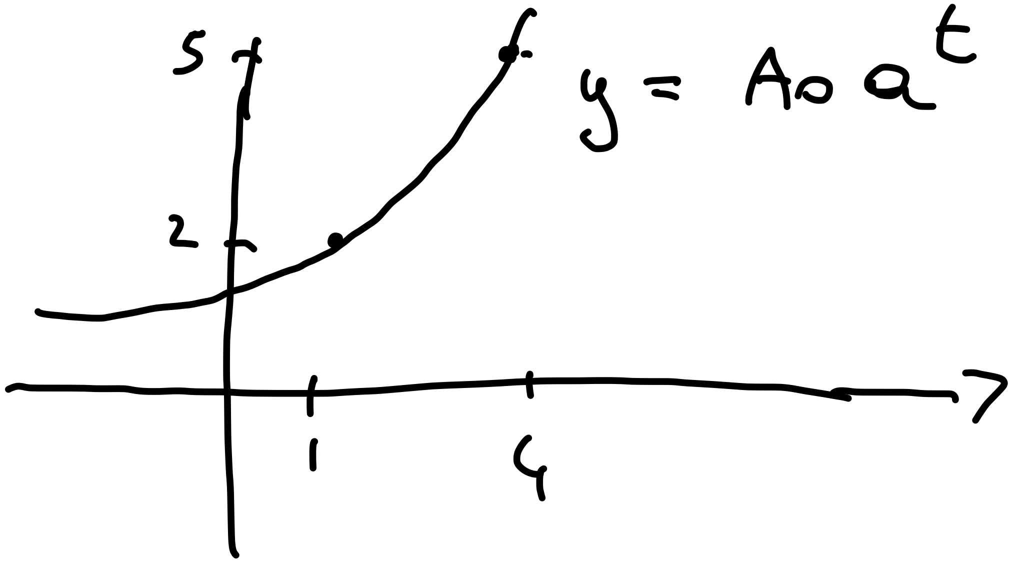
Lesson 16

Read Chapter 11

Exponential modelling

Find a formula for the exponential function that passes through the points $(1, 2)$ and $(\cancel{3}\,4, 5)$



$$y = A_0 a^t$$

$$\begin{cases} 2 = A_0 a^1 \\ 5 = A_0 a^4 \end{cases} \quad \begin{array}{l} \to \quad 2/a = A_0 \\ \to \quad 5 = \dfrac{2}{\cancel{a}} \cdot a^{\cancel{4}\,3} \to \dfrac{5}{2} = a^3 \end{array}$$

$$\sqrt[3]{\frac{5}{2}} = a$$

$$\frac{2}{\sqrt[3]{\frac{5}{2}}} = A_0 \qquad y = \frac{2}{\sqrt[3]{\frac{5}{2}}}\left(\sqrt[3]{\frac{5}{2}}\right)^t$$

Alternetive way

(1, 2) (4, 5) t

(0, 2) (3, 5) $t-1$

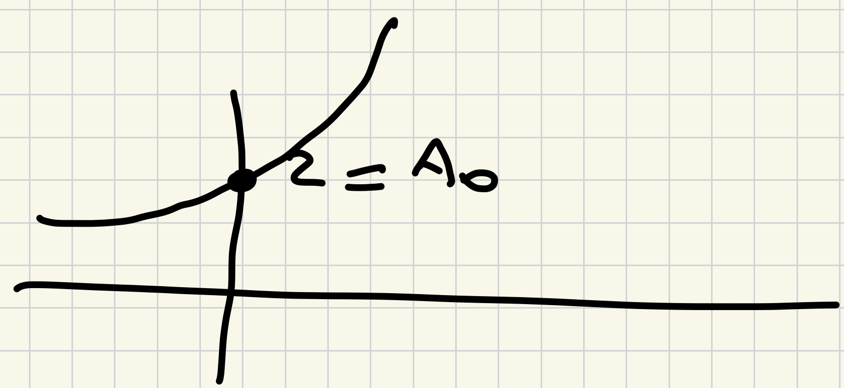
$$y = 2a^x$$

$$5 = 2a^3$$

$$\frac{5}{2} = a^3$$

$$\sqrt[3]{\frac{5}{2}} = a$$

$$y = 2\left(\sqrt[3]{\frac{5}{2}}\right)^{t-1}$$



Find a formula for the exponential function that passes through (1, 2) and has doubling time 80.

$$y = A_0 a^t$$

$$80 = \frac{\ln 2}{\ln a} \qquad \text{solve for } a$$

$$\ln a = \frac{\ln 2}{80} = \frac{1}{80} \ln 2 = \ln 2^{1/80}$$

$$e^{\ln a} = e^{\ln 2^{1/80}}$$

$$\boxed{y \ln x = \ln x^y}$$

$$a = \sqrt[80]{2}$$

$$y = A_0 \sqrt[80]{2}^{\,t}$$

$$2 = A_0 \sqrt[80]{2}^{\,1}$$

$$\frac{2}{\sqrt[80]{2}} = A_0$$

$$y = \frac{2}{\sqrt[80]{2}} \left(\sqrt[80]{2} \right)^t$$

OR

$(1, 2)$ t

$(0, 2)$ $t-1)$

$$y = 2\sqrt[80]{2}^{\,t-1}$$

Suppose a quantity is modelled by n exponential function
$q(t) = A_0 a^t$
if you know the quantity triples every 5 years:

$$a = \sqrt[5]{3}$$

quadruples every 7 years :

$$a = \sqrt[7]{4}$$

increase 5 % every 3 years:

$$A_0 \qquad 1 \cdot A_0 + 0.05\, A_0 = (1 + 0.05)\, A_0$$

$$a = \sqrt[3]{1.05}$$

increases 2 % every 7 years:

$$a = \sqrt[7]{1 + 0.02}$$

decreases 2 % every 7 years:

$$a = \sqrt[7]{1 - 0.02}$$

Flu and cold are spreading in Sickville. The number of cases of both diseases is growing exponentially. There were 100 flu cases a week ago and 1350 cases yesterday. There were also 100 cold cases a week ago, and the number of cold cases doubles every 5 days. When will there be 20 times as many people having the flu than a cold ?

$t=-7$ $\quad$ $t=-1$

$f(t) = $ # flu cases $\qquad t=0$ today

$c(t) = $ # cold cases

$(-7, 100) \qquad (-1, 1350) \qquad t$

$(0, 100) \qquad (6, 1350) \qquad t+7$

$$y = A_0 a^x$$

$$y = 100 a^x$$

$$1350 = 100 \cdot a^6$$

$$13.50 = a^6$$

$$\sqrt[6]{13.50} = a$$

$$f(t) = 100 \sqrt[6]{13.50}^{\,t+7}$$

$c(t)$

$t \quad (-7, \ 100)$

$a = \sqrt[5]{2}$

$t+7 \ (0, \ 100)$

$$c(t) = 100\left(\sqrt[5]{2}\right)^{t+7}$$

$$f(t) = 20 \cdot c(t) \qquad \text{Solve for } t$$

$$\cancel{100}\left(\sqrt[6]{13.5}\right)^{t+7} = 20 \cdot \cancel{100}\left(\sqrt[5]{2}\right)^{t+7}$$

$$\ln\left(\sqrt[6]{13.5}\right)^{t+7} = \ln\left(20 \cdot \left(\sqrt[5]{2}\right)^{t+7}\right)$$

$$(t+7)\ \ln\left(\sqrt[6]{13.5}\right) = \ln 20 + \ln\left(\sqrt[5]{2}\right)^{t+7}$$

$$(t+7)\ln\left(\sqrt[6]{13.5}\right) = \ln 20 + (t+7)\ln\left(\sqrt[5]{2}\right)$$

$$(t+7)\ln\sqrt[6]{13} - (t+7)\ln\sqrt[5]{2} = \ln 20$$

$$(t+7)\left(\ln 13.5^{1/6} - \ln 2^{1/5}\right) = \ln 20$$

$$(t+7) = \frac{\ln 20}{\frac{1}{6}\ln(13.5) - \frac{1}{5}\ln 2}$$

$$t = \frac{\ln 20}{\frac{1}{6}\ln(13.5) - \frac{1}{5}\ln 2} - 7 \simeq 3.14$$

exponential

The population of Arcadia increases by 8% every 10 years. The population of Brom triples every 120 years. Arcadia had 10,000 residents in 2000. The population of Brom was 10,000 in 2005. In what year will the city of Brom have twice as many residents as the city of Arcadia ?

$A(t) =$ population of Arcadia

$B(t) =$ population of Brom

$t = 0$ is year 2000

$(0 \quad 10{,}000)$ $\quad A(t) = 10{,}000\left(\sqrt[10]{1.08}\right)^t$

$t \quad (5, \; 10\,000)$

$t-5 \quad (0, \; 10{,}000)$ $\quad B(t) = 10{,}000\left(\sqrt[120]{3}\right)^{t-5}$

$B(t) = 2\ A(t)$ solve for t

$B(t) = 2\,A(t)$

$$\cancel{10{,}000}\,\sqrt[120]{3}^{\,t-5} = 2\cdot\cancel{10{,}000}\,\sqrt[10]{1.08}^{\,t}$$

$$\ln \sqrt[120]{3}^{\,t-5} = \ln 2\cdot\sqrt[10]{1.08}^{\,t}$$

$$(t-5)\ln\sqrt[120]{3} = \ln 2 + t\ln\sqrt[10]{1.08}$$

$$t\ln\sqrt[120]{3} - 5\ln\sqrt[120]{3} = \ln 2 + t\ln\sqrt[10]{1.08}$$

$$\underbrace{t}\ln\sqrt[120]{3} - \underbrace{t}\ln\sqrt[10]{1.08} = \ln 2 + 5\ln\sqrt[120]{3}$$

$$t\left(\ln\sqrt[120]{3} - \ln\sqrt[10]{1.08}\right) = \ln 2 + 5\ln\sqrt[120]{3}$$

$$t = \frac{\ln 2 + 5\ln\sqrt[120]{3}}{\ln\sqrt[120]{3} - \ln\sqrt[10]{1.08}} = 506 \quad \text{in } \boxed{2506}$$