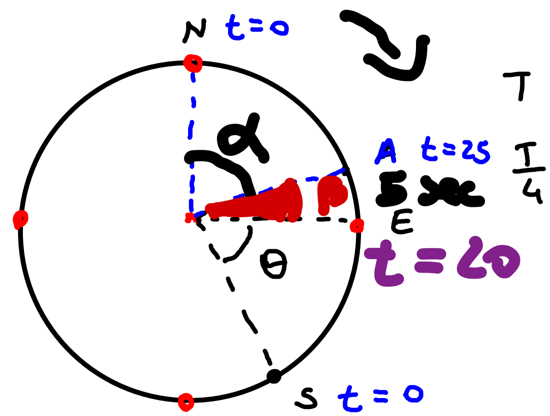
Lesson 21

Read Chapter 17

Trigonometric functions. Triangle definition.

Problems involving two triangles

Two objects move around a circle . They start at the same time. Object 1 moves in the counterclockwise direction, with angular speed of $\frac{\pi}{50}$ rad/sec; from where it starts it takes it 20 seconds to reach the easternmost part of the track. Object 2 moves in the clockwise direction, starting from the northernmost part of the track 's with a speed of 4 feet / sec . The two objects pass each other after 25 sec. What is object 1's starting position ? (Give your answer as an angle). What is the radius of the track ?



$T = \frac{2\pi}{\omega} = 100$ sec

$\frac{T}{4} = 25$

$\theta = \omega t$

$\theta = \frac{\pi}{50} \cdot 20 = \frac{2}{5}\pi$ rad

$$\boxed{\theta = -\frac{2\pi}{5}}$$

$V_2 = r\,\omega_2$

Find ω_2

$\frac{V_2}{\omega_2} = r$

$\frac{4}{\omega_2} = r$

$\alpha = \omega_2 \cdot 25$

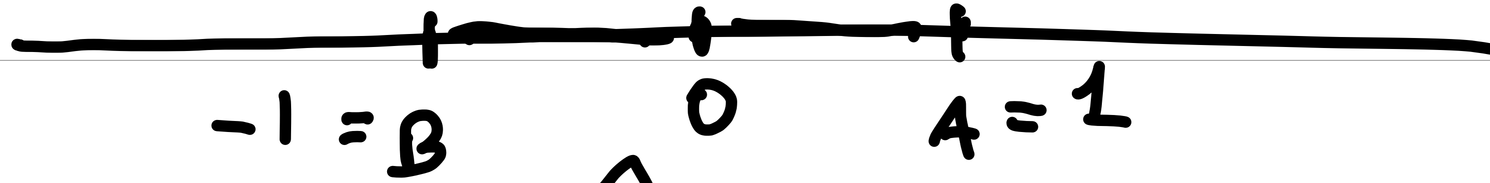
Want α. $\qquad \beta = \frac{\pi}{50}$. $S = \frac{\pi}{10}$

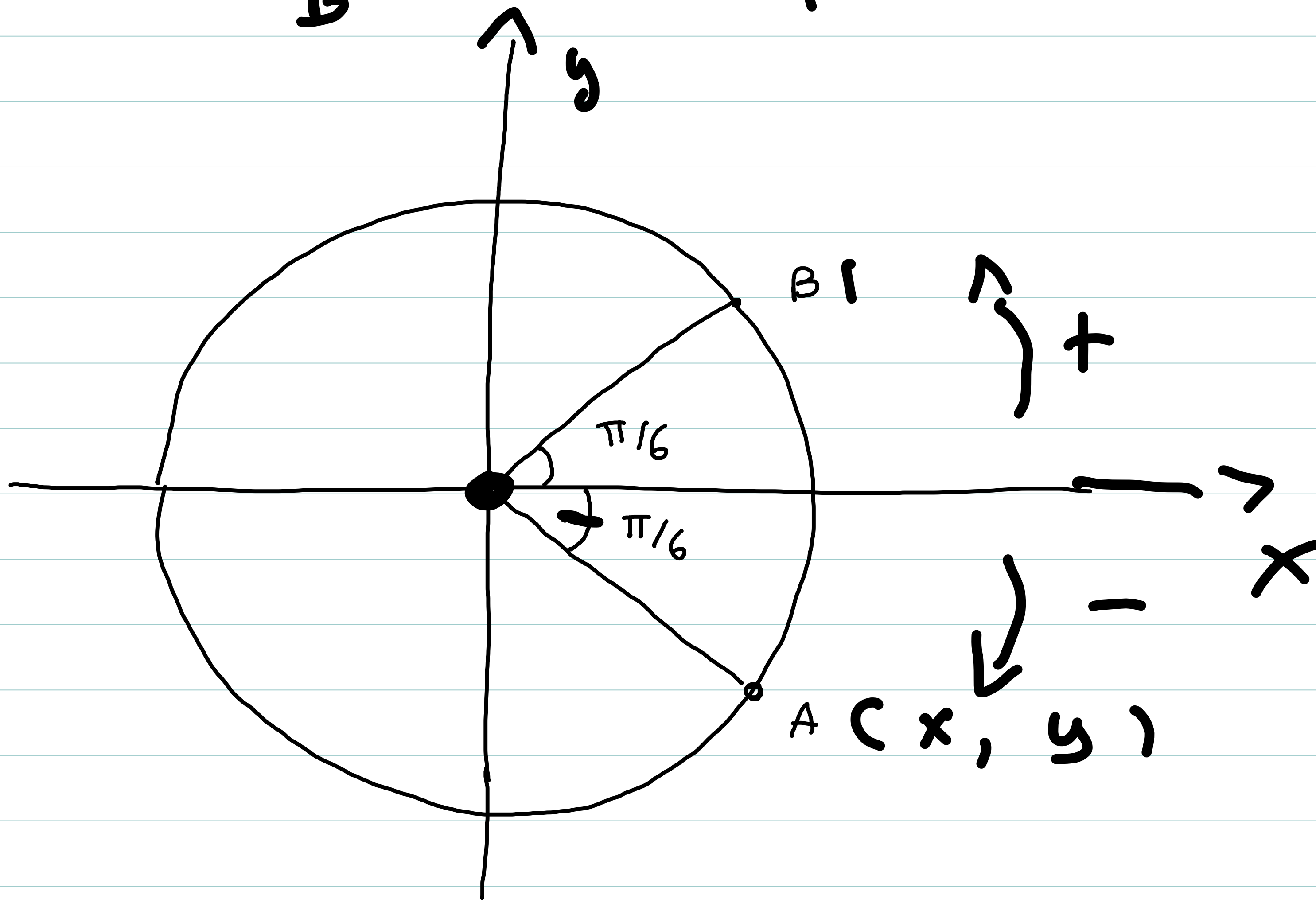
$$\alpha = \frac{\pi}{2} - \beta = \frac{\pi}{2} - \frac{\pi}{10} = \frac{4}{10}\pi$$

$$= \frac{2\pi}{5}.$$

$$\omega_2 = \frac{\alpha}{25} = \frac{2\pi}{5 \cdot 25}$$

$$r = \frac{4}{\frac{2\pi}{125}} = \frac{2\cancel{4} \cdot 125}{\cancel{2}\pi} \text{ feet.}$$

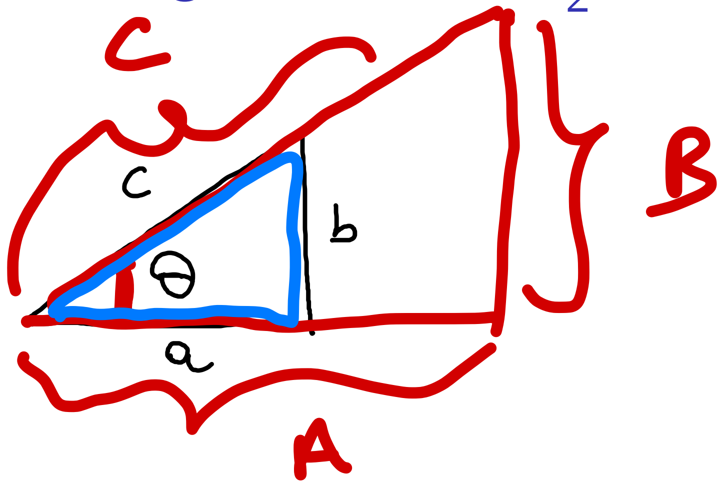




1) How do we distinguish the two angles?

2) What are the coordinates of A and B?

Trig for angles $0 < \theta < \frac{\pi}{2}$



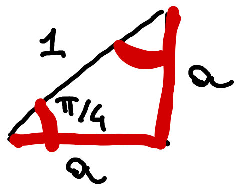
$$\sin\theta = \frac{b}{c} = \frac{B}{C}$$

$$\cos\theta = \frac{a}{c}$$

$$\tan\theta = \frac{b}{a} = \frac{\sin\theta}{\cos\theta} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{b}{\cancel{c}} \cdot \frac{\cancel{c}}{a}$$

$45^\circ \quad 60^\circ \quad 30^\circ$

$$\theta = \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}$$



$$a^2 + a^2 = 1 \; ; \; 2a^2 = 1 \; ; \; a^2 = \frac{1}{2} \; ; \; a = \frac{1}{\sqrt{2}}$$

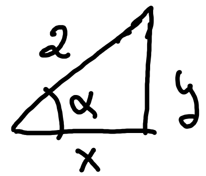
$$\sin\frac{\pi}{4} = \frac{a}{1} = \frac{1}{\sqrt{2}} \qquad \cos\frac{\pi}{4} = \frac{a}{1} = \frac{1}{\sqrt{2}} \qquad \tan\frac{\pi}{4} = \frac{a}{a} = 1$$

$$h^2 + \left(\frac{1}{2}\right)^2 = 1 \; ; \; h^2 = 1 - \frac{1}{4} \; ; \; h^2 = \frac{3}{4} \; ; \; h = \frac{\sqrt{3}}{2}$$

$$\sin\frac{\pi}{3} = \frac{h}{1} = \frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3} = \frac{\frac{1}{2}}{1} = \frac{1}{2} \qquad \tan\frac{\pi}{3} = \frac{h}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1}$$

$$\sin\frac{\pi}{6} = \frac{1}{2} \qquad \cos\frac{\pi}{6} = h = \frac{\sqrt{3}}{2} \qquad \tan\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$$

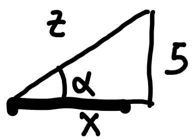
If $\alpha = 1.3$ rad, find x and y



$$\cos(1.3) = \frac{x}{2} \qquad \underbrace{2 \cdot \cos(1.3)}_{\text{use calculator}} = x$$

$$\sin(1.3) = \frac{y}{2} \quad = \quad 2\sin(1.3) = y$$

If $\alpha = 0.5$ rad, find x and z



$$\tan(0.5) = \frac{5}{x} \qquad x \tan(0.5) = 5$$

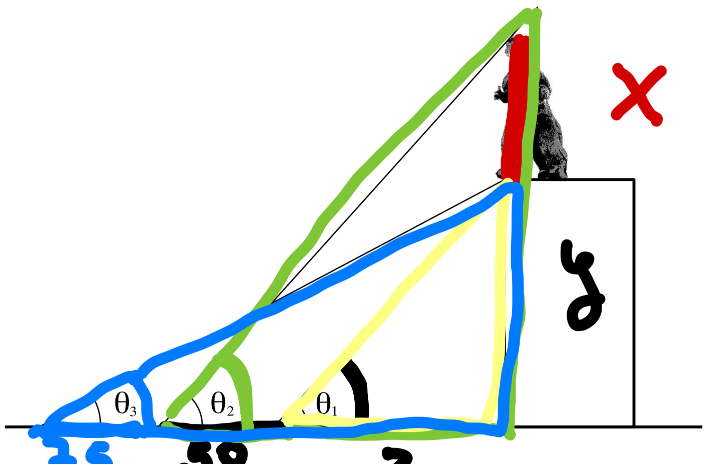
$$x = \frac{5}{\tan(0.5)}$$

$$\sin(0.5) = \frac{5}{z} \qquad z \cdot \sin(0.5) = 5$$

$$z = \frac{5}{\sin(0.5)}$$

3. Godzilla is attacking, but at the moment he is standing on top of a building downtown. You want to determine Godzilla's height, so you measure three angles. First, from a certain distance away from the building, you measure the angle the top of the building makes with the horizontal: $\theta_1 = 72°$. You then move 50 meters farther from the building and measure the angle Godzilla's head makes with the horizontal: $\theta_2 = 74°$. You then move 75 meters farther from the building and measure the angle the top of the building makes with the horizontal: $\theta_3 = 60°$.

The figure may not be to scale.



How tall is Godzilla?

$$\frac{y}{z} = \tan(72°)$$

$$\frac{x+y}{z+50} = \tan(74°)$$

$$\frac{y}{z+50+75} = \tan(60°)$$

$$\begin{cases} \dfrac{y}{z} = \tan(72^\circ) \\[2ex] \dfrac{x+y}{z+50} = \tan(74^\circ) \\[2ex] \dfrac{y}{z+125} = \tan(60^\circ) \end{cases}$$

$$\begin{cases} y = z\tan(72^\circ) \\[1ex] x+y = (z+50)\tan(74^\circ) \\[1ex] y = (z+125)\tan(60^\circ) \end{cases}$$

$$z\tan(72^\circ) = z\tan(60^\circ) + 125\tan(60^\circ)$$

$$z\left(\tan(72^\circ) - \tan(60^\circ)\right) = 125\tan(60^\circ)$$

$$z = \frac{125\tan(60^\circ)}{\tan(72^\circ) - \tan(60^\circ)} \approx 160.9$$

$$y = 160.9\tan(72^\circ) \approx 495.21$$

$$x = -495.21 + (160.9 + 50)\tan(74^\circ) \approx 240$$