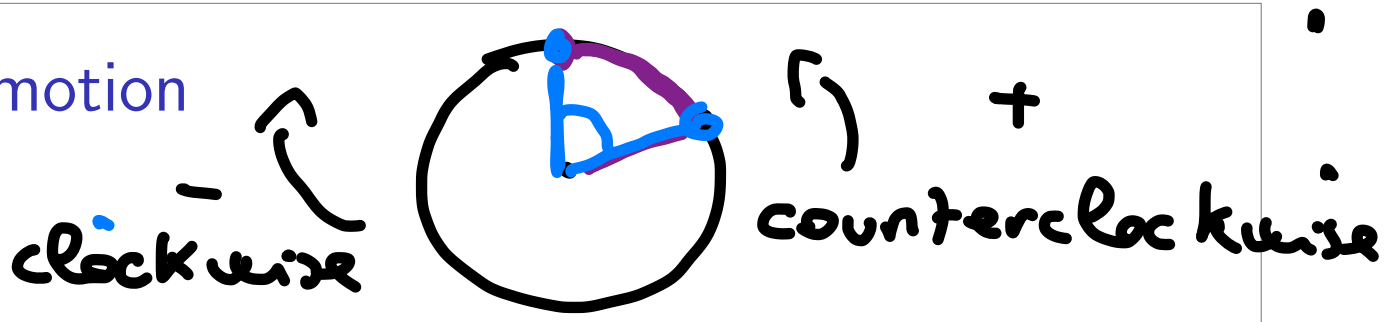
Lesson 20

Read Chapter 16

Circular motion

Uniform circular motion



We consider an object going around a circle with constant speed.

T. Period: time it takes to go around circle once

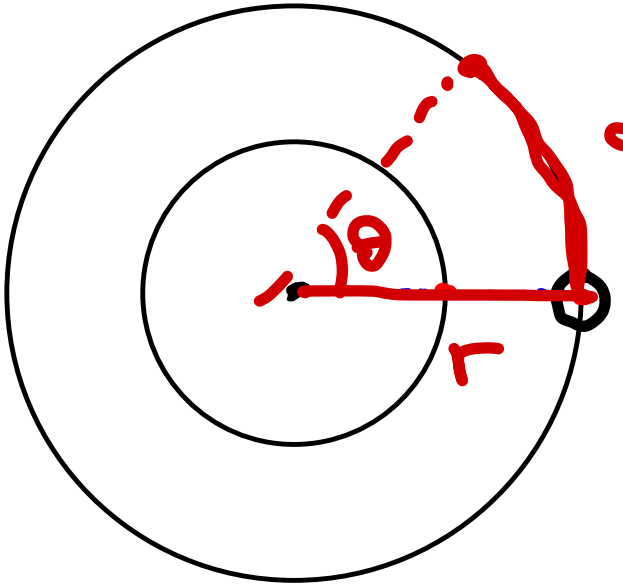
ω. Angular speed/velocity, measures how fast angles are swept. Units rad/time, RPM

v. Linear speed/velocity, measures how fast distances are covered. Units distance/time

Uniform circular motion formulas

1. $d = vt$
2. $\theta = \omega\, t$
3. $\omega = \frac{2\pi}{T}$
4. $T = \frac{2\pi}{\omega}$
5. $v = \omega\, r$
6. $1RPM = 2\pi$ rad /min

$2\pi = \omega \cdot T$

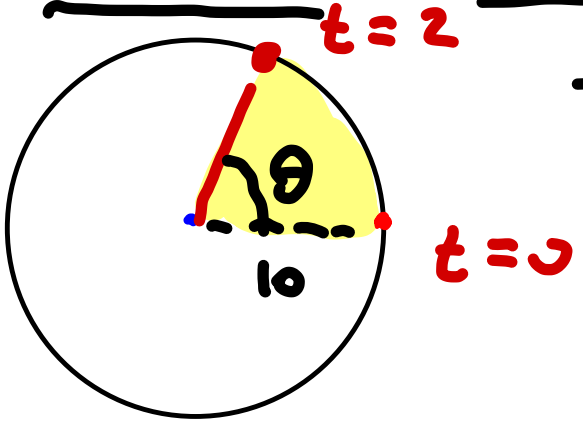


$d = r\theta$

$r\theta = vt$

$r\omega t = vt$

A rotating sprinkle reaches 10 m far and completes a full revolution in 5 min. How much area does it irrigate in 2 min ? How long does it take the sprinkle to irrigate 50 square meters ?



$T = 5$ min $\quad \omega = \frac{2\pi}{5}$ rad/min

Formula from last week

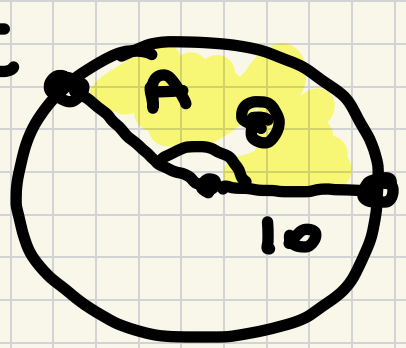
$A = \frac{1}{2}\theta r^2$

1) Need θ $\quad \theta = \omega \cdot t$

$\theta = \frac{2\pi}{5} \cdot 2 = \frac{4\pi}{5}$

$$A = \frac{1}{2} \cdot \frac{4\pi}{5} \cdot 10^2 \quad m^2$$

2)



$t = 0$

$A = 50$

find t

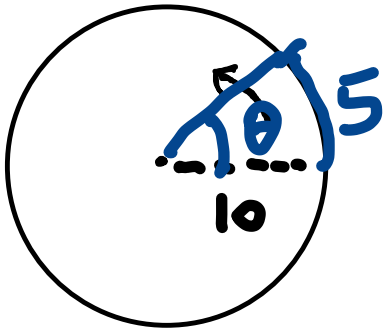
$$A = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2} r^2 \omega t$$

$$50 = \frac{1}{2} \cdot 10^2 \cdot \frac{2\pi}{5} \cdot t$$

$$\frac{5}{2\pi} = t \quad min$$

An object moves around a circle of radius 10 ft with $\omega = 12$ RPM. Find its linear velocity in feet/sec. How many radians does the object turn in 3 sec ? What distance does it cover in 3 min ? How long does it take the object to move 3 feet ? How long does it take the object to turn an angle of 45° ? When the object has moved a distance of 5 feet , what angle has it turned ?



$$\omega = 12 \cdot 2\pi \text{ rad/min}$$

$$\omega = \frac{24\pi}{60} \text{ rad/sec}$$

① $v = \omega r$

$$v = \frac{24}{60}\pi \cdot 10 = 4\pi \text{ feet/sec}$$

② $\theta = \omega \cdot t$

$$\theta = \frac{24\pi}{60} \cdot 3 \quad \text{rad}$$

(3) $d = v \cdot t$

$d = 4\pi \cdot 3 \cdot 60$ feet

(4) $d = vt$

$3 = 4\pi t$

$\frac{3}{4\pi} = t$ sec

(5) $\theta = \omega t$

$$45^\circ \cdot \frac{2\pi}{360} = \frac{\pi}{4}$$

$$\frac{\pi}{4} = 24\pi \cdot t$$

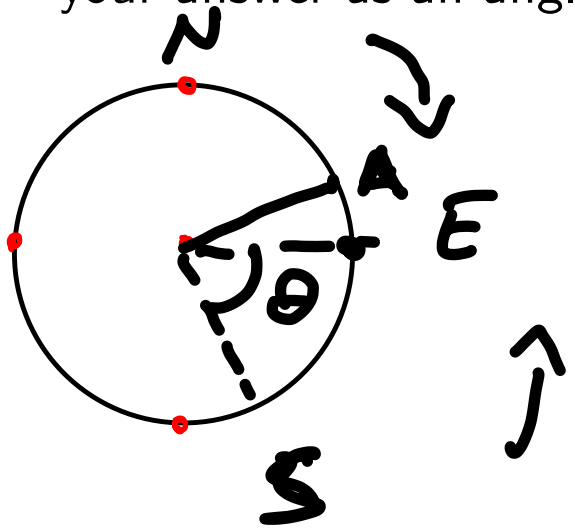
$$\frac{1}{4 \cdot 24} = t \quad \text{minutes}$$

(6) $d = r \cdot \theta$

$5 = 10 \cdot \theta$

$\frac{5}{10} = \frac{1}{2} = \theta$ rad

Two objects move around a circle . They start at the same time. Object 1 moves in the counterclockwise direction, with angular speed of $\frac{\pi}{50}$ rad/sec; from where it starts it takes it 20 seconds to reach the easternmost part of the track. Object 2 moves in the clockwise direction, starting from the northernmost part of the track 's with a speed of 4 feet / sec . The two objects pass each other after 25 sec. What is object 1's starting position ? (Give your answer as an angle). What is the radius of the track ?



$\omega_1 = \frac{\pi}{50}$ rad/sec

$T_1 = \frac{2\pi}{\frac{\pi}{50}} = 100$ sec

$\theta = \frac{\pi}{50} \cdot 20 = \frac{2}{5}\pi$ rad

$V_2 = G \, dt \, xc$

Finish next time