Lesson 15

Week 6
Midterm info
Extra credit quiz

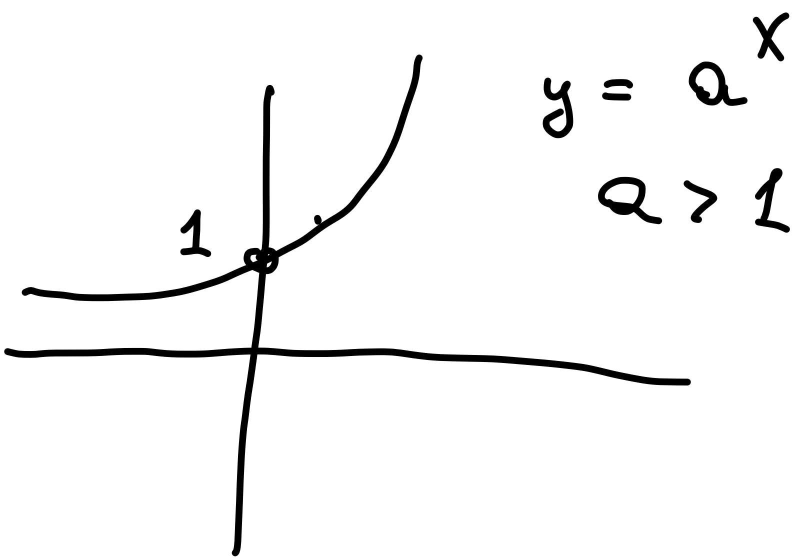
Read Chapter 12

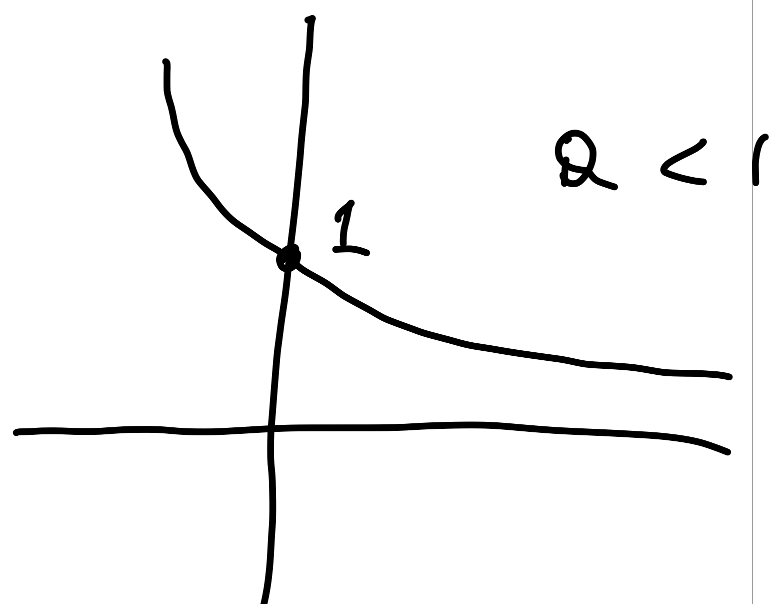
[redacted]

Logarithms

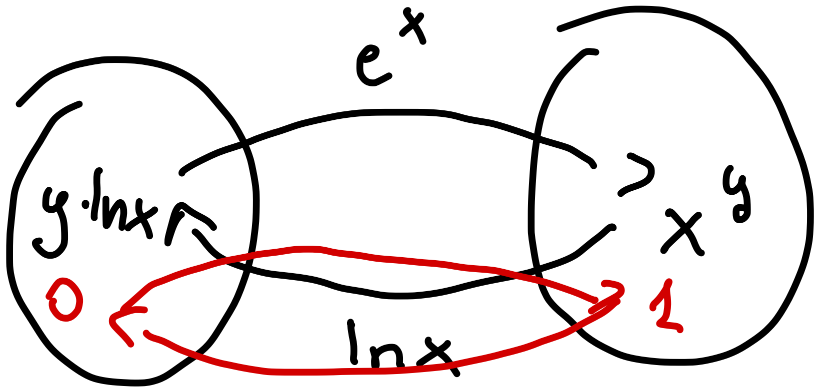
Other log functions

$\log_a x$ is the inverse of a^x





properties of log



- $\ln x^y = y \ln x$
- $log_b x = \frac{\ln x}{\ln b}$
- $a^x = e^{(\ln a)x}$
- $\ln(xy) = \ln(x) + \ln(y)$
- $\ln \frac{x}{y} = \ln x - \ln y$
- $\ln 1 = 0$
- $\ln \frac{1}{x} = -\ln x$

$$\ln(x^{-1}) = (-1)\ln x$$

$$e^{y \cdot \ln x} = e^{(\ln x)\cdot y}$$

$$x^y$$

Solve the following equations

1. $5e^{x-4} = 2$

$$e^{x-4} = \frac{2}{5}$$

$$\ln e^{x-4} = \ln \frac{2}{5}$$

$$x - \cancel{4} = \ln\left(\frac{2}{5}\right) + 4$$

$$\ln(5 \cdot e^{x-4}) = \ln 2$$

$$\ln(5) + \ln e^{x-4} = \ln 2$$

$$x - \cancel{4} = \ln 2 - \ln(5) + 4$$

2. $5\,3^{x-4} = 2$

$$3^{x-4} = 2/5$$

$$\ln 3^{x-4} = \ln\left(\frac{2}{5}\right)$$

$$(x-4) \cdot \ln 3 = \ln\left(\frac{2}{5}\right)$$

$$x - \cancel{4} = \frac{\ln(2/5)}{\ln(3)} + 4$$

$$3^{x-4} = 2/5$$

$$\log_3 3^{x-4} = \log_3\left(\frac{2}{5}\right)$$

$$x - \cancel{4} = \log_3\left(\frac{2}{5}\right) + 4$$

Solve the following equations

1. $5\ln(5x+2)=3$

$$e^{\ln(5x+2)} = e^{3/5}$$

$$5x+2 = e^{3/5}$$

$$\cancel{5}x = \frac{\left(e^{3/5} - 2\right)}{5}$$

2. $log_2(5x+2)=3$

$$2^{\log_2(5x+2)} = 2^{3}$$

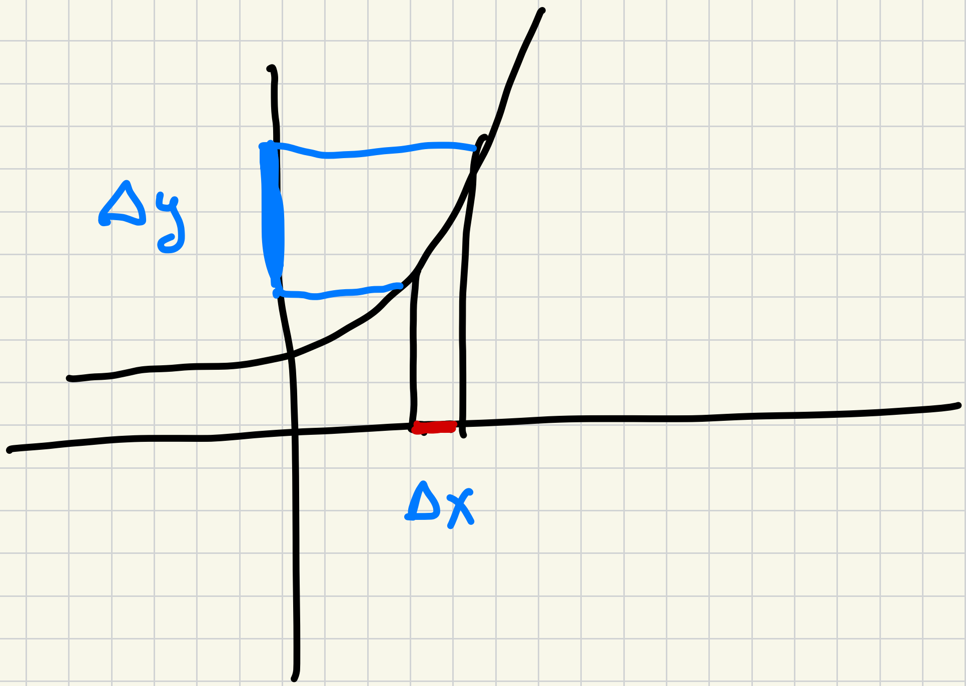
$$5x+2 = 8$$

$$\cancel{5}x = \frac{6}{5}$$

$\log_2 x$

inverse

2^x



Δy
Δx

Exponential functions in standard form

$$f(x) = A_0\, a^x = A_0 \left(e^{\ln a}\right)^x = A_0 e^{\ln a \cdot x}$$

or

$$f(x) = A_0 e^{(\ln a)\,x} = A_0 e^{kx}$$

Rewrite in e form $\quad A_0 e^{kt}$

- $y = 5\,7^t$ $= 5 \cdot (e^{\ln 7 \cdot})^t = 5 e^{\ln 7 \cdot t}$

- $y = \frac{3}{2^{3t-1}}$ $= A_0 e^{kt}$

1) Put in standard form $A_0 a^t$

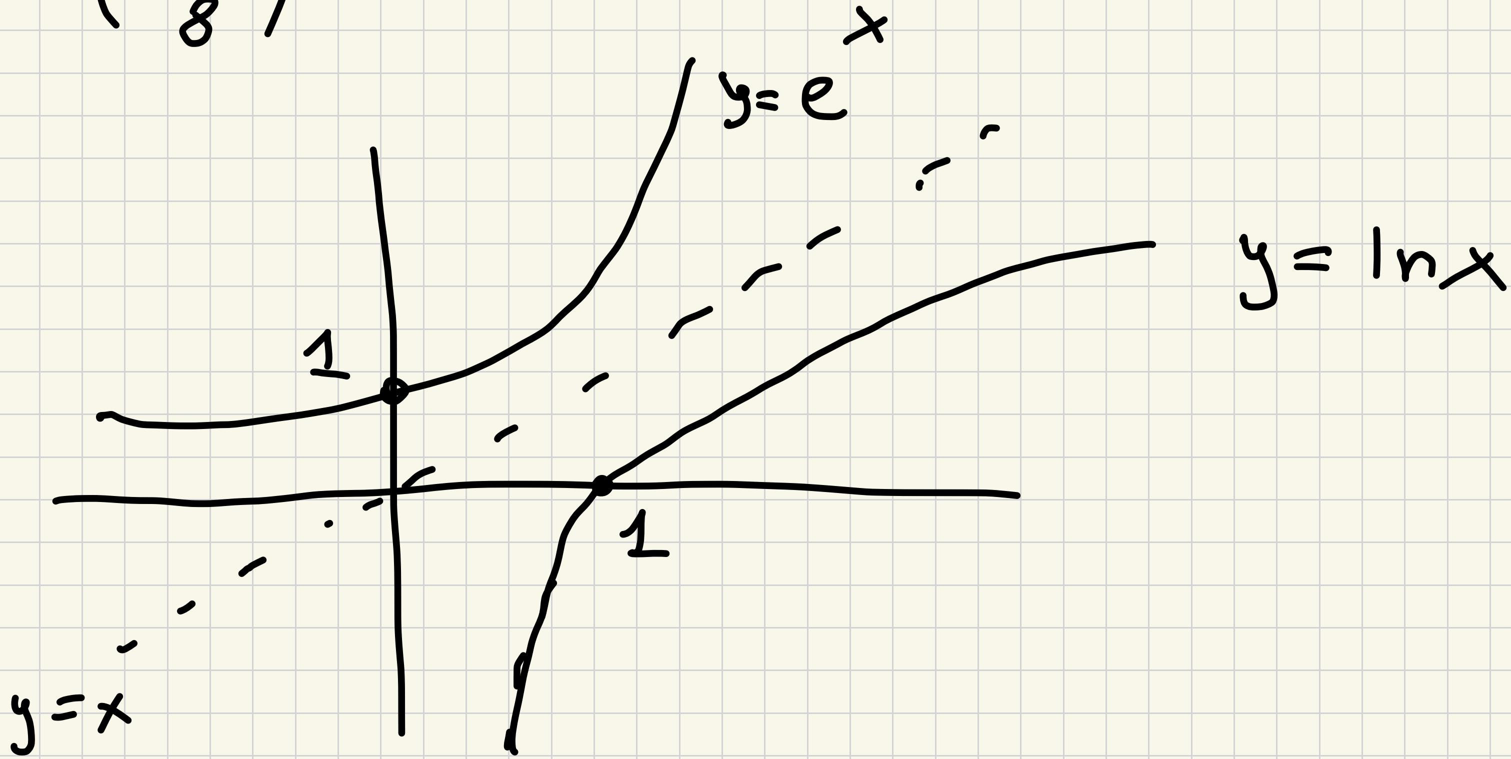
$$3 \cdot \left(\frac{1}{2}\right)^{3t-1} = 3 \cdot \left(\frac{1}{2}\right)^{3t} \cdot \left(\frac{1}{2}\right)^{-1}$$

$$= 3 \cdot 2 \cdot \left[\left(\frac{1}{2}\right)^3\right]^t = 6 \left(\frac{1}{8}\right)^t$$

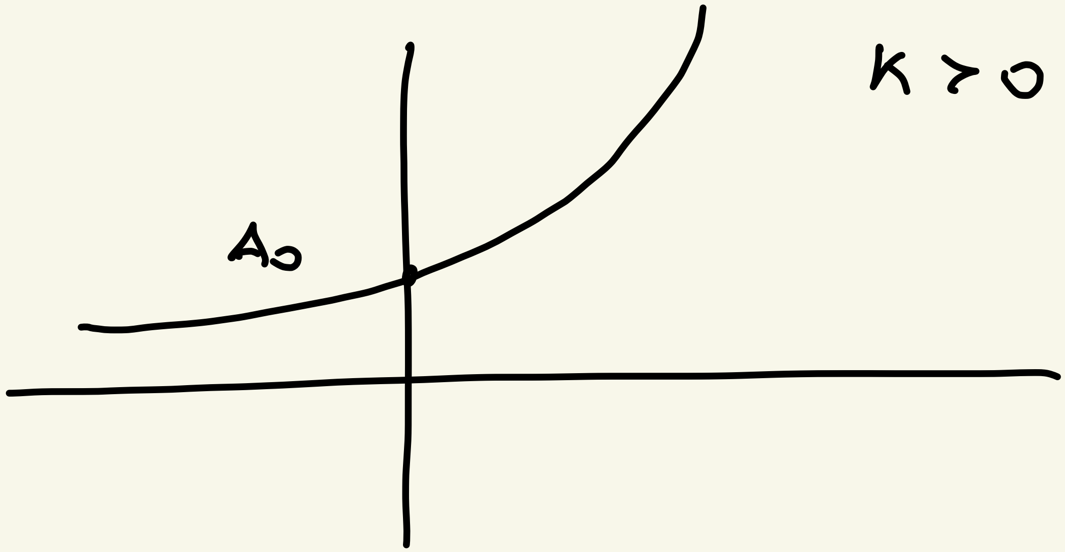
2) Put in "e form"

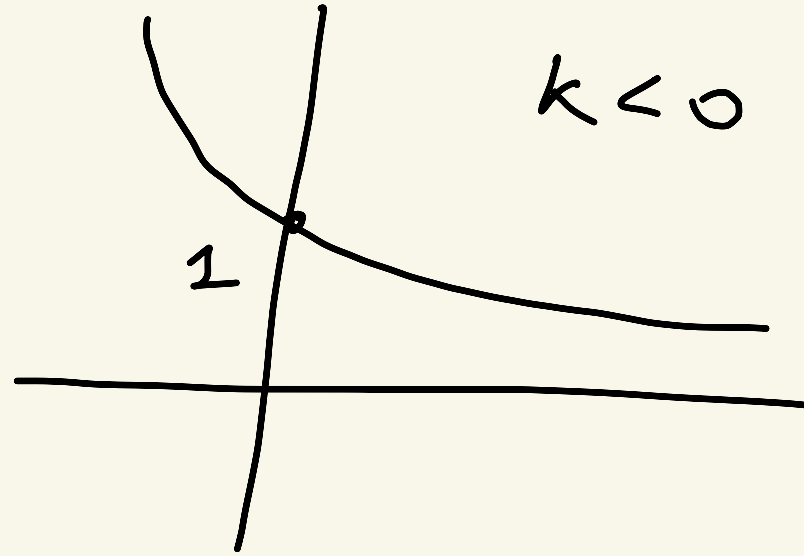
$$6 \cdot \left(e^{\ln 1/8}\right)^{t} = 6\, e^{(\ln 1/8)\cdot t}$$

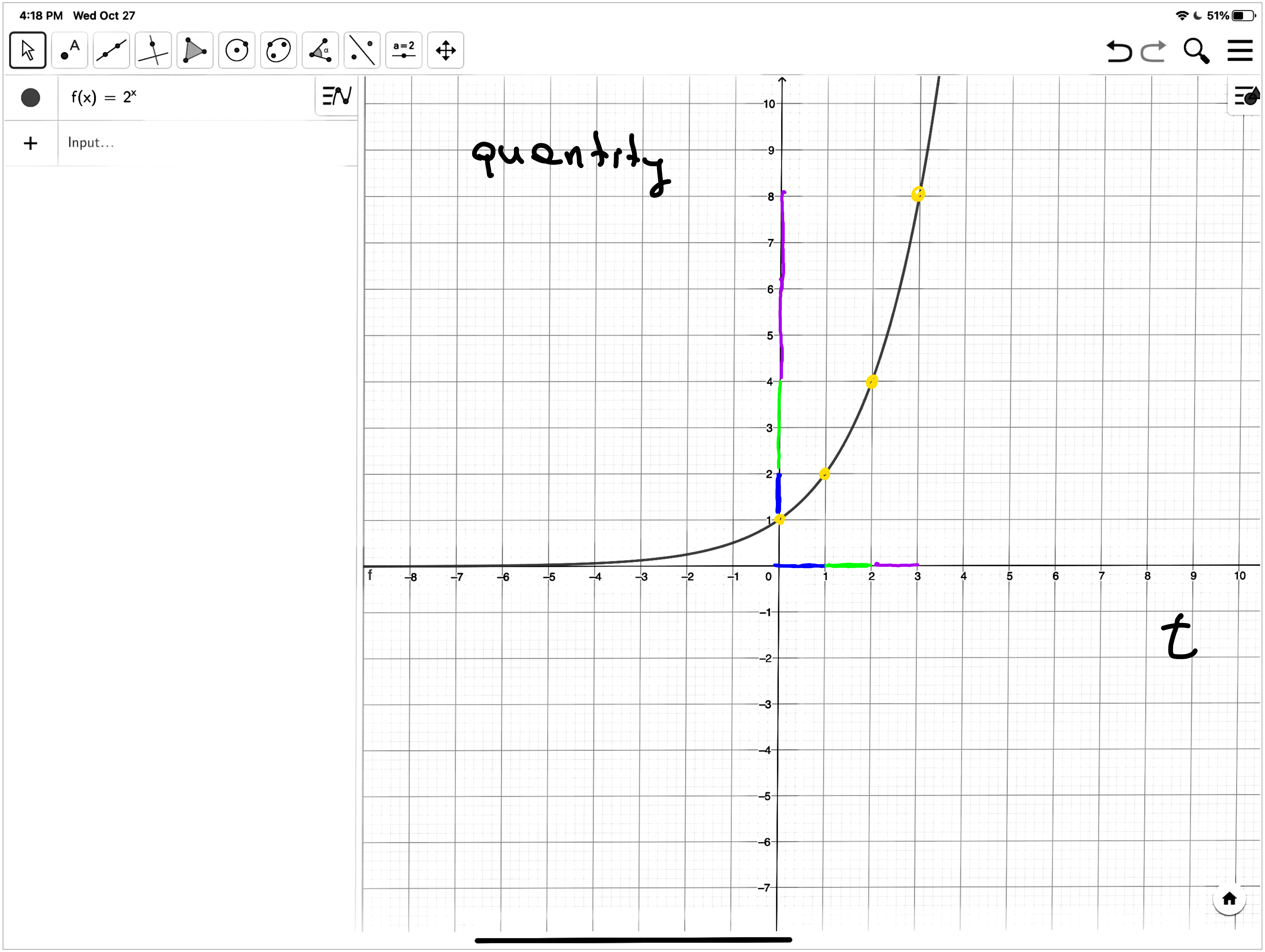
$$\ln\left(\frac{1}{8}\right) < 0$$



$$A_0 e^{kt} = A_0 e^{\ln a \, t} = A_0 a^t$$






4:18 PM Wed Oct 27
51%
f(x) = 2^x
Input...
quantity
t

Doubling time

Given an exponential function $f(t) = A_0\, a^t$, its doubling time is the period of time required for f to double in value.

$f(0) = A_0 a^0 = A_0$

Want function to double in value, that is reach $2 \cdot A_0$

$2 \cancel{A_0} = \cancel{A_0} a^t$

$\ln 2 = \ln a^t$

$\ln 2 = t \ln a$

$\ln 2 / \ln a = t$

The doubling time for $f(x) = A_0\, a^x$ is $\frac{\ln 2}{\ln a}$

Tripling time

Given an exponential function $f(t) = A_0\, a^t$, its tripling time is the period of time required for f to double in value.

The tripling time for $f(x) = A_0\, a^x$ is

$$\frac{\ln 3}{\ln a}$$

Half life

Given an exponential function $f(t) = A_0\, a^t$, its half life is the period of time required for f to half in value.

The ~~tripling time~~ Half life for $f(x) = A_0\, a^x$ is

$$\frac{\ln 1/2}{\ln a}$$

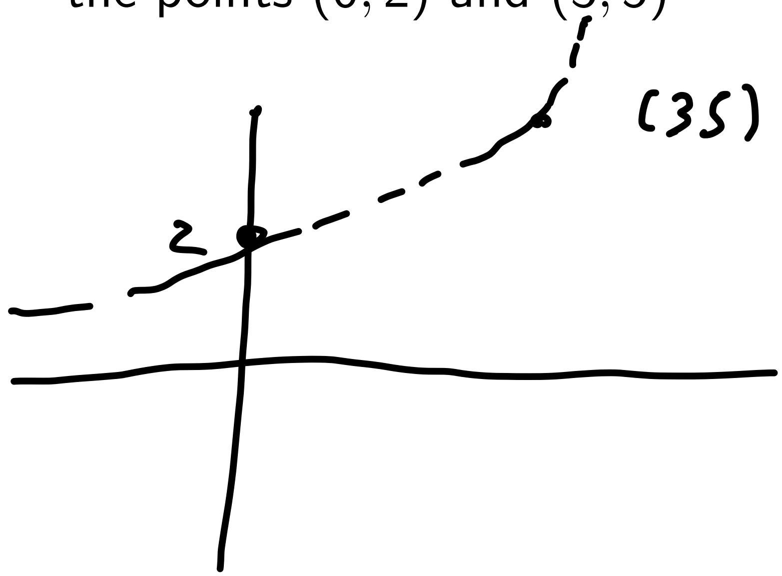
Exponential modelling problems

Exponential modelling problems are problems that talk about a quantity that grows or decays exponentially. Your task is to find a formula $f(x) = A_0 a^x$ for the quantity as a function of some variable x and use the formula to answer the questions in the problem.

Usually given

1) 2 points
2) 1 point and doubling / tripling... time

Find a formula for the exponential function that passes through the points $(0,2)$ and $(3,5)$



$$y = A_0 a^x$$

$$2 = A_0 a^0 \rightarrow A_0 = 2$$

$$5 = A_0 a^3 \rightarrow 5 = 2a^3$$

$$\frac{5}{2} = a^3$$

$$\sqrt[3]{\frac{5}{2}} = a$$

$$\boxed{y = 2\sqrt[3]{\frac{5}{2}}^{\,x}}$$