Lesson 25

Read Chapter 19 and 20

Inverse trigonometric functions

Sinusoidal functions
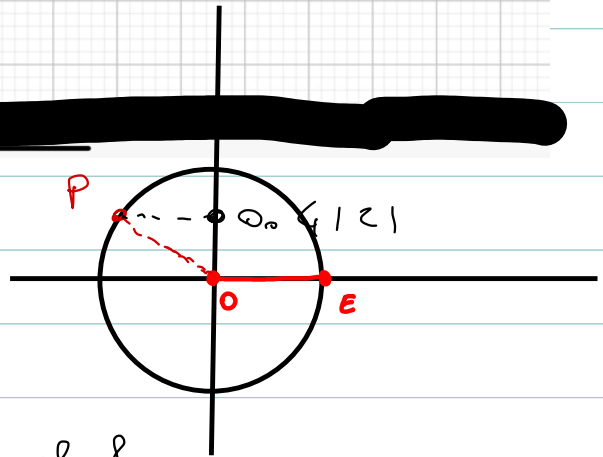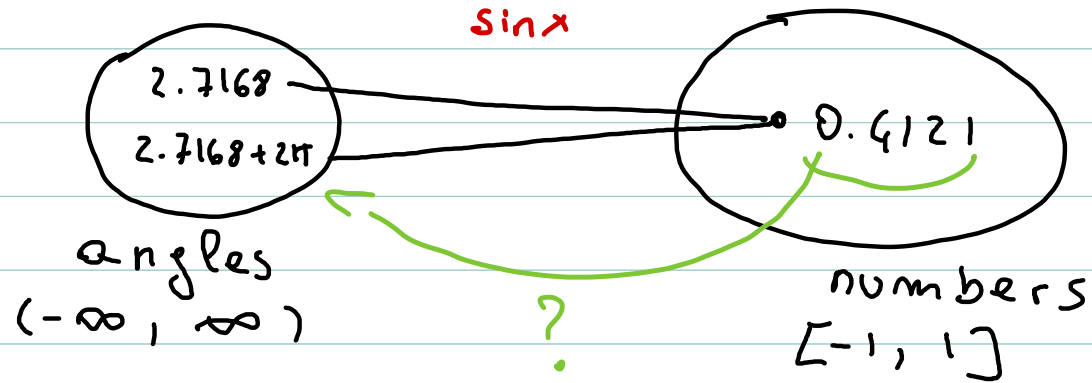
$y = \sin x$

y values
are numbers

Fails the horizontal
line test

x values are
angles in rad
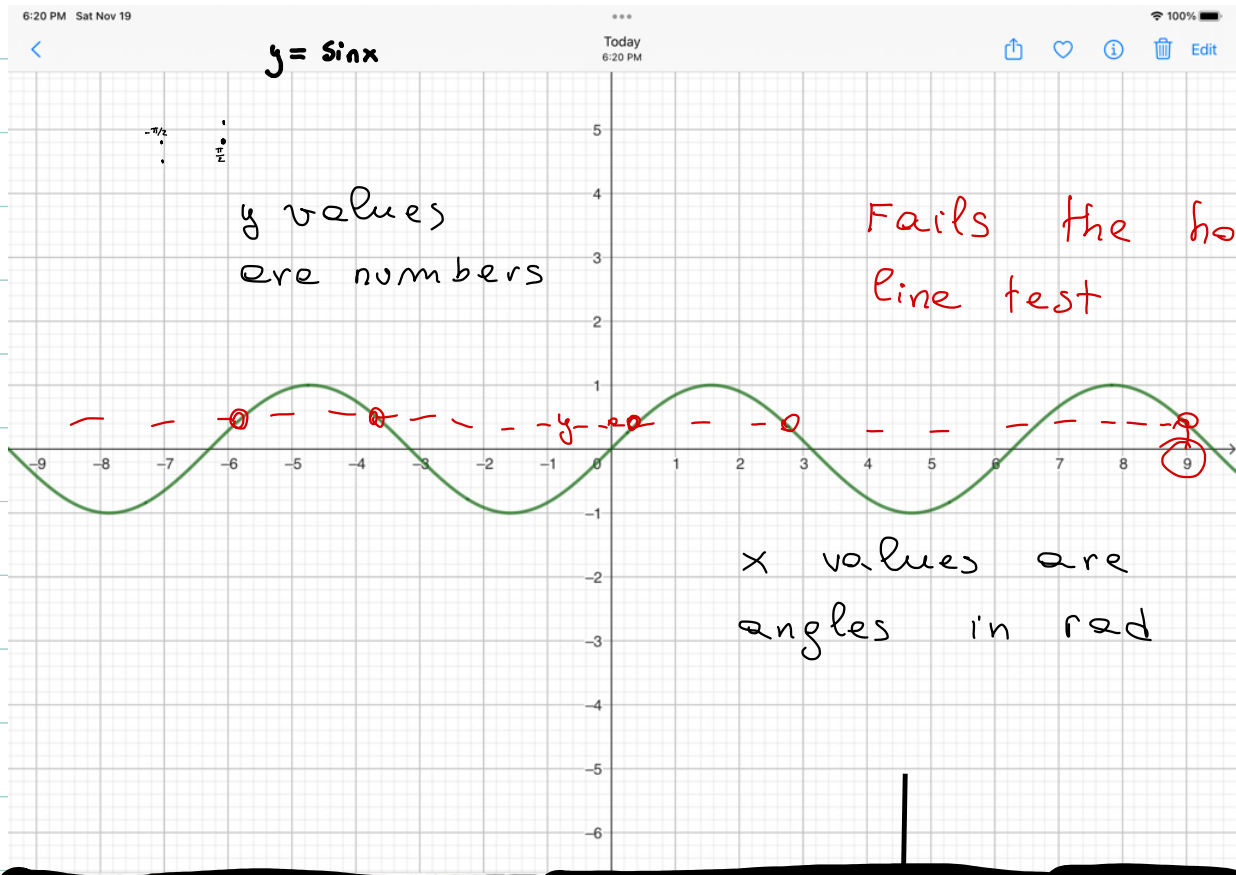
$\sin(q) = 0.4121$

$q - 2\pi = 2.7168$

$q = 2\pi + 2.7168$

y coordinate of P

P

O   E

$\sin x$

2.7168

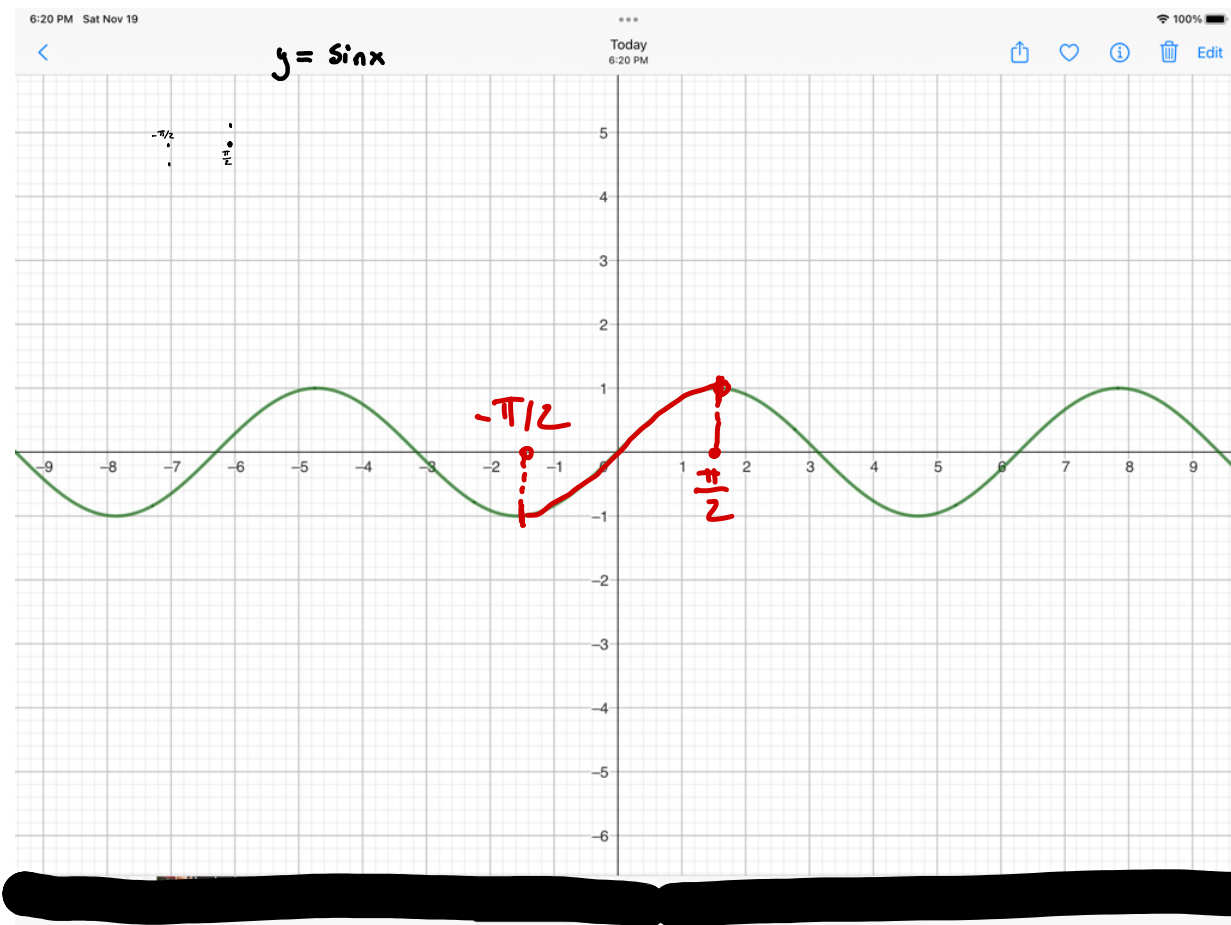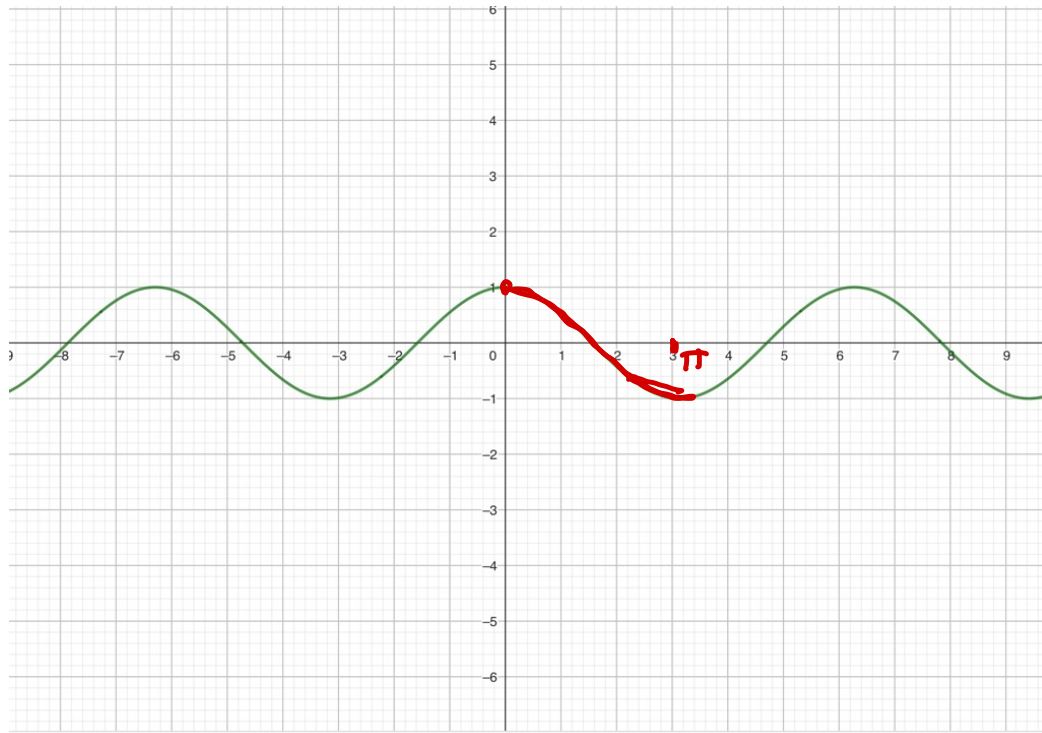2.7168 + 2π

angles
(−∞, ∞)

0.4121

numbers
[−1, 1]

?

NOT invertible

Trick : restrict domain

$\sin^{-1}(x)$

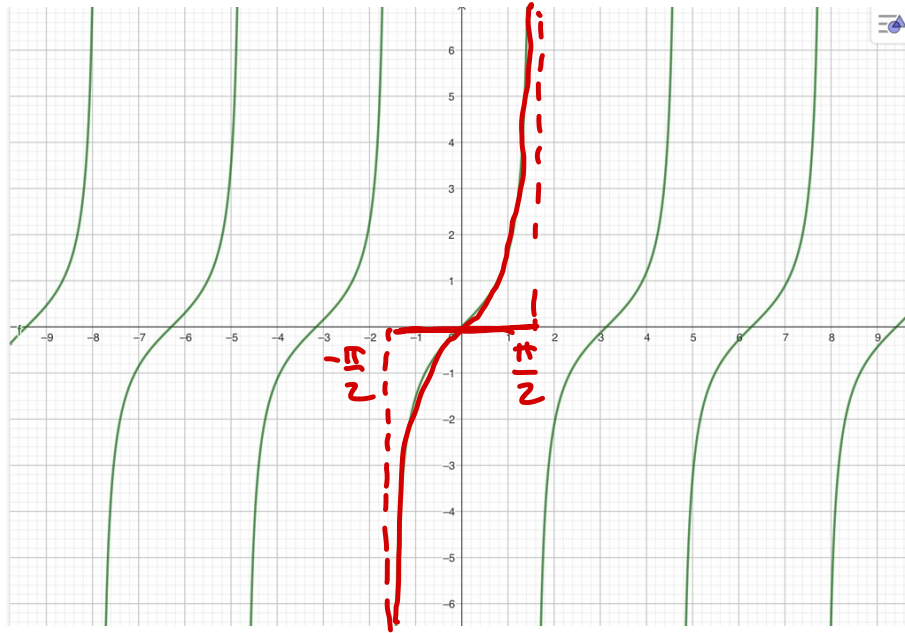$\arcsin(x)$ is the inverse of $\sin x$ restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \sin x$

$-\pi/2$

$-\frac{\pi}{2}$

$\frac{\pi}{2}$

$\frac{\pi}{2}$

$Cos^{-1}(x)$

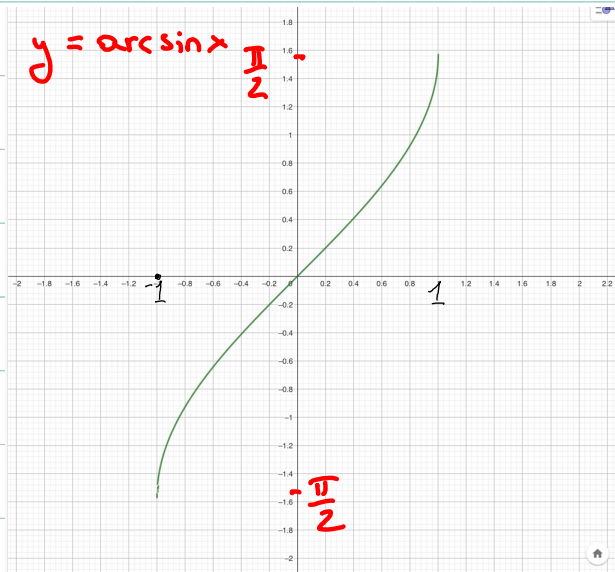arccos(x)   is the inverse of cosx restricted to   [0, π]

arctan(x) is the inverse of tan x restricted to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$y = \tan x$
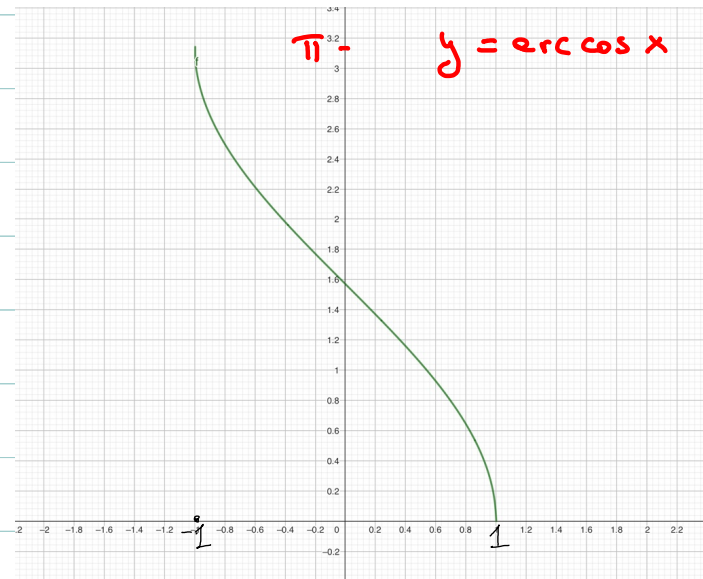
$\tan x = \dfrac{\sin x}{\cos x}$
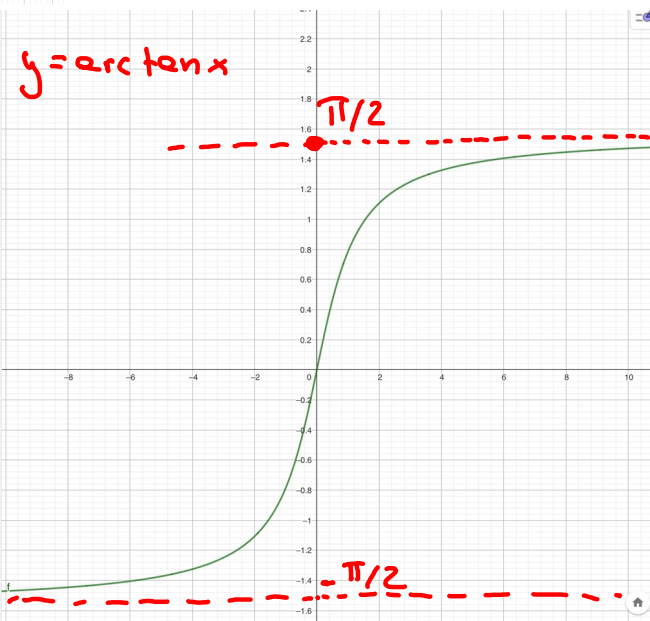
$\cos \dfrac{\pi}{2} = 0$

$y = \arcsin x$

$\frac{\pi}{2}$

$-\frac{\pi}{2}$

Domain $[-1, 1]$
Range $[-\frac{\pi}{2}, \frac{\pi}{2}]$

arc sin x

sin⁻¹ x

$y = \arccos x$

π

Domain $[-1, 1]$
Range $[0, \pi]$

$y = \arctan x$

$\pi/2$

$-\pi/2$

Domain $(-\infty, +\infty)$
Range $(-\frac{\pi}{2}, \frac{\pi}{2})$

Sope   sin (x) = $\frac{1}{2}$

$y = \frac{1}{2}$

$-\frac{\pi}{2}$ $\frac{\pi}{6}$ $\frac{\pi}{2}$ S π $\frac{3}{2}\pi$ 2π

-1

$S = \frac{\pi}{2} + \left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

sin x

? $\frac{1}{2}$

angles   numbers

$\left[-\frac{\pi}{2} \quad \frac{\pi}{2}\right]$ $\left[-1 \quad 1\right]$
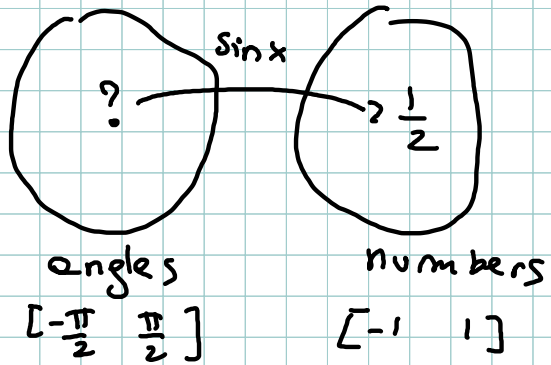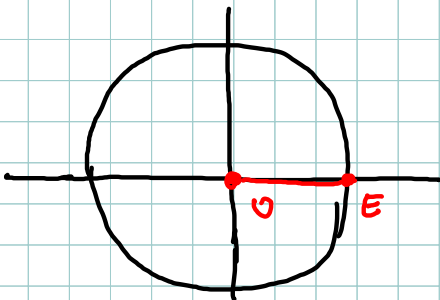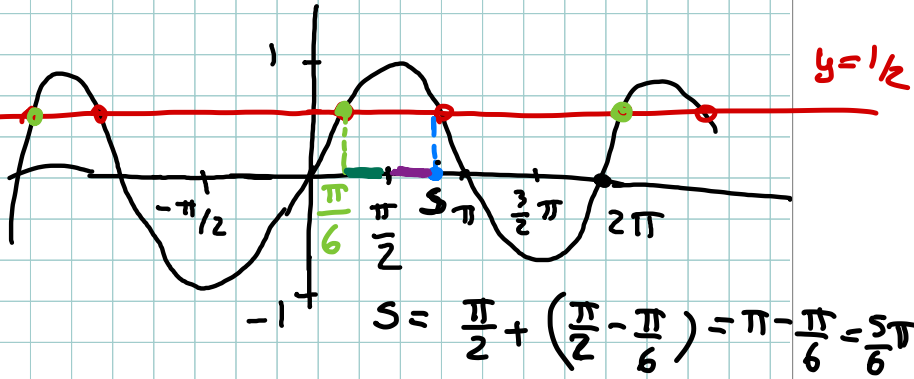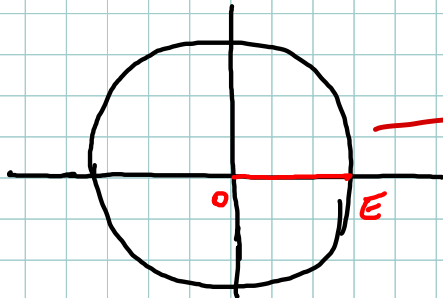
Find ALL angles X s.t

$$\sin x = \frac{1}{2}$$

1) Use $\sin^{-1} x$

$$\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \quad \text{principal}$$
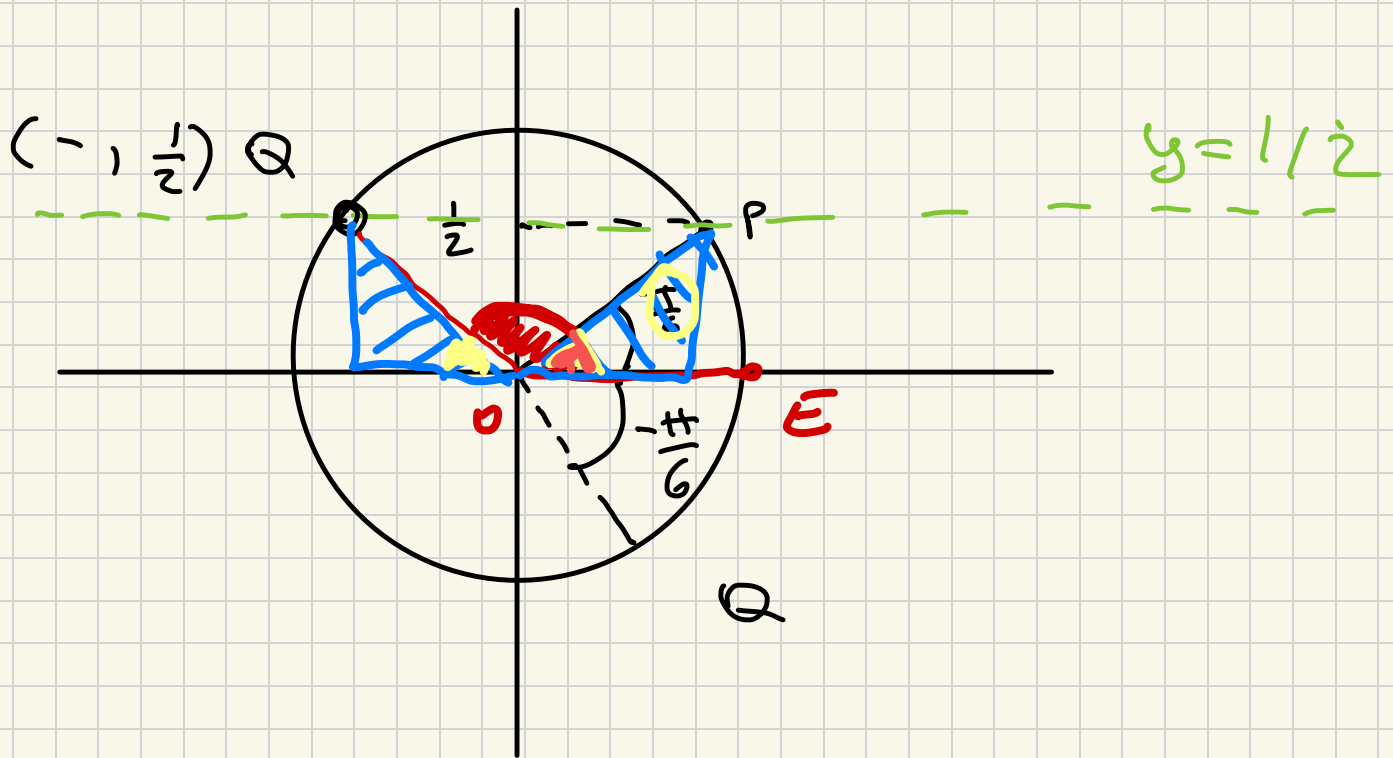
2) $\pi - \frac{\pi}{6}$ is another solution

$$= \frac{5}{6}\pi \quad \text{symmetry}$$

3) $\boxed{\dfrac{\pi}{6} + 2\pi k}$

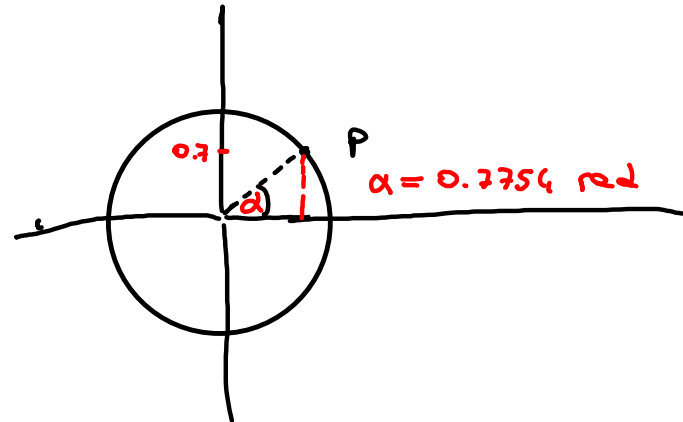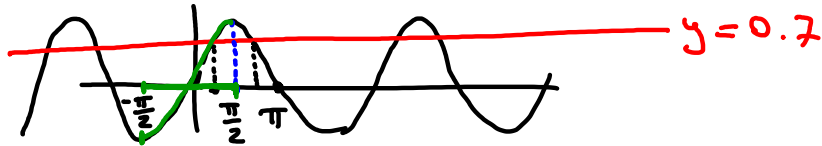4) $\boxed{\dfrac{5}{6}\pi + 2\pi k}$

$$k = 0, \pm 1, \pm 2, \ldots$$

$\sin x = \dfrac{1}{2} \rightarrow y$ coordinate of point on unit circle

Angle in red $\pi - \dfrac{\pi}{6} = \dfrac{5}{6}\pi$

Solve $\sin(x) = 0.7$

$y = 0.7$

$a = 0.7754 \text{ red}$

1) $\sin^{-1}(0.7) = 0.7754$ CALCULATOR MODE IS RAD

$\left(\frac{\pi}{4} \approx 0.7854\right)$ Principal

2) $\pi - 0.7754 = 2.3662$ red   symmetry

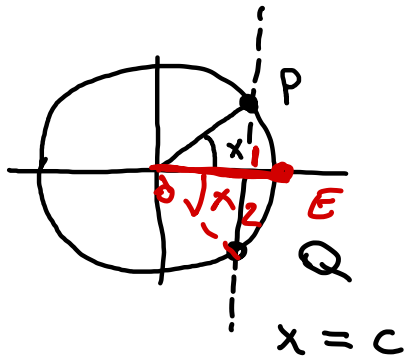ALL sol $0.7754 + 2k\pi$

            $2.3662 + 2k\pi$

How to solve $\sin(x) = c$

▶ $x_1 = \arcsin(c)$ This is the principal solution. $-\frac{\pi}{2} \leq x_1 \leq \frac{\pi}{2}$

▶ All values $x_1 + 2\pi k$, $k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.

▶ $x_2 = \pi - x_1$. This is the symmetry solution. $\frac{\pi}{2} \leq x_2 \leq \frac{3\pi}{2}$

▶ All values $x_2 + 2\pi k$ $k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.
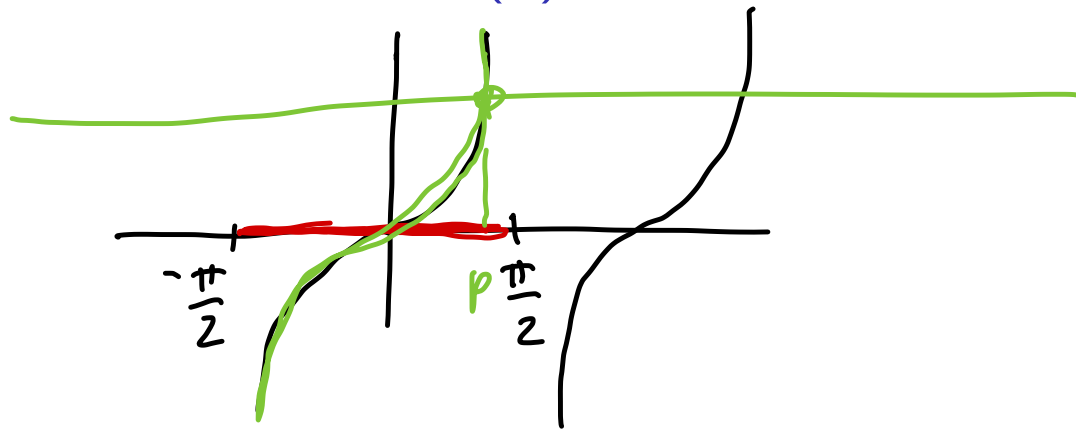
How to solve $\cos(x) = c$

$\mathrm{arc\ cos}\ (\cos x) = \mathrm{arc\ cos}\ c$

$$x_1 = \mathrm{arc\ cos}\ c$$

$x = c$

- ▶ $x_1 = \arccos(c)$ This is the principal solution. $0 \leq x_1 \leq \pi$
- ▶ All values $x_1 + 2\pi k, \ k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.
- ▶ $x_2 = -x_1$. This is the symmetry solution. $-\pi \leq x_2 \leq 0$
- ▶ All values $x_2 + 2\pi k \ \ k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.
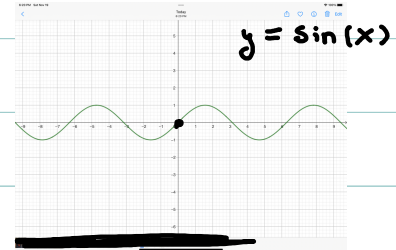
How to solve ~~arc~~tan (x)=c

- $x_1 = \arctan(c)$ This is the principal solution. $-\frac{\pi}{2} < x_1 < \frac{\pi}{2}$
- All values $x_1 + \pi k$ $k = 0, \pm 1, \pm 2, \pm 3, \cdots$ are also solutions.

Sinusoidal functions
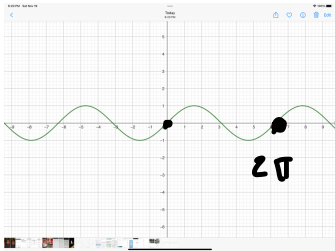
$$f(x) = A \sin\left(\frac{2\pi}{B}(x-c)\right) + D$$
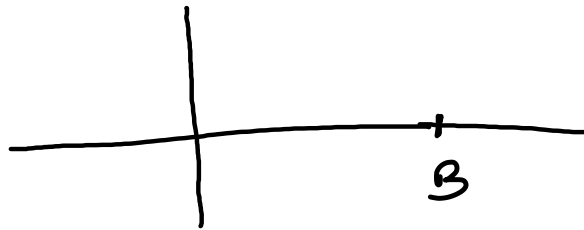
$$A, B > 0$$

$y = \sin(x)$

1) h scale of $\dfrac{B}{2\pi} = \left(\dfrac{\frac{1}{2\pi}}{B}\right)$

h

2) Shift right C units (if C is positive)
       left $|C|$ units (if C is negative)

3) V scale of A

V

4) Shift up D units (if D is positive)
       down $|D|$ units (if D is negative)
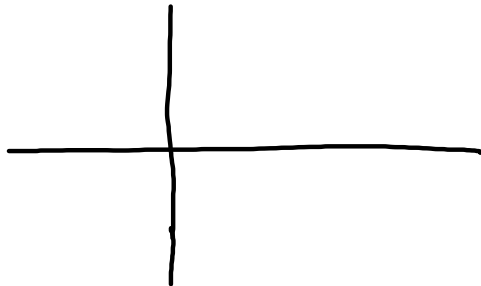
Graph $f(x) = A\sin\left(\frac{2\pi}{B}(x - C)\right) + \underline{D}$
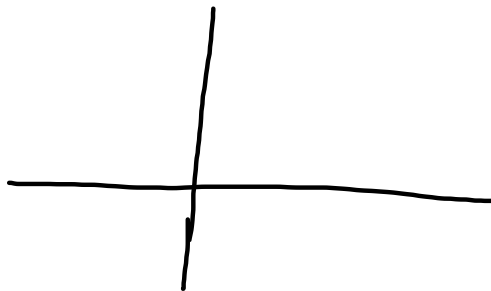
$A, B > 0$

①

$\frac{-B}{2\pi}$

2π

B

②

DRAW   AT

HOME

③

④