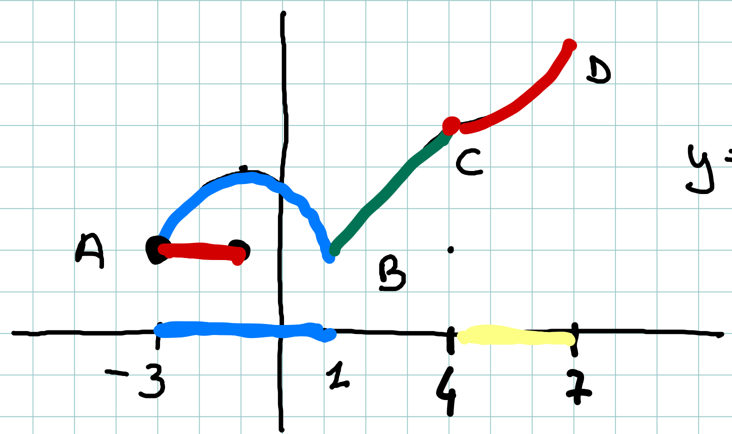
Lesson 18

Chapter 13

Midterm review



$y = f(x)$

A $(-3, 2)$

B $(1, 2)$

C $(4, 5)$

D $(7, 7)$

$\left.\begin{array}{l} \text{B} \\ \text{C} \end{array}\right\} y = x + 1$

AB semicircle

BC line

CD arc of parabola with vertex at C

Write multipart formula

a) $y = y_0 + \sqrt{r^2 - (x - x_0)^2}$

$(x_0\ y_0)$ center $(-1, 2)$

r radius 2

diameter $= 1 + 3 = 1 - (-3) = 4$

radius $= 4/2 = 2$

b) $y = y_0 + m(x - x_0)$

$$y = 2 + \frac{5-2}{4-1}(x-1) = 2 + x - 1$$

c) $y = a(x-4)^2 + 5$

$$7 = a(7-4)^2 + 5$$

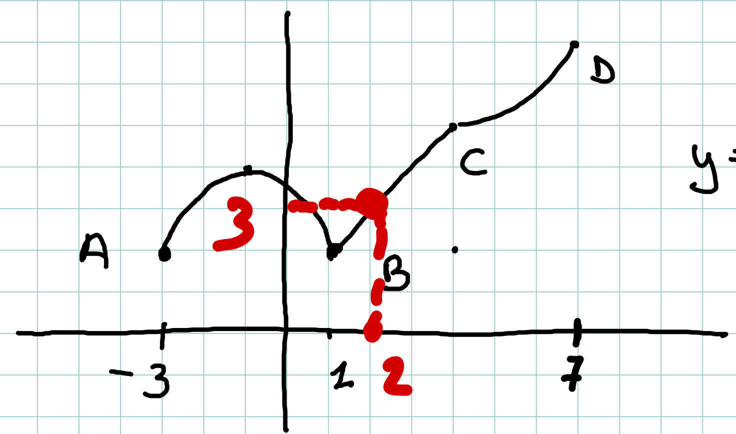
$$7 = a \cdot 9 + 5$$

$$\frac{2}{9} = a$$

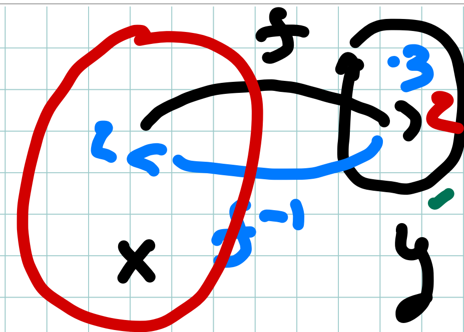
center $x_0 = -3 + 2 = -1$

$y_0 = 2$

$$f(x) = \begin{cases} 2 + \sqrt{4 - (x+1)^2} & \text{if } -3 \le x \le 1 \\ x + 1 & \text{if } 1 < x \le 4 \\ \frac{2}{9}(x-4)^2 + 5 & \text{if } 4 < x \le 7 \end{cases}$$



$y = f(x)$



$f(x)$ on $[1, 7]$

Find $f^{-1}(3) = 2$

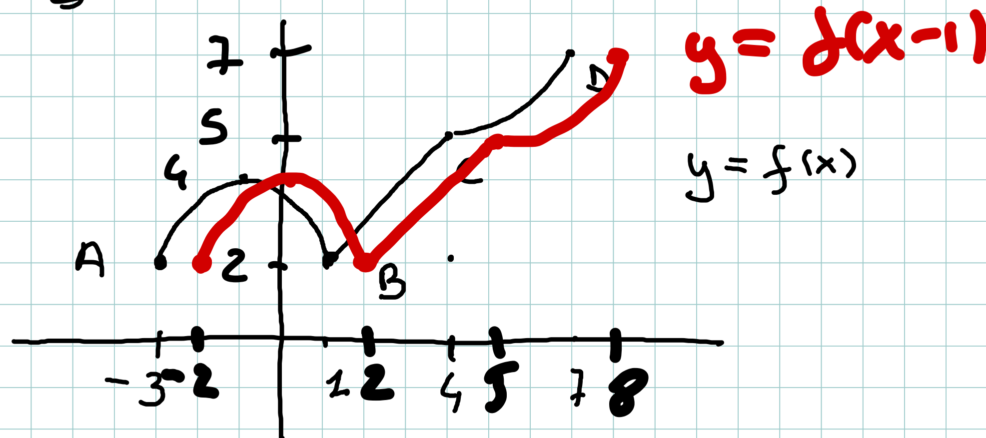
Domain of f^{-1} = Renge of f $[2, 7]$

Renge of f^{-1} = Domain of f $[1, 7)$

Find $f(\underbrace{f(1)}) = f(2) = 3$

graph $y = -2f(3x-1) + 5$ [redacted]

[redacted]



horizontal

a) shift

$f(x) \to f(x-1)$

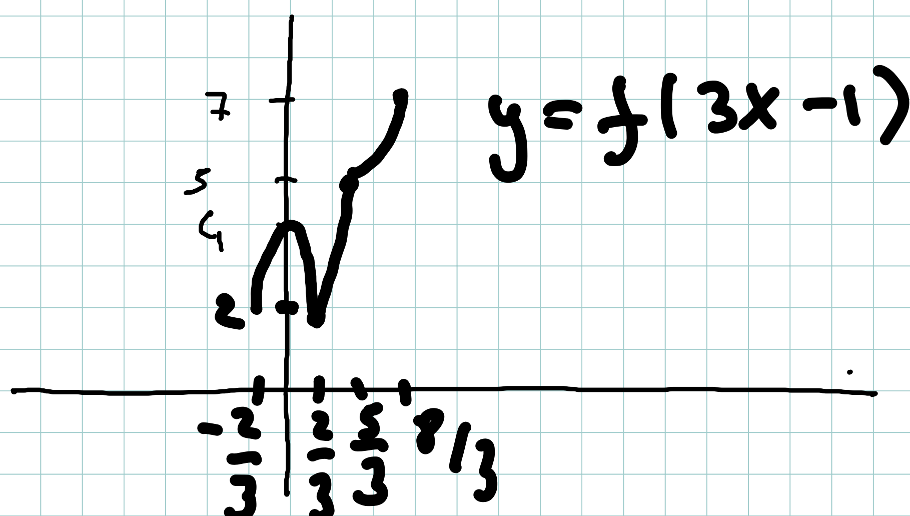
shift right 1 unit

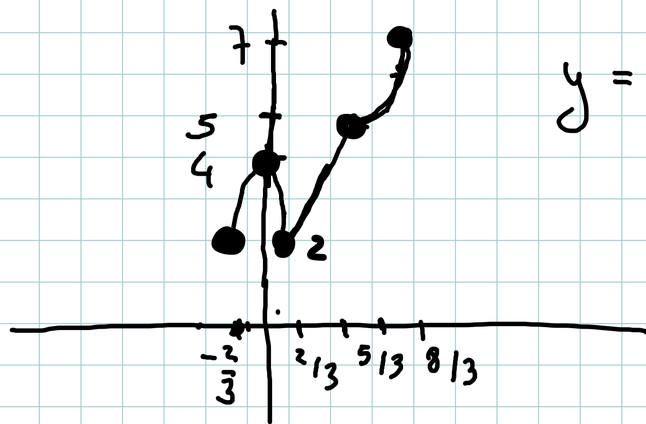
b) scaling

$f(x-1) \to f(3x-1)$

horizontally of factor $c = \frac{1}{3}$

$3x = \frac{x}{\frac{1}{3}}$



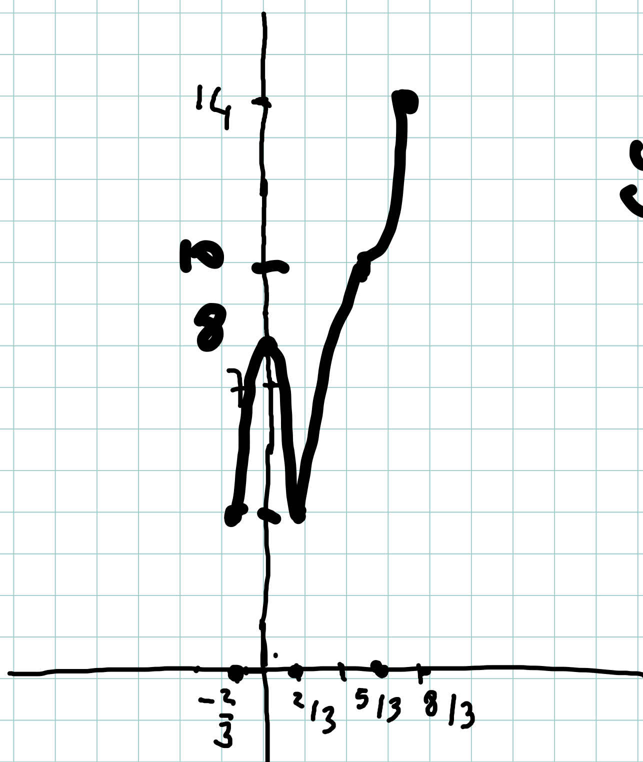


$y = f(3x-1)$

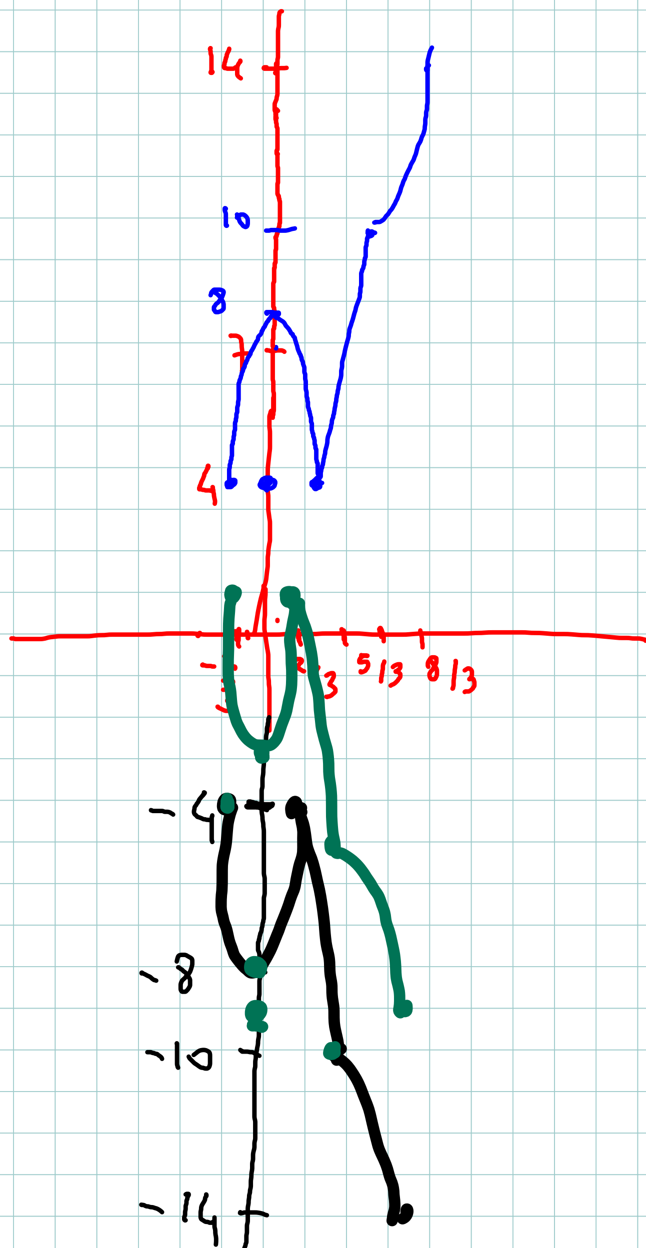
vertical

$y = -2f(3x-1)+5$

a) scaling of a factor $c=2$



$y = 2f(3x-1)$



$y = 2f(3x-1)$

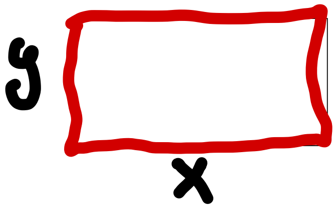
$y = -2f(3x-1)+5$

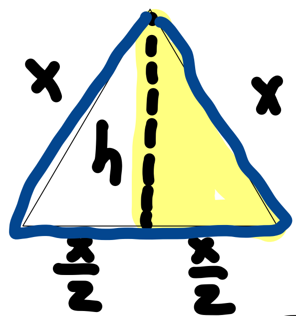
$y = -2f(3x-1)$

$$f(x) = \begin{cases} 2 + \sqrt{4 - (x+1)^2} & \text{if } -3 \leq x \leq 1 \\ x - 1 & \text{if } 1 < x \leq 5 \\ -\frac{2}{9}(x-4)^2 + 5 & \text{if } 5 < x \leq 7 \end{cases}$$

$$y = -2f(3x-1) + 5$$

1. You want to build two enclosures using exactly 3000 feet of fencing. One enclosure will be an equilateral triangle, the other a rectangle. The basis of the rectangle has the same length as the sides of the triangle. What should the side of the triangle be in order to maximize the area of the two combined enclosures ?





$$h^2 + \left(\frac{x}{2}\right)^2 = x^2$$

$$h^2 = x^2 - \frac{x^2}{4} = \frac{3}{4}x^2$$

$$h = \sqrt{\frac{3}{4}x^2} = \frac{\sqrt{3}}{2}x$$

$$A = x \cdot y + \frac{1}{2} \cdot \underbrace{x}_{b} \cdot \underbrace{\frac{\sqrt{3}}{2}x}_{h}$$

$$A = xy + \frac{\sqrt{3}}{4}x^2$$

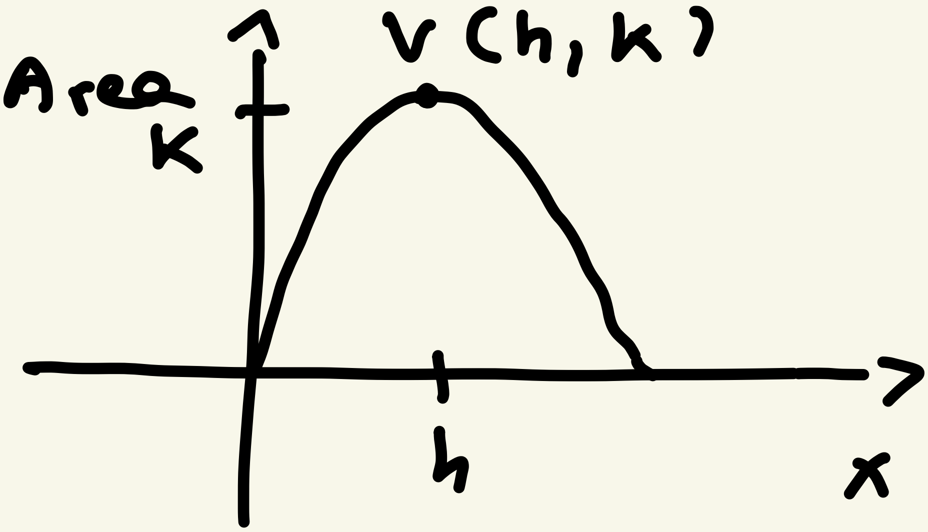
$$3000 = 2x + 2y + 3x = 5x + 2y$$

$$\frac{3000 - 5x}{2} = y$$

$$A = x\,\frac{3000 - 5x}{2} + \frac{\sqrt{3}}{4}x^2$$

$$A = 1500x - \frac{5}{2}x^2 + \frac{\sqrt{3}}{4}x^2$$

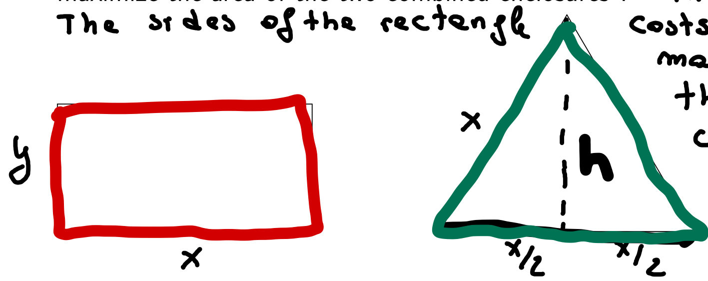
$$= \underbrace{\left(\frac{\sqrt{3}}{4} - \frac{5}{2}\right)}_{a<0} x^2 + \underbrace{1500}_{b} x$$



The length of the side of triangle that maximizes Area is $x = h = -\dfrac{1500}{2\left(\frac{\sqrt{3}}{4} - \frac{5}{2}\right)}$

$$\| \quad -\frac{b}{2a}$$

1. You want to build two enclosures ~~using~~ spending exactly \$3000 ~~feet of fencing~~. One enclosure will be an equilateral triangle, the other a rectangle. The basis of the rectangle has the same length as the sides of the triangle. What ~~should the side of the triangle be in order to maximize the area~~ is max. area of the two combined enclosures ? The material for building the sides of the rectangle costs \$2 per foot. The material for building the sides of the triangle costs \$3 per foot.



$$\frac{x^2}{4} + h^2 = x^2 \qquad h^2 = \frac{3}{4}x^2 \qquad h = \frac{\sqrt{3}}{2}x$$

$$A = xy + \frac{1}{2}\underbrace{x}_{b}\underbrace{\frac{\sqrt{3}}{2}x}_{h} = xy + \frac{\sqrt{3}}{4}x^2$$

$$3000 = (2x+2y)\cdot 2 + 3x\cdot 3$$

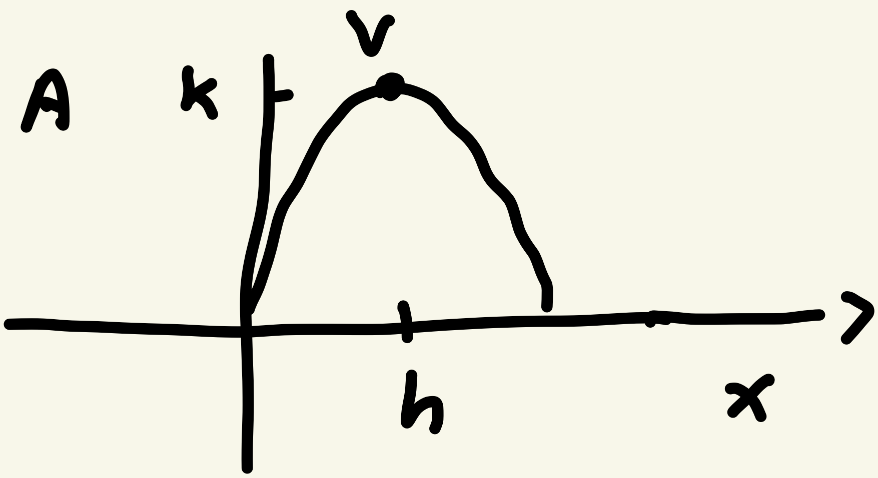
$$3000 = 4x + 4y + 9x = 13x + 4y$$

$$\frac{3000 - 13x}{4} = y$$

$$A(x) = x\,\frac{3000-13x}{4} + \frac{\sqrt{3}}{4}x^2$$

$$= \frac{3000}{4}x - \frac{13}{4}x^2 + \frac{\sqrt{3}}{4}x^2$$

$$= \underbrace{\left(\frac{\sqrt{3}}{4} - \frac{13}{4}\right)}_{a\,<\,0}x^2 + \underbrace{750}_{b}\,x$$



Problem wants maximum Area

so k :

OPTION 1) $k = -\dfrac{b^2 - 4ac}{4a}$

OPTION 2) $h = -\dfrac{b}{2a}$.

$$= -\frac{750}{\cancel{4}\left(\frac{\sqrt{3}}{\cancel{4}_{2}} - \frac{13}{\cancel{4}_{2}}\right)} = \frac{1500}{\sqrt{3}-13}$$

$$k = A(h) =$$

$$\frac{\cancel{\sqrt{3}-13}}{4} \cdot \frac{(1500)^2}{(\sqrt{3}-13)^{\cancel{2}}} \cancel{+} \frac{750 \cdot 1500}{\sqrt{3}-13}$$

$$= \frac{(1500)^2 - 4 \cdot 750 \cdot 1500}{4(\sqrt{3}-13)}$$

$$= \frac{1500(1500-3000)}{4(\sqrt{3}-13)}$$

$$= \frac{1500 \quad 1500}{4\,(13-\sqrt{3})} =$$

$$= \frac{750^2}{13-\sqrt{3}}$$