Lesson 22

Read Chapter 17

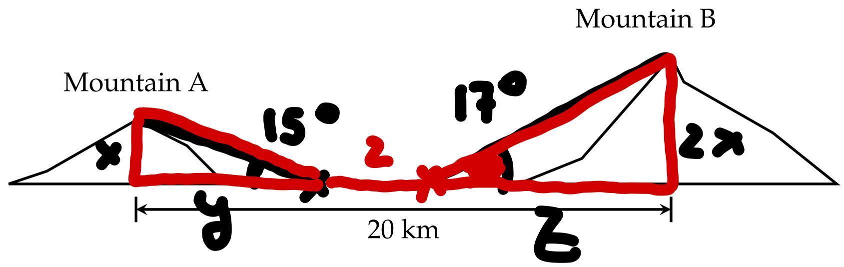
Trigonometry. Uniform circular motion.

4. You are on a road connecting the bases of Mountain A and Mountain B.

   You look at Mountain A and measure the angle of elevation to the top of Mountain A to be $15°$.

   You then travel 2 km toward Mountain B.

   You measure Mountain B's angle of elevation from your new location to be $17°$.



   Mountain A and Mountain B are 20 km apart as shown in the figure, and Mountain B is exactly twice as tall as Mountain A.

   What is the height of Mountain A?

$$\frac{x}{y} = \tan(15°)$$

$$\frac{2x}{z} = \tan(17°)$$

$$y + 2 + z = 20$$

$$z = 18 - y$$

$$\begin{cases} \dfrac{x}{y} = \tan(15°) \\ \dfrac{2x}{18-y} = \tan(17°) \end{cases} \qquad \begin{cases} x = y\tan(15°) \\ 2x = (18-y)\tan(17°) \end{cases}$$

$$2y\tan(15°) = 18\cdot\tan(17°) - y\tan(17°)$$

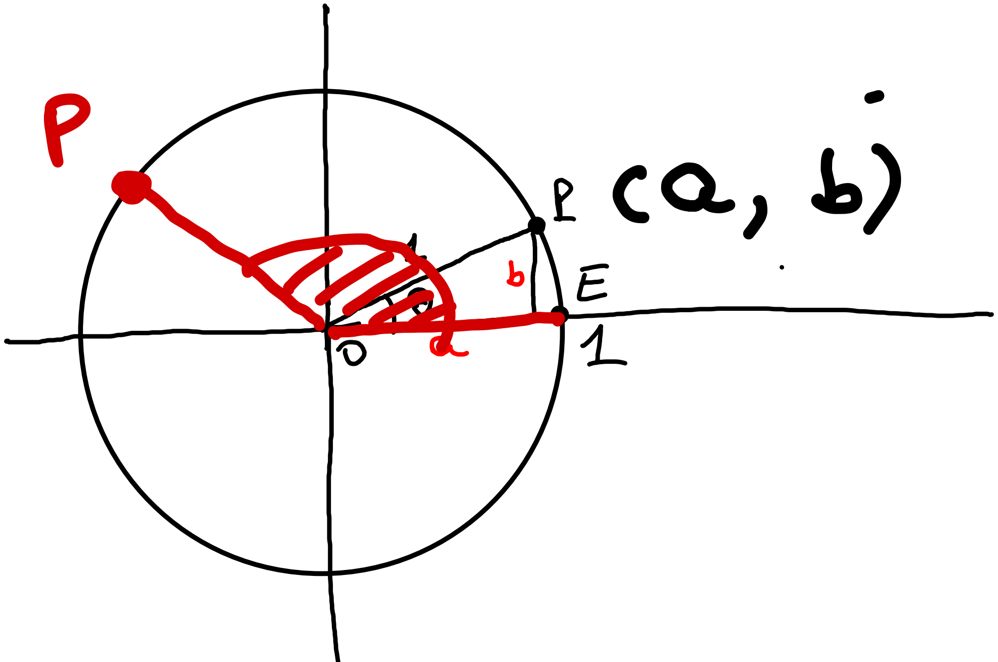
$$2y\tan(15°) + y\tan(17°) = 18\tan(17°)$$

$$y(2\tan(15°) + \tan(17°)) = 18\tan(17°)$$

$$y = \frac{18\tan(17°)}{2\tan(15°)+\tan(17°)}$$

$$x = \frac{18\tan(17°)}{2\tan(15°)+\tan(17°)}\cdot\tan(15°) \approx 1.75 \text{ km}$$

Trig definitions. Unit circle approach

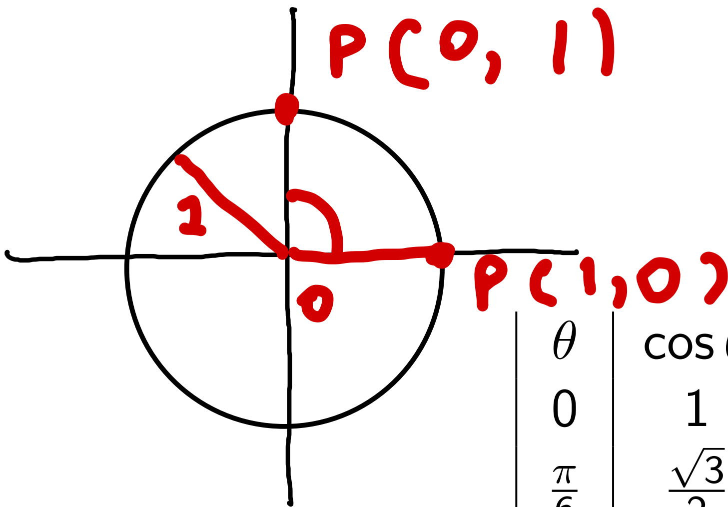


$$\cos\theta = \frac{a}{1} = a$$

$$\sin\theta = \frac{b}{1} = b$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

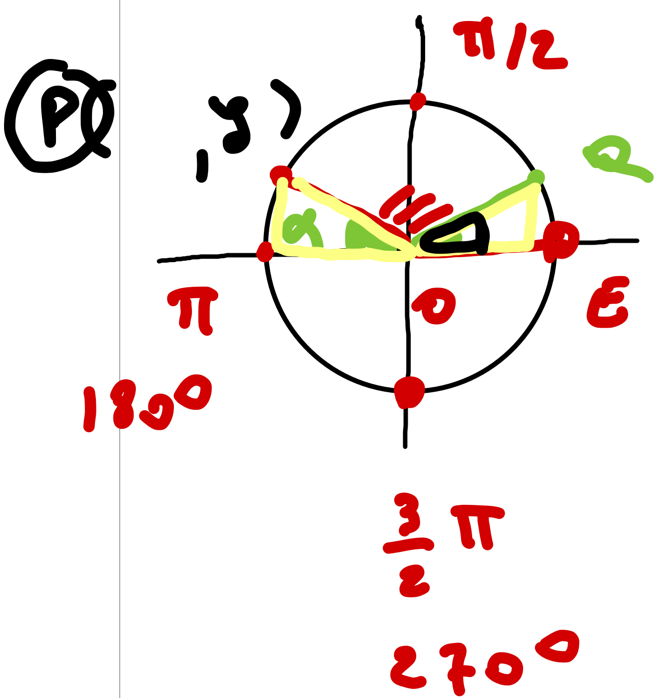
$\cos\theta$ and $\sin\theta$ are the x and y coordinates of the point P on the unit circle such that the radius OP forms an angle θ with the horizontal. This means : if the radius OE moves in the counterclockwise direction, it has to sweep an angle θ for E to reach P.



θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
0	1	0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	0	1	-

Compute $\sin(\frac{2}{3}\pi)$ and $\cos(-\frac{\pi}{6})$

$$\alpha = \pi - \frac{2}{3}\pi = \frac{\pi}{3}$$



90°

$\left(- \,,\ \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \right)$

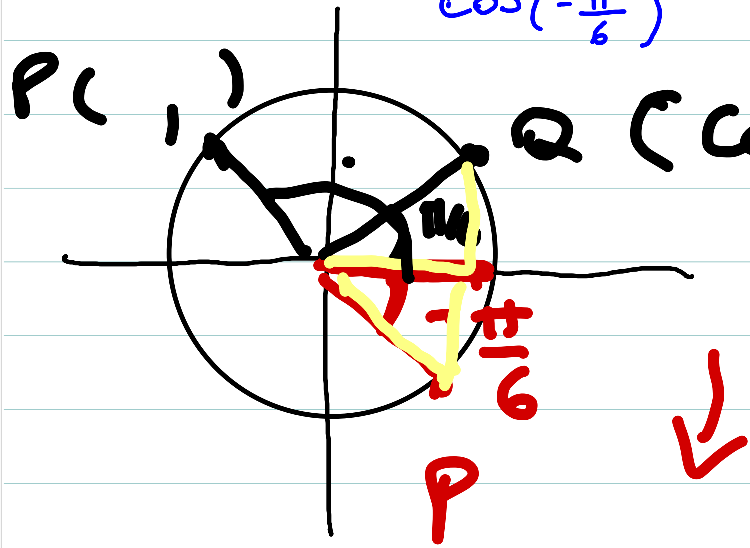
2π 360

$\sin(\frac{2}{3}\pi)$ is y coordinate of P which is equal as y coordinate of Q which is $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{2}{3}\pi\right)$ is x coordinate of P which is $-$ x coordinate of Q which is $\cos\frac{\pi}{3} = \frac{1}{2}$

$$\cos\left(\frac{2}{3}\pi\right) = -\frac{1}{2}$$

$\cos(-\frac{\pi}{6})$



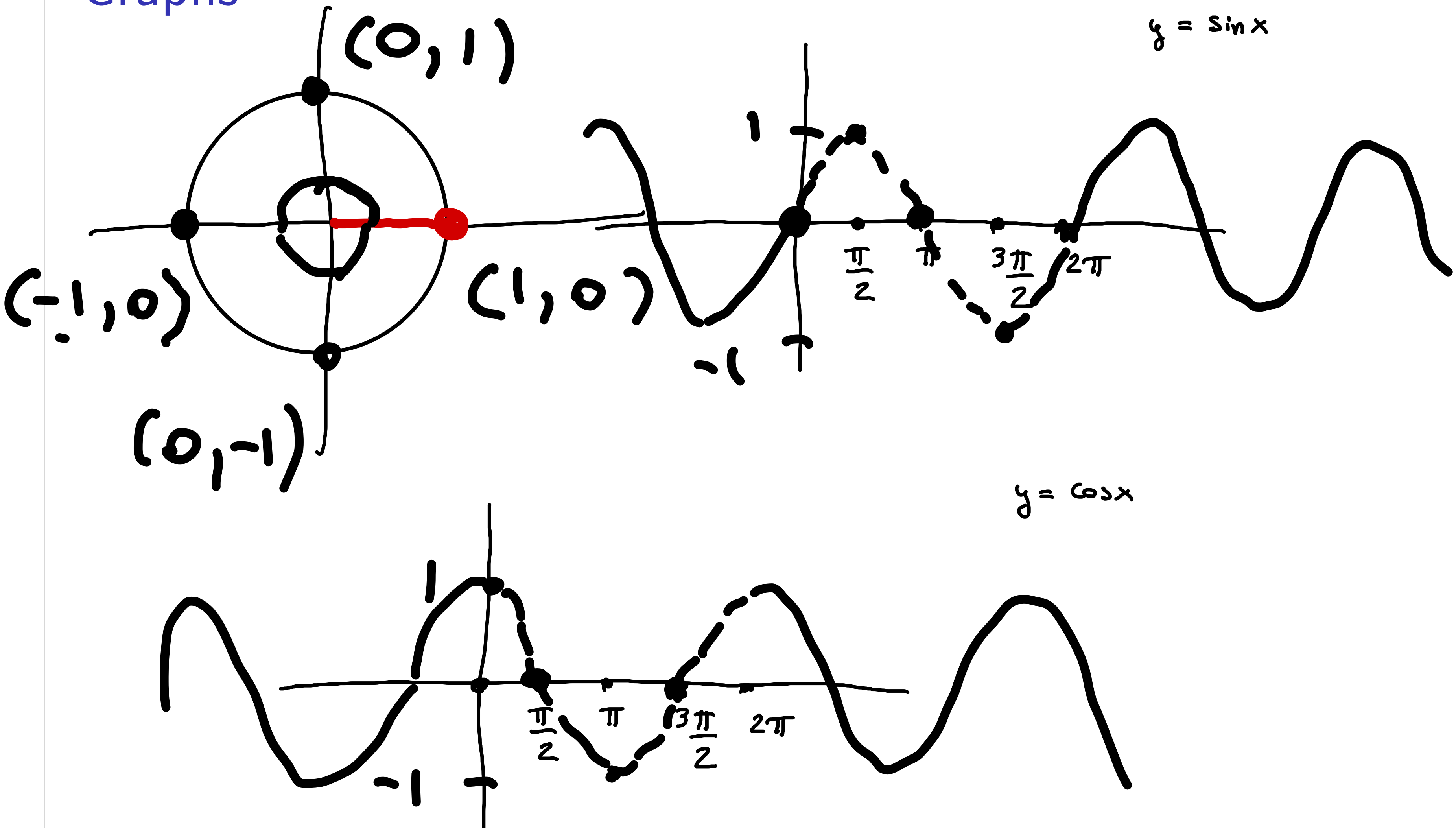
$Q\left(\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2},\ - \quad\right)$

$\cos(-\frac{\pi}{6})$ = x coordinate of P

= x coordinate of Q which

is $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Graphs

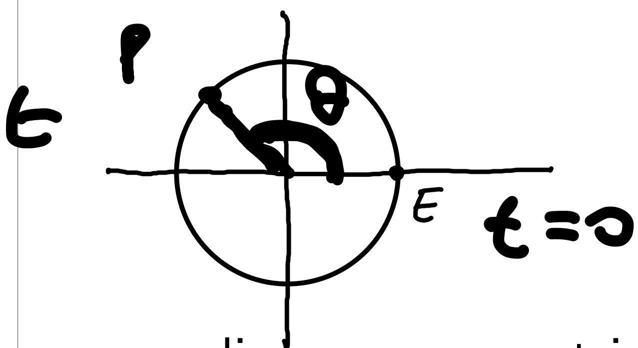


x is angle in radians

Suppose an object moves around a circle with angular velocity ω, starting at E

$$\theta = \underbrace{\omega t}$$

radius $=1$, parametric equations are :

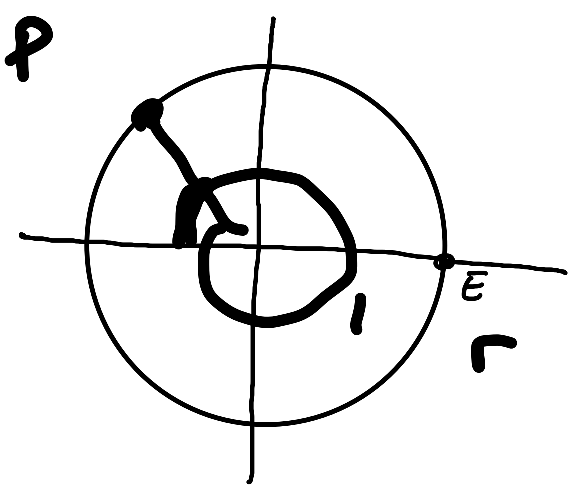


$$P(x(t) \quad y(t))$$

$$x(t) = \cos(\omega t)$$

$$y(t) = \sin(\omega t)$$

radius $=$r, parametric equations are :

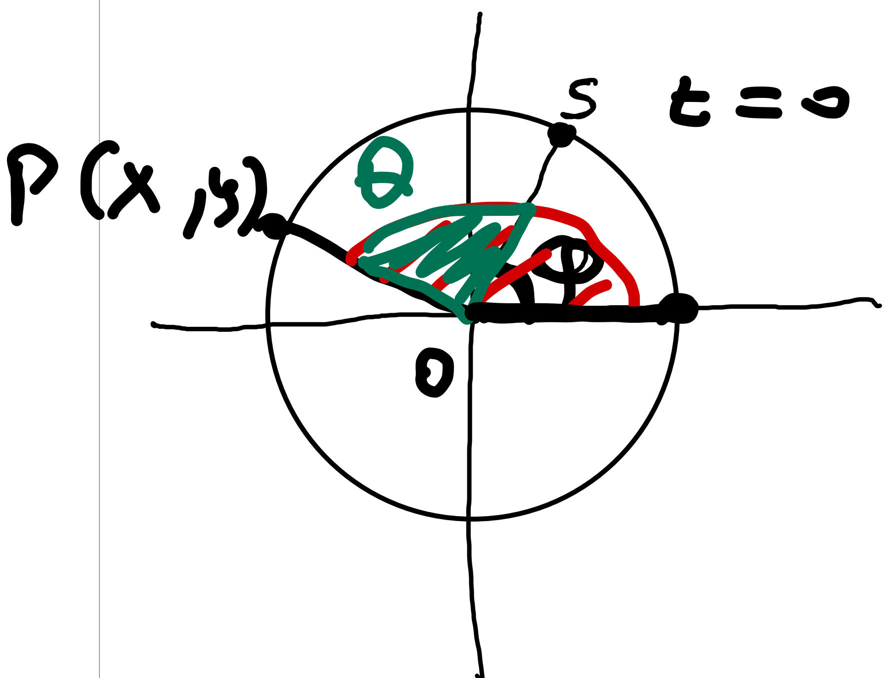


$$x(t) = r\cos(\omega t)$$

$$y(t) = r\sin(\omega t)$$

Suppose an object moves around a circle with angular velocity ω, starting at S

radius $=$r, parametric equations are :



coordinates of P are

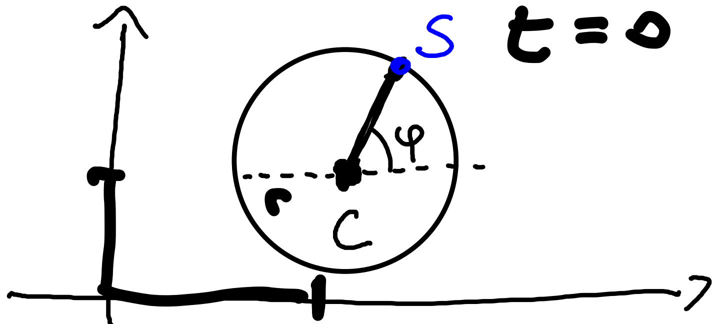
$$x(t) = r\cos(\omega t + \varphi$$

$$y(t) = r\sin(\omega t + \varphi)$$

$$\theta = \omega t$$

$$\theta + \varphi$$

Parametric equations of motion for uniform circular motion



The parametric equation of motion of an object that moves on a circle of radius r centered at $\underline{C(x_0, y_0)}$ with angular velocity ω and starting at S are

$$x(t) = x_0 + r\cos(\omega\, t + \phi)$$

$$y(t) = y_0 + r\sin(\omega\, t + \phi)$$

Spring 2012 Final

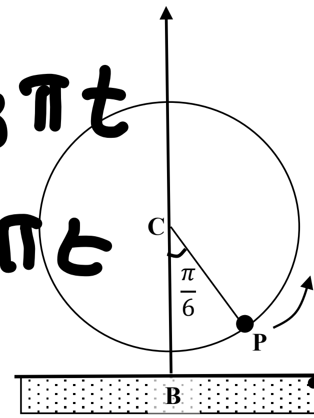
$C(0, 52)$

Problem 6. (16 pts) Percy is riding on a ferris wheel of radius 50 feet, whose center C is 52 feet above ground. The wheel rotates at a constant rate in the direction shown by the arrow, taking 1.5 minutes for each full revolution. The wheel starts turning when Percy is at the point P, making an angle of $\frac{\pi}{6}$ radians with the vertical, as shown. *(Make sure your calculator is in radian mode)*

a) (5 pts) How high is Percy above ground when the wheel starts turning?

$x(t) = 0 + 50\cos(4/3\,\pi t$

$y(t) = 52 + 50\sin(\frac{4}{3}\pi t$



$t = 0$

x

b) ~~(4 pts) Impose a coordinate system **with the origin at the base point B**. What is the equation of the line CP?~~

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.5} = \frac{4\pi}{3}\ \frac{\text{rad}}{\text{min}}$$

c) (7 pts) Percy drops his ice cream cone 1.25 minutes after the wheel starts moving. If the cone falls straight down from Percy's position at that time, where does it land with respect to the base point B?

We will finish next time