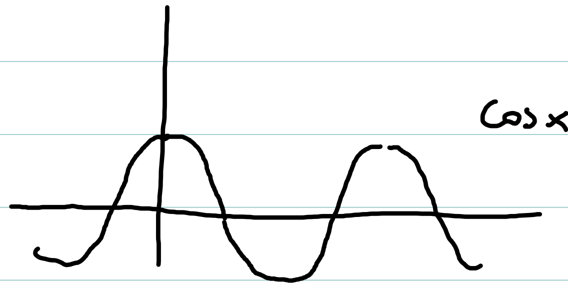
$C(x_0\ y_0)$

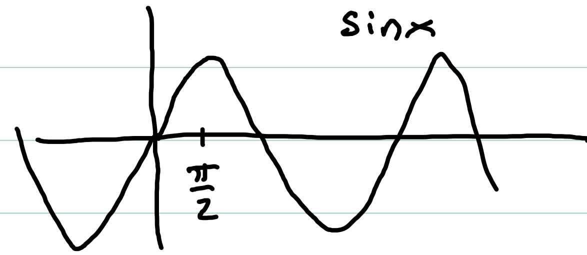
Angular speed ω

The height of the rotating object above the x axis is a sinusoidal function

See Video on Week 10 module of canvas

Note :





$$\cos x = \sin\left(x + \frac{\pi}{2}\right)$$

$$\cos x = \sin\left(-x + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - x\right)$$