Lesson 23

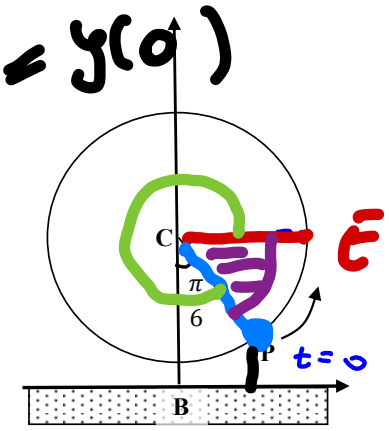
Read Chapter 18

trigonometric functions

Spring 2012 Final

C(0, 52)

Problem 6. (16 pts) Percy is riding on a ferris wheel of radius 50 feet, whose center C is 52 feet above ground. The wheel rotates at a constant rate in the direction shown by the arrow, taking 1.5 minutes for each full revolution. The wheel starts turning when Percy is at the point P, making an angle of $\frac{\pi}{6}$ radians with the vertical, as shown. *(Make sure your calculator is in radian mode)*



a) (5 pts) How high is Percy above ground when the wheel starts turning?

$y(0)$

$T = 1.5 \qquad \omega = \frac{2\pi}{T} = \frac{2\pi}{1.5} = \frac{4\pi}{3}$

$x(t) = 0 + 50\cos\left(\frac{4\pi}{3}t + \varphi\right)$

$y(t) = 52 + 50\sin\left(\frac{4\pi}{3}t + \varphi\right)$

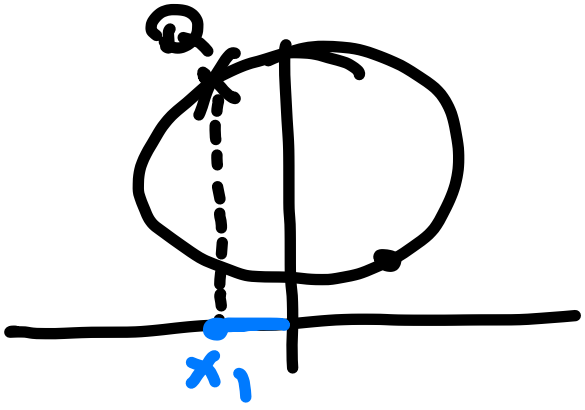
b) (4 pts) Impose a coordinate system **with the origin at the base point B**. What is the equation of the line CP?

$\varphi_1 = \frac{3}{2}\pi + \frac{\pi}{6} = \frac{10}{6}\pi$ OR

$\varphi_2 = -\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = -\frac{\pi}{3}$

c) (7 pts) Percy drops his ice cream cone 1.25 minutes after the wheel starts moving. If the cone falls straight down from Percy's position at that time, where does it land with respect to the base point B?

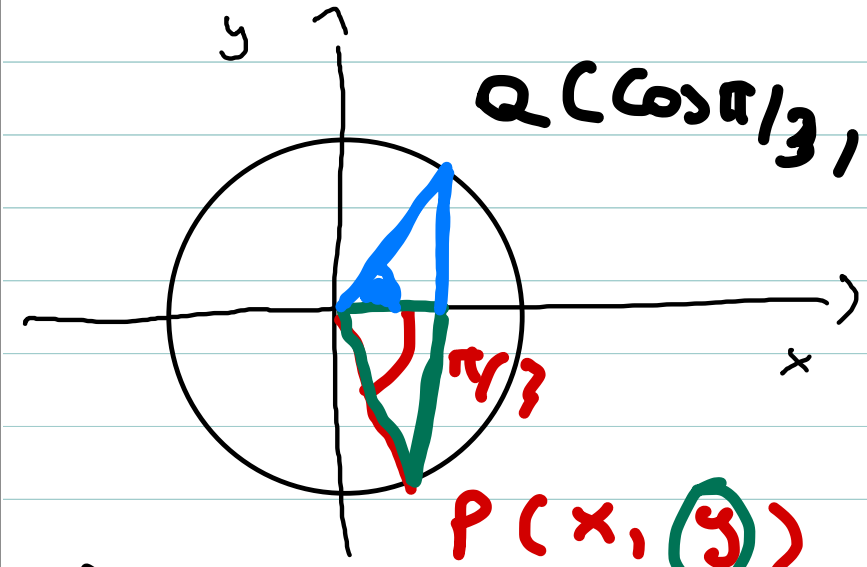
Note $\varphi_1 = \varphi_2 + 2\pi$



$x(1.25)$

$$y(t) = 52 + 50 \sin\left(\frac{4}{3}\pi t - \pi/3\right)$$

Part a) wants $y(0) = 52 + 50 \sin\left(-\frac{\pi}{3}\right) = 52 - 50\,\frac{\sqrt{3}}{2}$



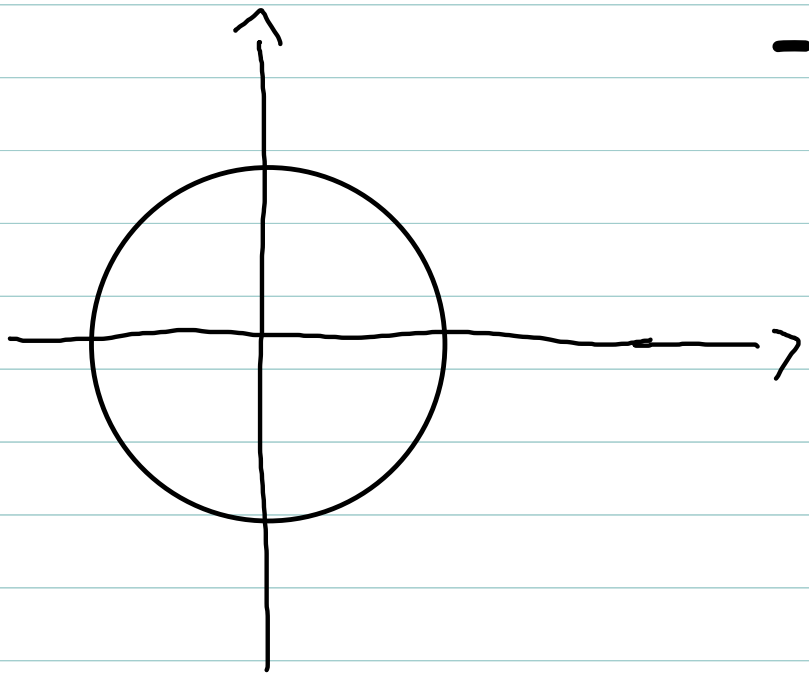
$Q\left(\cos \pi/3, \underbrace{\sin \frac{\pi}{3}}_{\frac{\sqrt{3}}{2}}\right)$

$P(x, y)$ $\quad y = -\frac{\sqrt{3}}{2}$

$$x(t) = 50 \cos\left(\frac{4\pi}{3} t - \pi/3\right)$$

part c) wants $x(1.25) = 50 \cos\left(\frac{4\pi}{3} \times 1.25 - \frac{\pi}{3}\right)$

$= 50 \cos\left(\frac{5}{3}\pi - \frac{1}{3}\pi\right) = 50 \underbrace{\cos\left(\frac{4}{3}\pi\right)}_{-1/2} = -25$



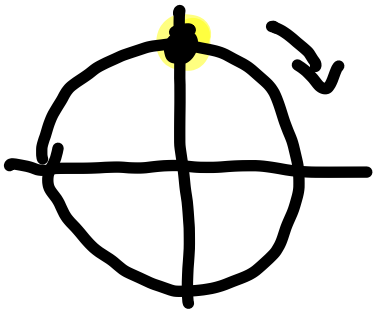
2. Polya and Baxter begin running around a circular racetrack at the same time.

   Polya starts at the northernmost point of the track and runs clockwise at a speed of 6 meters per second. It takes him 50 seconds to run one complete lap.

   Baxter runs counterclockwise at a speed of 4 meters per second.

   Polya and Baxter first pass each other after 10 seconds.

   (a) **[3 points]** Find Baxter's angular speed in radians per second.



$$\omega_B = \frac{v_B}{r} = \frac{4}{r}$$

$$v_P = \omega_P \cdot r$$

$$6 = \frac{2\pi}{50} \cdot r$$

   (b) **[7 points]** Find Baxter's coordinates after two minutes.

   (Place the origin at the center of the track, and measure units in meters.)

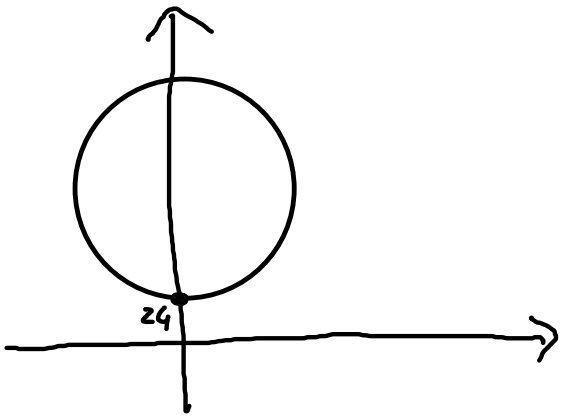
$$r = \frac{50 \cdot 6}{2\pi} \text{ meters}$$

Win 2007 final

7. You are going to ride a ferris wheel that has a radius of 160 feet and rotates at a constant 1.57 revolutions per minute. The lowest point on the wheel is 24 feet off the ground.

   From the point where you start your ride, it will take you 21 seconds to reach the highest point of the wheel.

   (a) How far off the ground will you be when the ride starts?



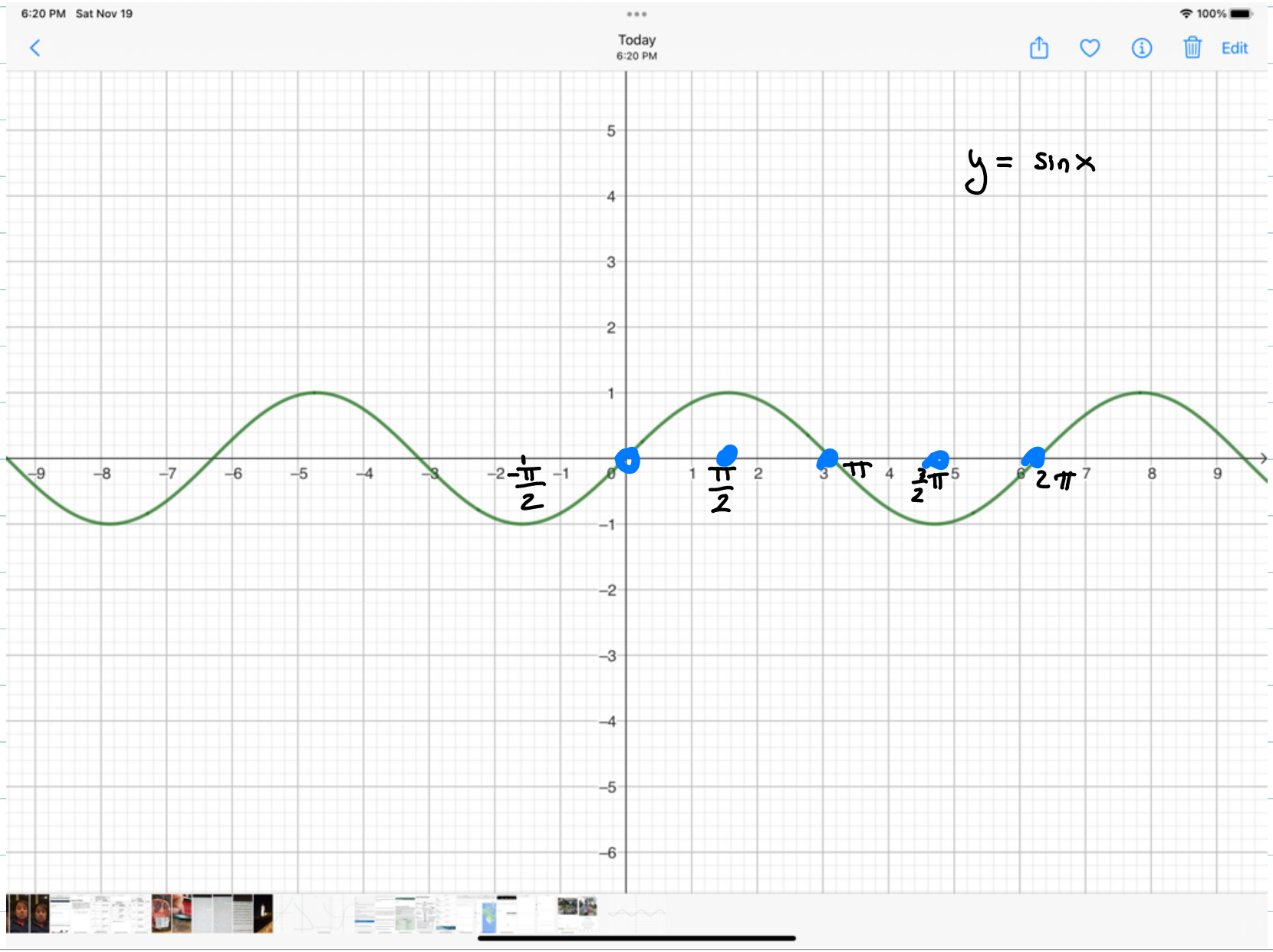
   (b) How far off the ground will you be after 5.6 minutes on the ride?

Watch video recording

Ch 18

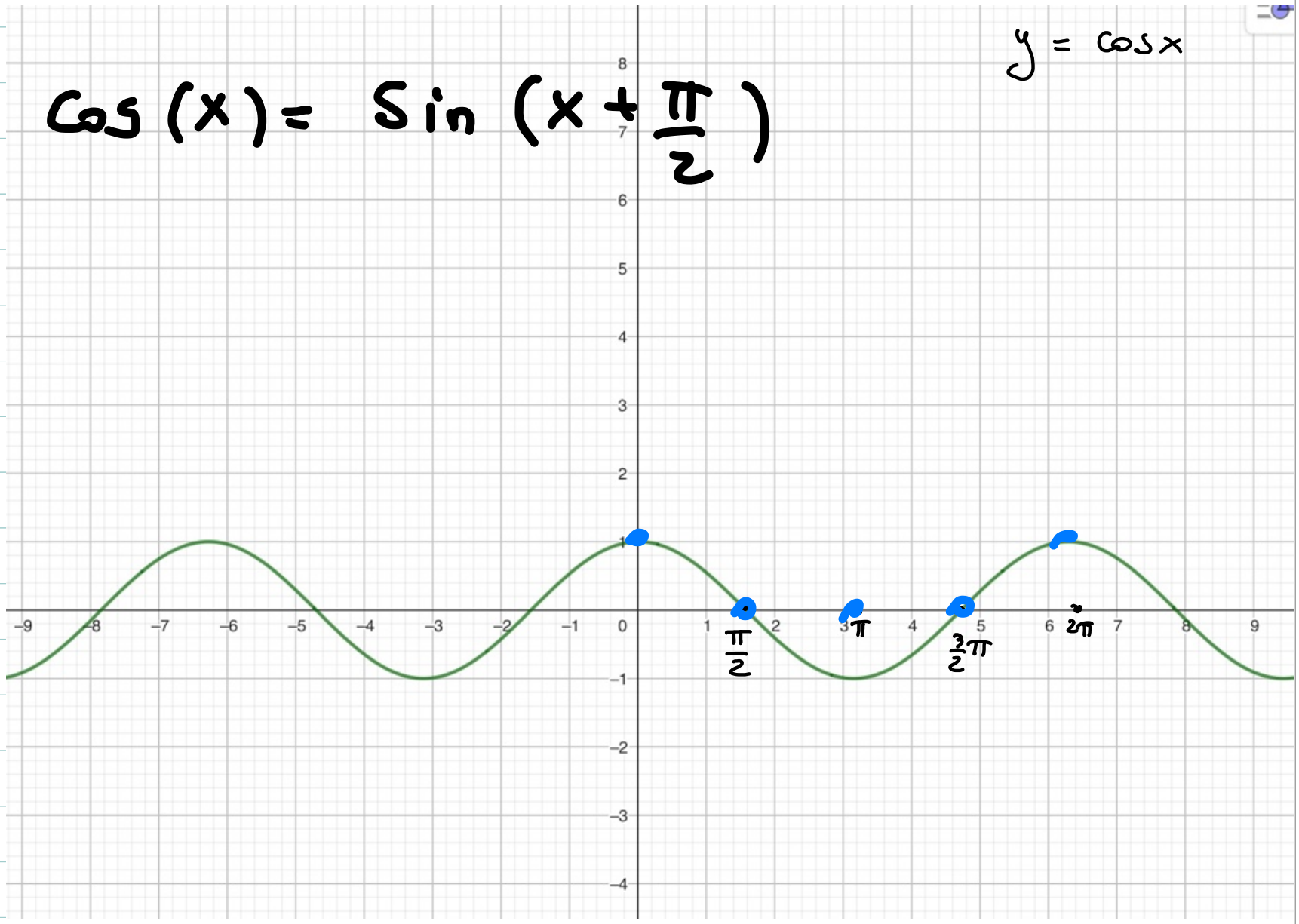
Trig functions

$\sin x$ $\cos x$ $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$ $\sec x = \frac{1}{\cos x}$ $\csc x = \frac{1}{\sin x}$


6:20 PM Sat Nov 19
100%
Today
6:20 PM
Edit
y = sin x
5
4
3
2
1
−1
−2
−3
−4
−5
−6
−9
−8
−7
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
7
8
9
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3}{2}\pi$
2π

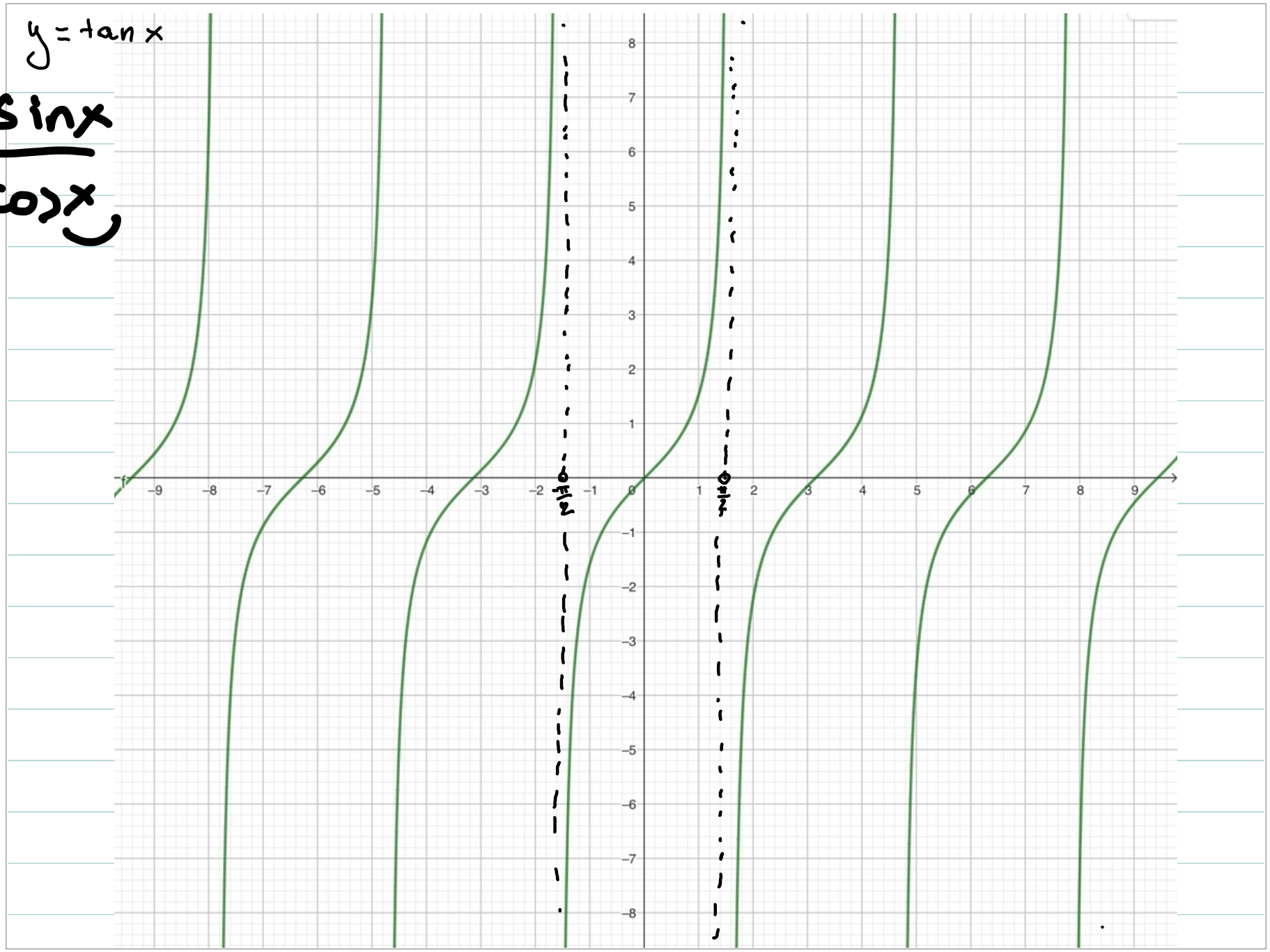
$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$

$y = \cos x$

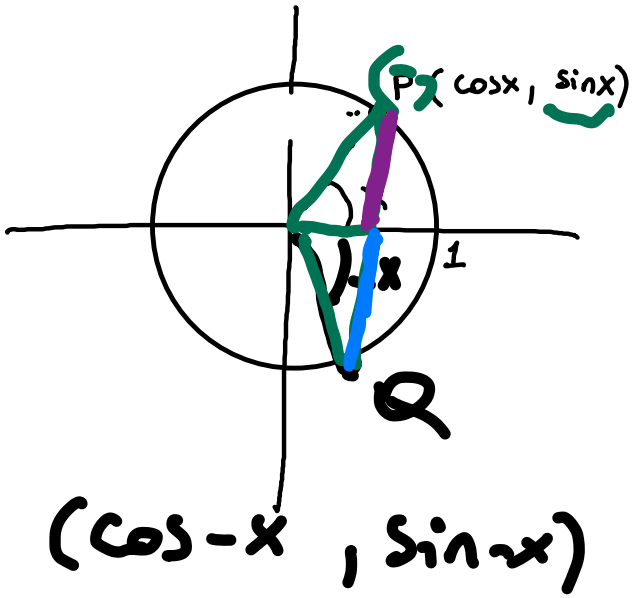


$y = \tan x$

$$\frac{\sin x}{\cos x}$$



useful trigonometric formulas



$$\sin^2 x + \cos^2 x = 1$$

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin(\pi + x) = -\sin x$$

$$\cos(\pi + x) = -\cos x$$

$$\sin(\pi - x) = \sin x$$

$$\cos(\pi - x) = -\cos x$$

harder to justify

$$\begin{cases} \sin(2x) = 2\sin x \cos x \\ \cos(2x) = 1 - 2\sin^2 x \end{cases}$$

$\sin^2\theta + \cos^2\theta = 1$

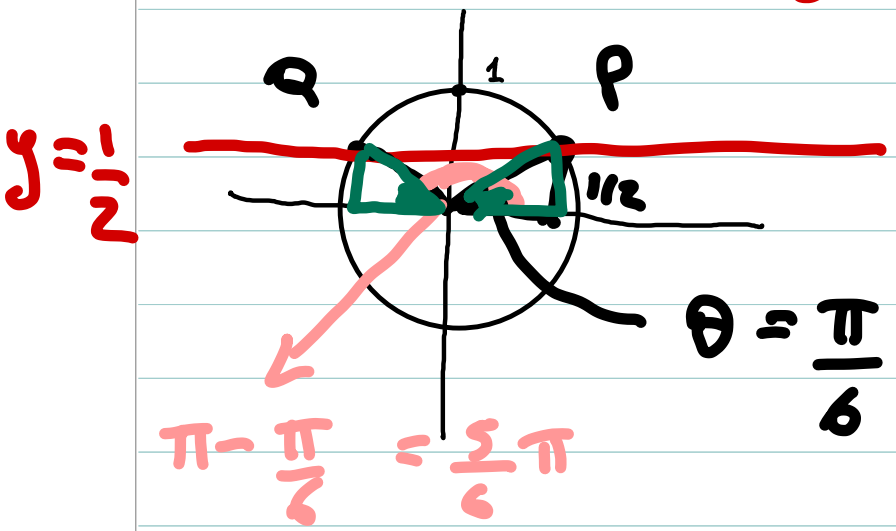
$\cos^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$

$\frac{1}{4} + \cos^2\theta = 1$

$\cos\theta = \pm\sqrt{3}/2$

If $\sin\theta = \frac{1}{2}$ what could $\cos\theta$ be?

y coordinate of a point P on the unit circle.



$\theta = \frac{\pi}{6}$

$\pi - \frac{\pi}{6} = \frac{5}{6}\pi$

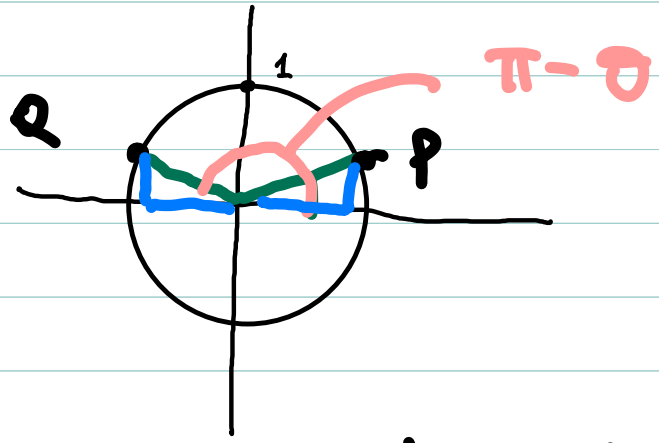
$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\cos\frac{5}{6}\pi = -\frac{\sqrt{3}}{2}$

What is $\sin(-\theta)$? $= -\frac{1}{2}$

What is $\cos(\pi - \theta)$ if θ is in the first quadrant?

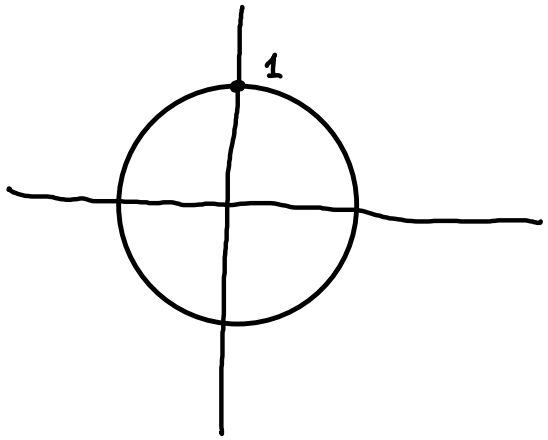
$\theta = \frac{\pi}{6}$

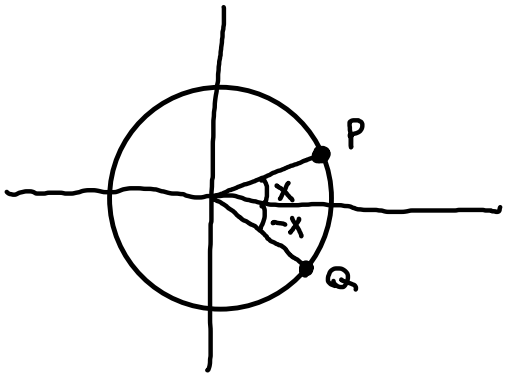


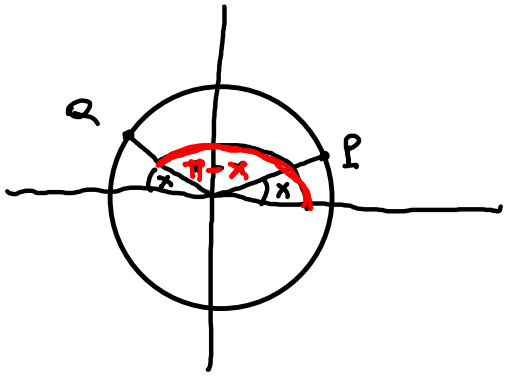
x coordinate of P is $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

x coordinate of Q $= -\frac{\sqrt{3}}{2}$

If $\sin x = \frac{1}{2}$ what could $\cos x$ be ?
What is $\sin(-x)$?
What is $\cos(\pi - x)$ if x is in the first quadrant ?
What could x be ?

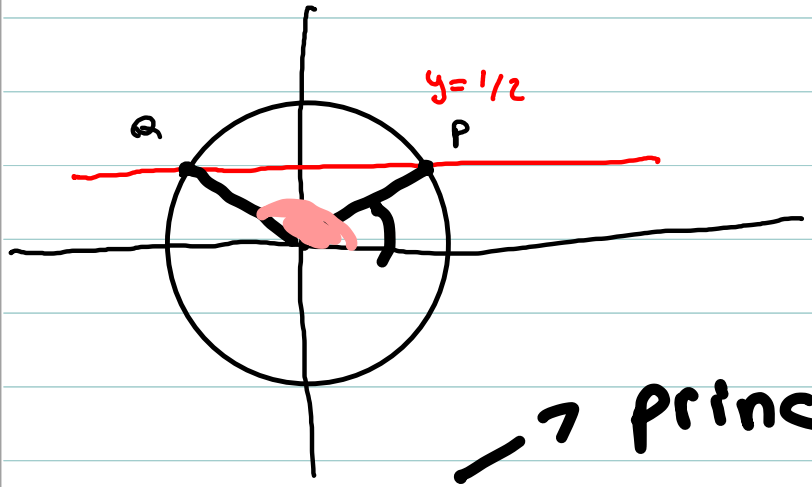


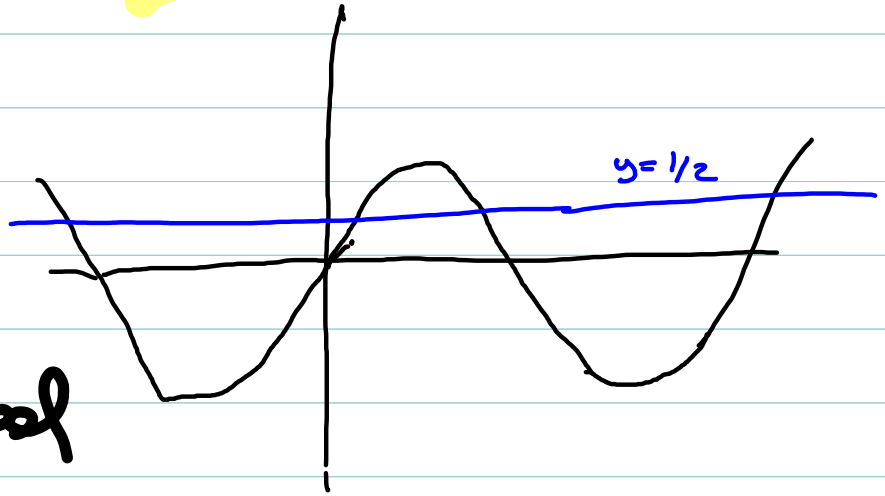




What is θ ?

Solve $\sin\theta = \frac{1}{2}$ or $\sin x = \frac{1}{2}$





principal

$$\theta = \frac{\pi}{6} + 2k\pi \qquad k = 0, \pm 1, \pm 2, \ldots$$

$$\theta = \pi - \frac{\pi}{6} + 2k\pi$$

symmetry