Lesson 17

Read Chapter 13

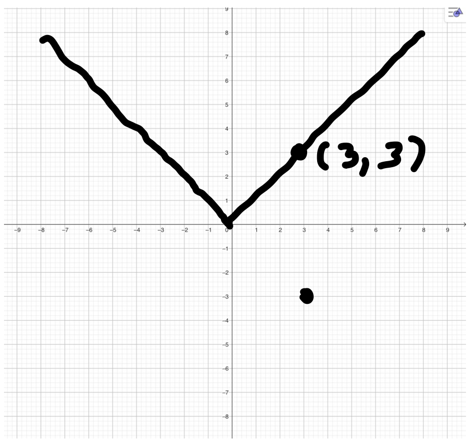
Graphical tools

Motivation

$y = |\ 1 - |x|\ |$

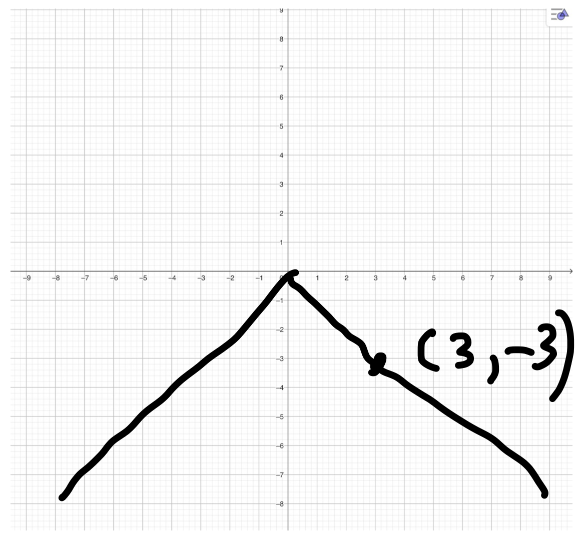
In an old problem we considered graphing $y = 1 - |x|$.

1



$y = |x|$

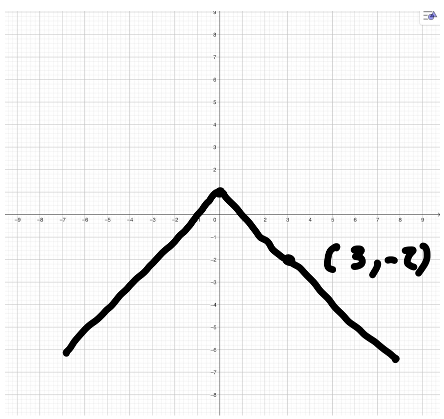
2



$y = -|x|$

$y_{new} = -y_{old}$

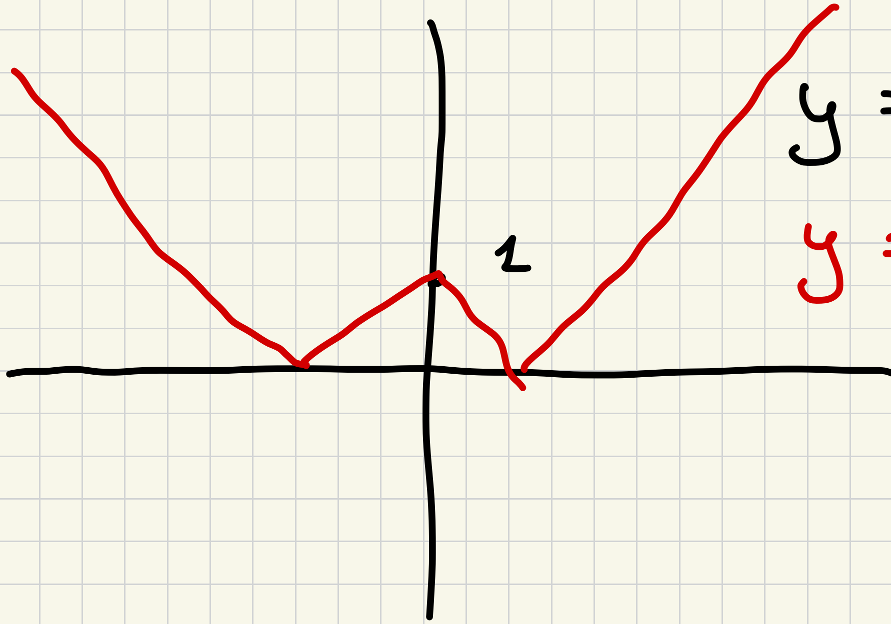
3



$y = 1 - |x|$

$y_{new} = 1 + y_{old}$

$$y = |1 - |x||$$



$y = 1 - |x|$

$y = |1 - |x||$

$\underbrace{y_{new}} = |\underline{y_{old}}|$

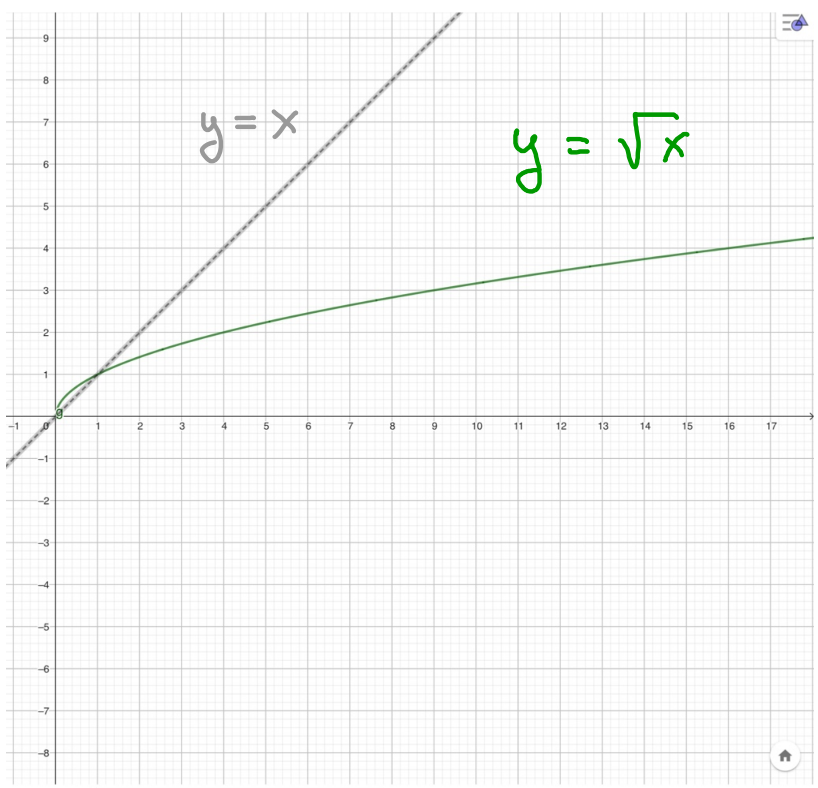
Goals

Draw the graph of af(bx+c)+d from the graph of f(x)

Find the formula for the function whose graph is obtained from the graph of $f(x)$ by performing a series of graphical operations (shifts, reflections and scalings)

Which graphs should you know to start with?

Linear functions, quadratic functions, exponential functions, $\ln x$, $\sqrt{x}$, $|x|$



Domain $[0, +\infty)$

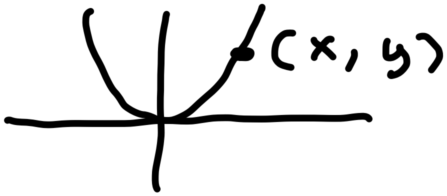
Range $[0, +\infty)$

Vertical translation

$c > 0$ $\quad f(x) \to f(x) + c$

$f(x) \to f(x) - c$

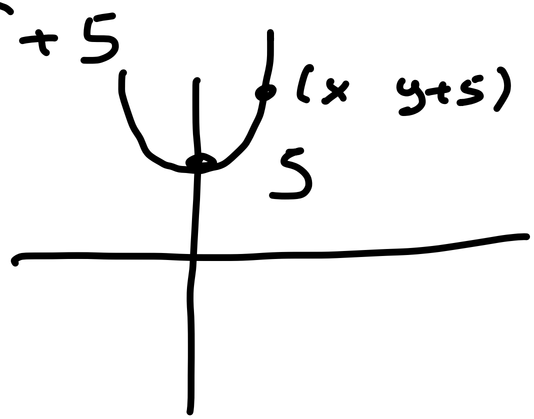
Given $f(x) = x^2$, what do the graph of



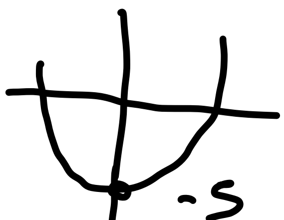
$y = f(x) + 5$ and

$x^2 + 5 \qquad 1 \cdot (x-0)^2 + 5$

$y_{new} = y_{old} + 5$



$y = f(x) - 5$

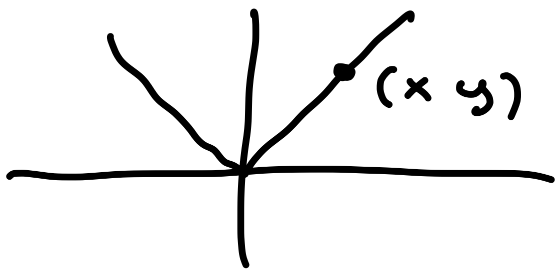


look like ?

Vertical Reflections

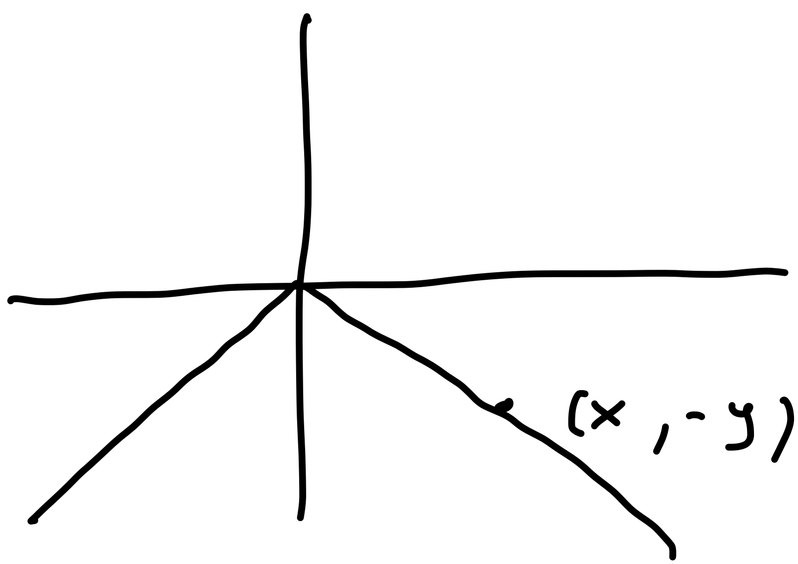
$f(x) = -f(x)$

Given $f(x) = |x|$



what does the graph of $y = -|x|$ looks like ?

$y_{new} = -y_{old}$



Vertical scaling (expansion or compression)

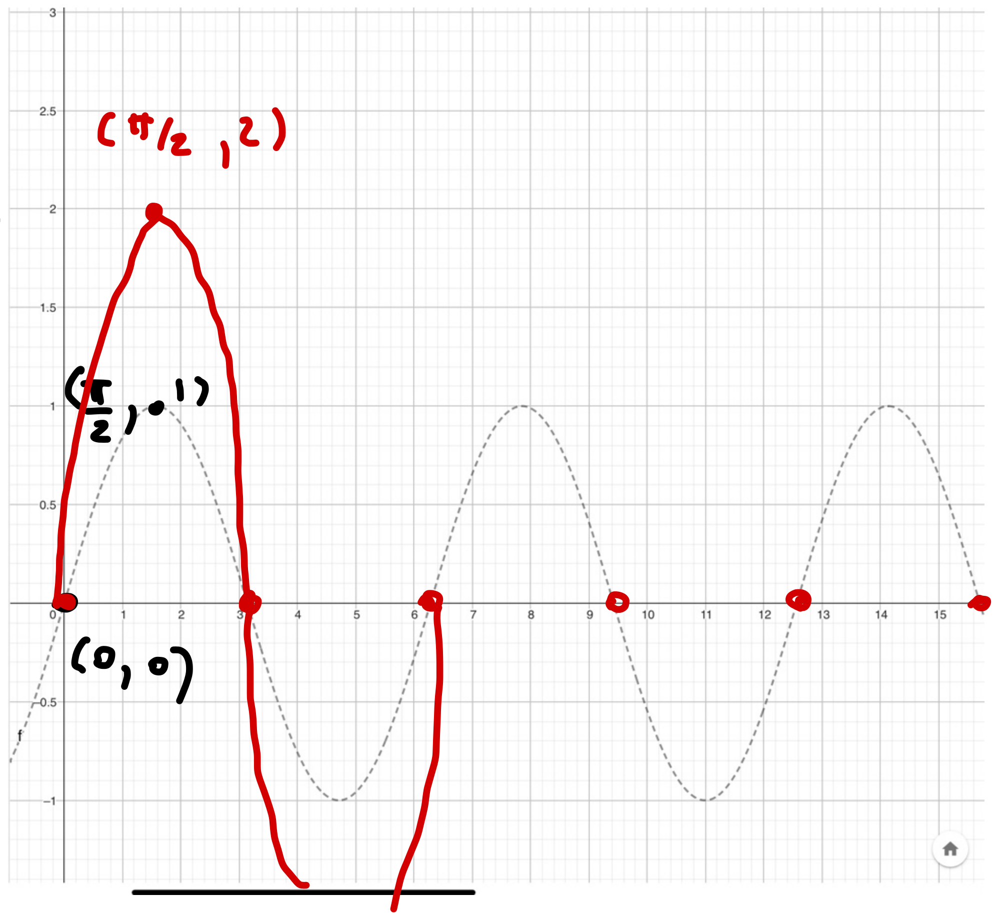
$f(x) \to c f(x)$

$c>1, \; 0<c<1$

Given $f(x) = \sin x$ what do the graphs of

$y = 2f(x)$ and of

$y_{new} = 2 \cdot y_{old}$



$y = \frac{1}{2}f(x)$

$y_{new} = \frac{1}{2} y_{old}$

look like ?

$y = -2f(x) + 3$

$y_{new} = -2\, y_{old} + 3$

Vertical translation up 3 units

reflection across x axis

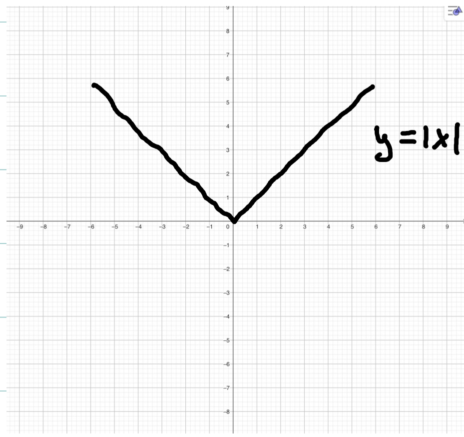
vertical scaling of a factor of 2

In which order?

1) First scale by 2
2) Then reflect across x axis
3) Then shift up 3 units

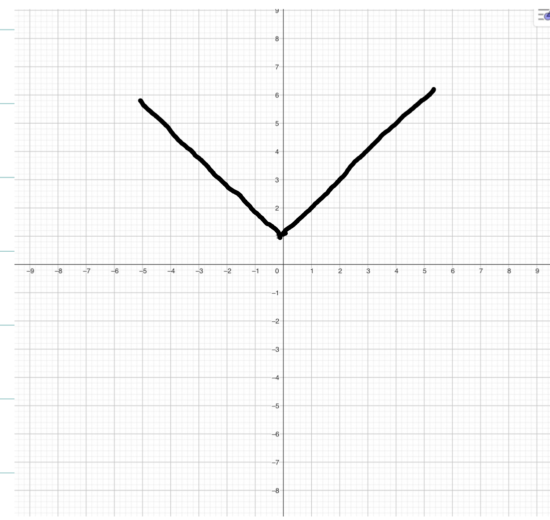
$y = 1 - |x|$ WRONG ORDER

1

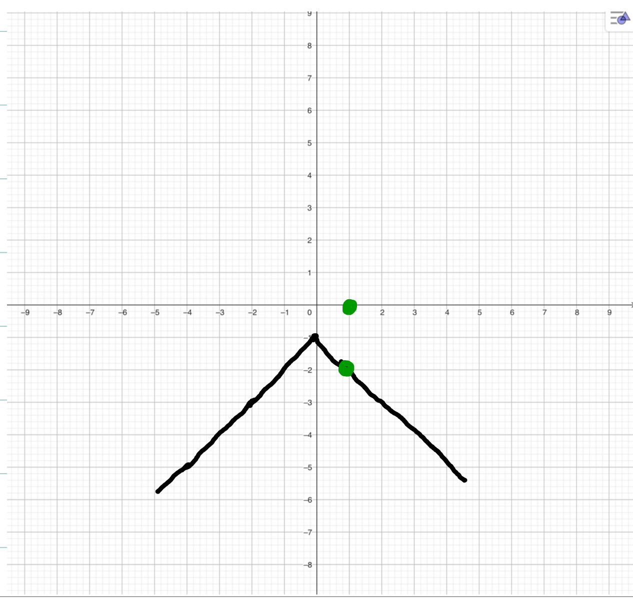


2

Shift up 1 unit



3



reflect across x axis

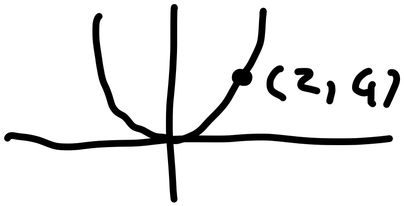
WRONG GRAPH !

when $x = 1$ y should be 0

Horizontal translation

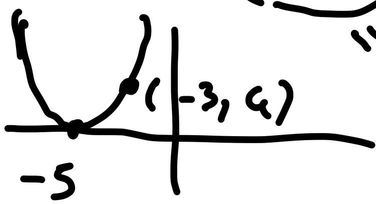
replace x with $x+c$
$x-c$
$c>0$

Given $f(x) = x^2$ what do the graph of

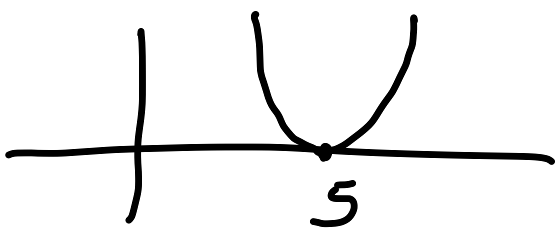


$$g(x) = f(x+5) = (\underbrace{x+5}_{=2})^2 = 1\cdot(x-(-5))^2 + 0$$

$$x+5 = 2$$
$$x = 2-5$$



and $f(x-5) = (x-5)^2 + 0$

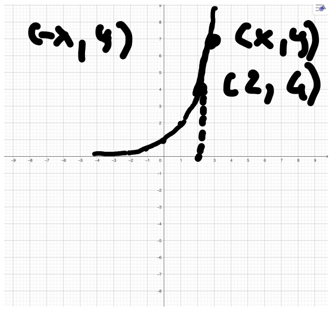


look like ?

Horizontal Reflections

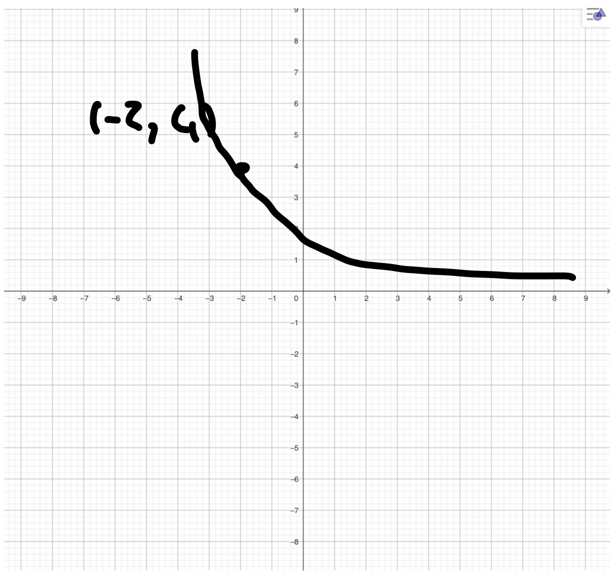
replacing x with $-x$

Given $f(x) = 2^x$ what does the graph of



$g(x) = f(-x) = 2^{-x}$ looks like ?

$g(-2) = 2^{-(-2)} = 4$



replace $x \to \frac{x}{c}$
$c > 0$ scaling factor

Horizontal scaling (expansion or compression)

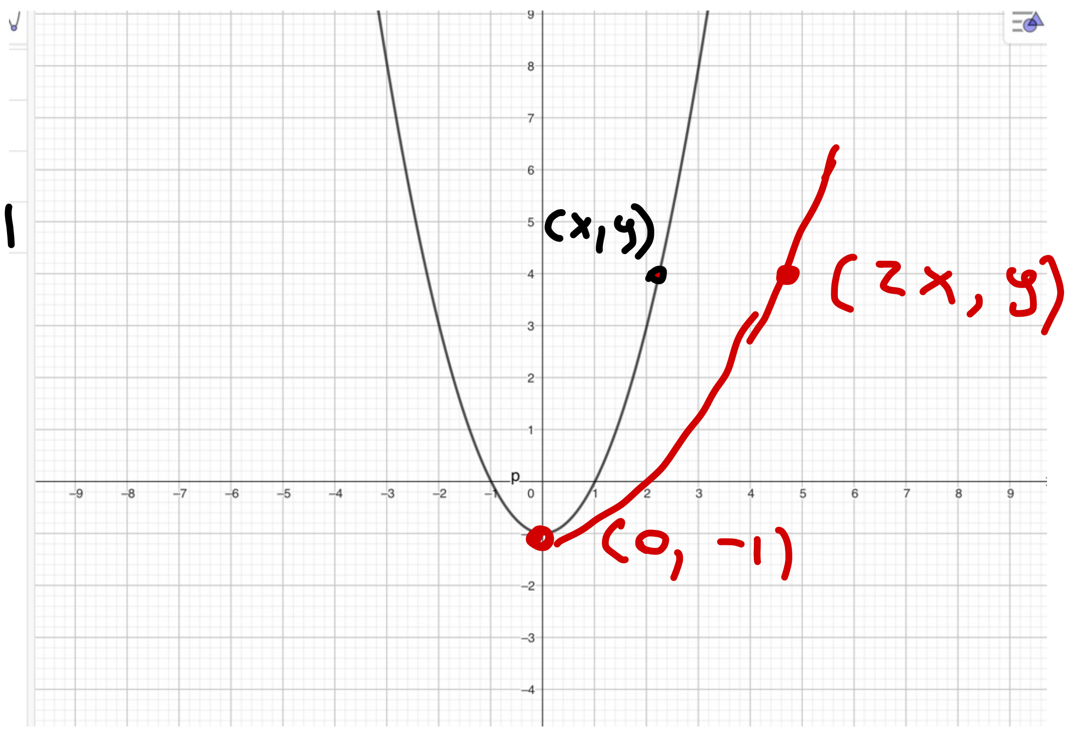
Given $f(x) = x^2 - 1$ what do the graphs of

$p(x) = f(\frac{x}{2}) = \frac{x^2}{4} - 1$

$c = 2$

and of $q(x) = f(\frac{x}{\frac{1}{2}})$

$c = \frac{1}{2}$



look like ?

so if I replace x with $2 \cdot x$

I need to say $2x = \frac{x}{\frac{1}{2}}$

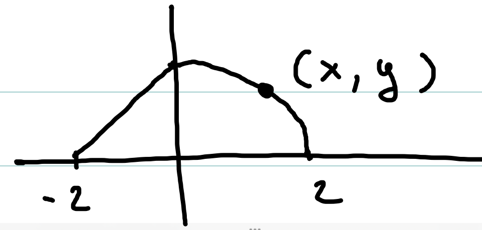
$$\frac{d}{1} x = \frac{x}{\frac{1}{d}}$$

$f\left(\frac{2}{3}x\right)$ what is my scaling factor?

$$\frac{2}{3} x = \frac{x}{\frac{1}{\frac{2}{3}}} = \frac{x}{\frac{3}{2}}$$

$$\frac{3}{2} = c$$

original function $y = f(x)$



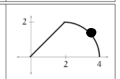
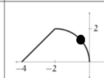
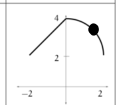
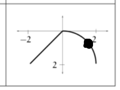
	Shifting (Assume $c > 0$)				$(x\ y)$ moves to
	Symbolic Change	**New Equation**	**Graphical Consequence**	**Picture**	
Horizontal	Replace x with $(x - c)$.	$y = f(x - c)$	A shift to the right c units.		$(x+c, y)$
	Replace x with $(x + c)$.	$y = f(x + c)$	A shift to the left c units.		$(x-c, y)$
Vertical	Replace $f(x)$ with $(f(x) + c)$.	$y = f(x) + c$	A shift up c units.		$(x, y+c)$
	Replace $f(x)$ with $(f(x) - c)$.	$y = f(x) - c$	A shift down c units.		$(x, y-c)$

Table 13.2: Shifting $y = f(x)$.

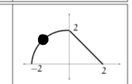
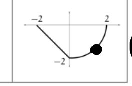
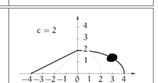
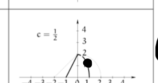
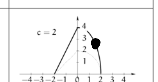
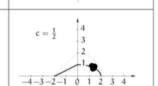
Reflection				$(x\ y)$ moves to
Symbolic Change	**New Equation**	**Graphical Consequence**	**Picture**	
Replace x with $-x$.	$y = f(-x)$	A reflection across the y-axis.		$(-x, y)$
Replace $f(x)$ with $-f(x)$.	$y = -f(x)$	A reflection across the x-axis.		$(x, -y)$

Table 13.1: Reflecting $y = f(x)$.

	Dilation				(x, y) moves to
	Symbolic Change	**New Equation**	**Graphical Consequence**	**Picture**	
horizontal	If $c > 1$, replace x with $\left(\frac{x}{c}\right)$.	$y = f\left(\frac{x}{c}\right)$	A horizontal expansion.		(cx, y)
	If $0 < c < 1$, replace x with $\left(\frac{x}{c}\right)$.	$y = f\left(\frac{x}{c}\right)$	A horizontal compression.		(cx, y)
Vertical	If $c > 1$, replace $f(x)$ with $(cf(x))$.	$y = cf(x)$	A vertical expansion.		(x, cy)
	If $0 < c < 1$, replace $f(x)$ with $(cf(x))$.	$y = cf(x)$	A vertical compression.		(x, cy)

How to graph $a\,f(bx+c)+d$

1. Graph $y = f(x)$
   Horizontally :
2. Shift $|c|$ units, left if c is positive, right if c is negative . $x + c$
3. Scale horizontally of a factor $\frac{1}{|b|}$ (compression if $|b| > 1$, expansion if $|b| < 1$) $\frac{x}{\frac{1}{|b|}} + c$
4. Reflect across y axis if b is negative . Skip this step if b is positive. $-\frac{x}{\frac{1}{|b|}} + c = bx + c$
   Vertically:
5. Scale by a factor of $|a|$ (compression if $|a| < 1$, expansion if $|a| > 1$)
6. Reflect across x axis if a is negative . Skip this step if a is positive.
7. Shift $|d|$ units, up if c is positive, down if c is negative .

Note: the order is important.