Lesson 14

Read Chapter 10

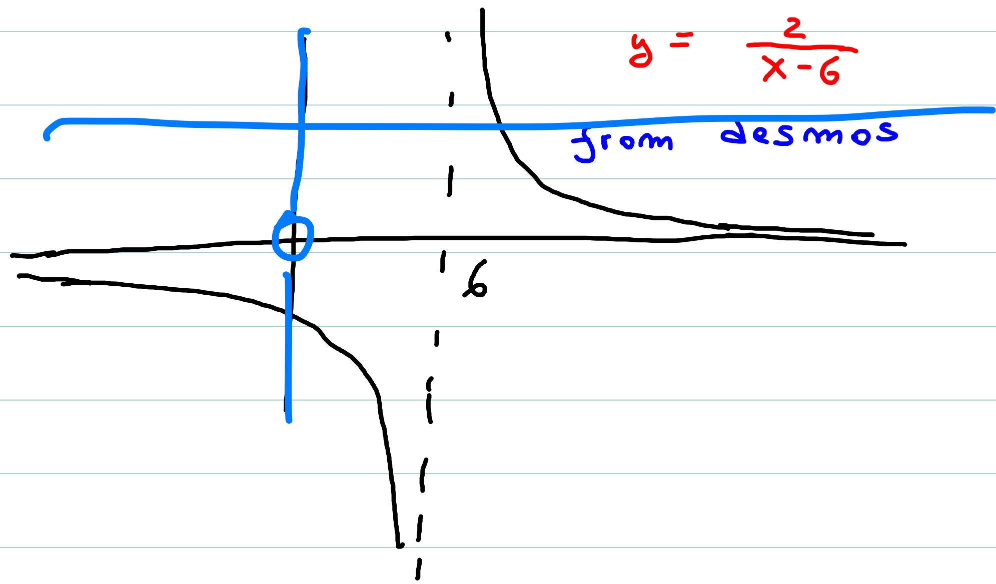
Exponential functions

From Past time

$f(x) = y = \dfrac{2}{x-6}$ $\quad x \neq 6$

DOMAIN $(-\infty, 6) \cup (6, +\infty)$

RANGE $(-\infty, 0) \cup (0, +\infty)$ $\quad y \neq 0$



Invertible? yes $\quad f^{-1}$: Domain $(-\infty, 0) \cup (0, +\infty)$

Range $(-\infty, 6) \cup (6, +\infty)$

Formula: $y = \dfrac{2}{x-6}$ $\quad$ solve for x

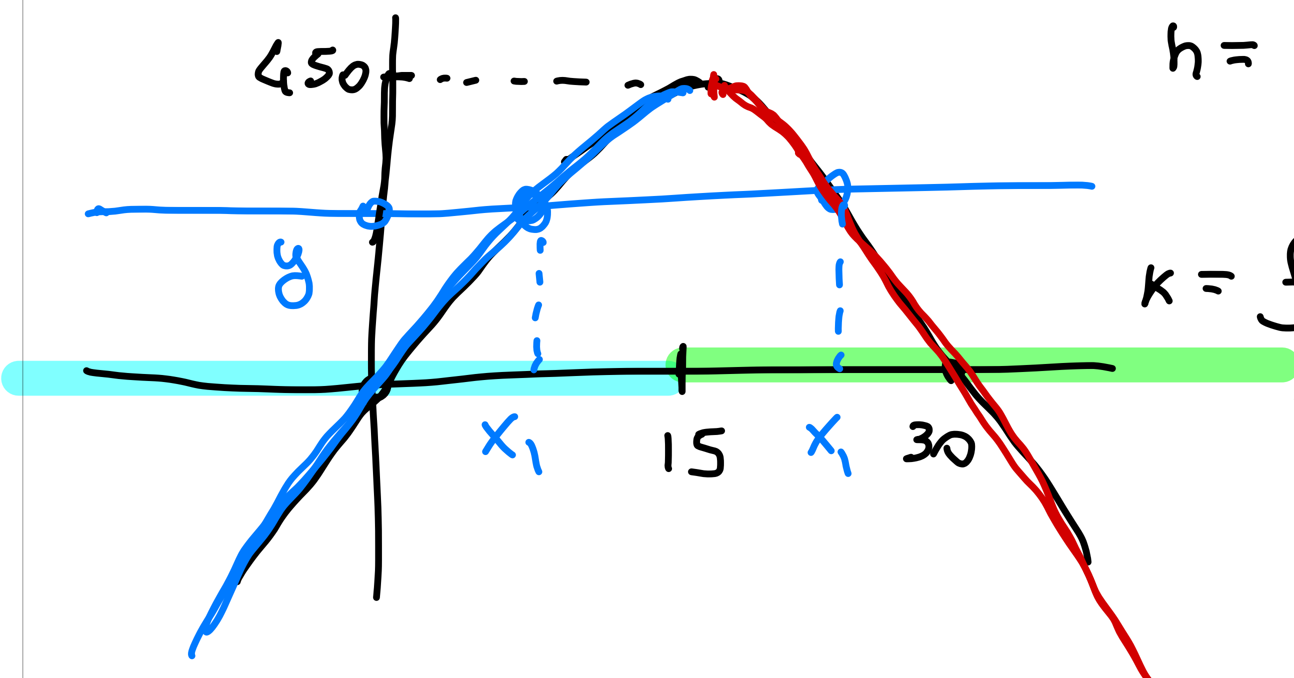
$(x-6)\cdot y = 2$

$(x-6) = \dfrac{2}{y}$

$x = 6 + \dfrac{2}{y}$ $\quad f^{-1}(y) = 6 + \dfrac{2}{y}$ $\quad f^{-1}(x) = 6 + \dfrac{2}{x}$

Explain why $f(x) = -2x^2 + 60x$ is not invertible.

- Graph parabola. Show:
  a) shape b) x, y intercepts c) vertex



$$h = \frac{-60}{-2(-2)} = 15$$

$$k = f(15) = -2 \cdot 15^2 + 60 \cdot 15$$
$$= 450$$

NOT INVERTIBLE

- Algebra $y = -2x^2 + 60x$

solve for x

$$2x^2 - 60x + y = 0$$

$$x = \frac{60 \pm \sqrt{60^2 - 4 \cdot 2 \cdot y}}{2 \cdot 2}$$

$$x = \underbrace{15 \pm \frac{\sqrt{3600 - 8y}}{4}}_{\text{NOT FORMULA FOR A FUNCTION}}$$

What is the inverse of $g(x) = -2x^2 + 60x$ on $[15, +\infty)$

For g^{-1} DOMAIN $(-\infty, 450]$ RANGE $[15, +\infty)$

FORMULA $15 + \dfrac{\sqrt{3600 - 8y}}{4} = g^{-1}(y) = x$

What is the inverse of $h(x) = -2x^2 + 60x$ on $(-\infty, 15]$

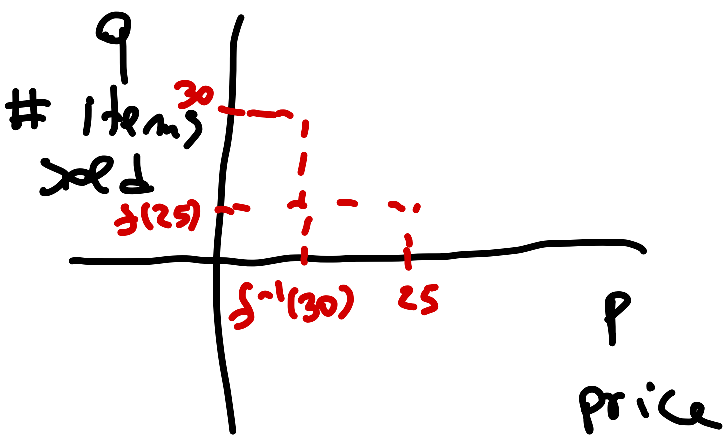
h^{-1} Domain $(-\infty, 450]$ Range $(-\infty, 15]$

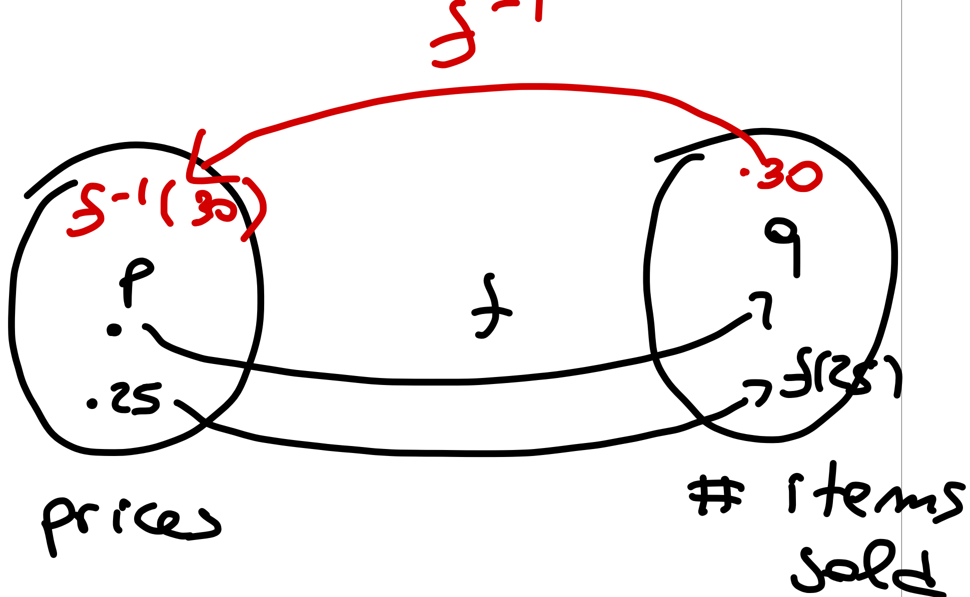
$$h^{-1}(y) = 15 - \frac{\sqrt{3600 - 8y}}{4}$$

Suppose p is the price of an item and $q = f(p)$ is the number of items sold at that price. Explain in words the meaning of:

$f(25)$ = # items sold if price of an item is $25

$f^{-1}(30)$ = price of item when 30 are sold





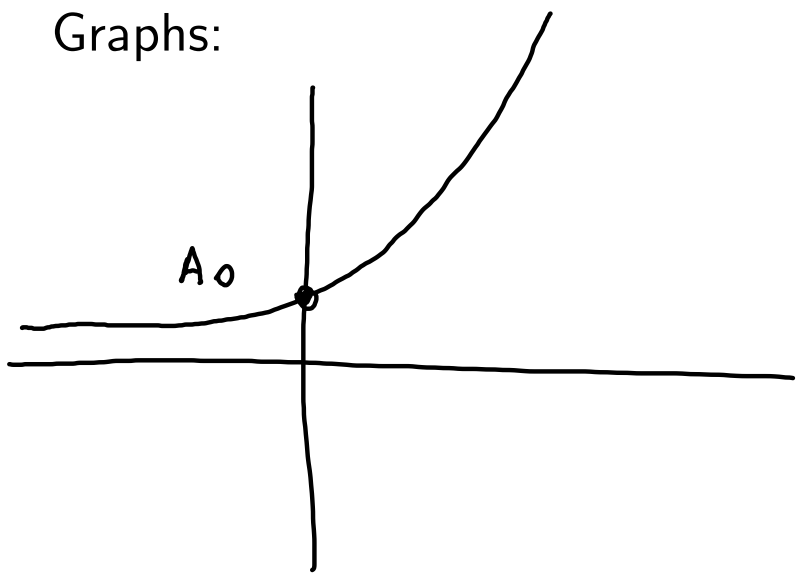
Function in standard exponential form : $\boxed{f(x) = A_0\, a^x}$, $a > 0$ and $a \neq 1$

$a^{1/2} = \sqrt{a}$

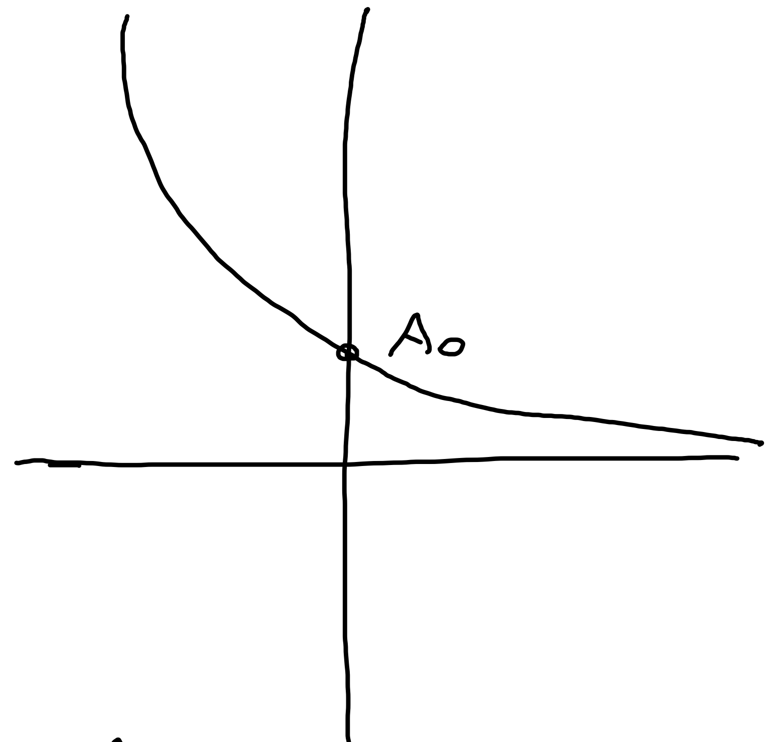
1^x ok but boring!

$f(0) = A_0$ y. intercept

Graphs:



$A_0 > 0 \quad a > 1$



$A_0 > 0 \quad a < 1$

$$2^3 \cdot 2^5 = 2^8 \quad \text{BUT}$$

$$2^3 \cdot 3^5 \neq 6^8 \quad \text{NO}$$

Useful algebra

1. $a^{x+y} = a^x \cdot a^y$
2. $a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x$
3. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
4. $a^{xy} = (a^x)^y$

Put $f(x) = 3 \cdot 2^{-x+\frac{1}{2}}$ in standard exponential form

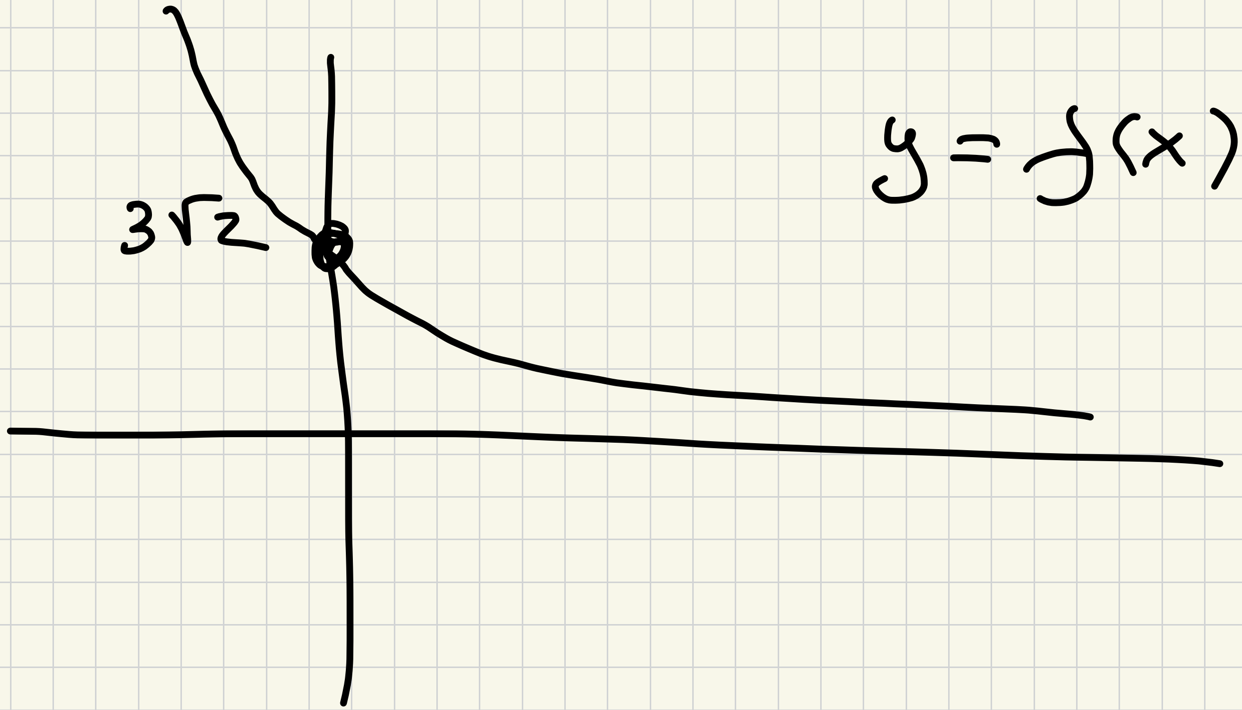
$$f(x) = A_0 \, a^{x}$$

$$3 \cdot 2^{-x} \cdot 2^{1/2}$$

$$3 \cdot 2^{1/2} \quad 2^{-x}$$

$$3\sqrt{2} \quad \left(\frac{1}{2}\right)^{x}$$

$$A_0 = 3\sqrt{2} \qquad a = \frac{1}{2}$$



$$\left(\frac{2}{3}\right)^{-x} \qquad \left(\frac{3}{2}\right)^{x}$$

$$\left(\frac{1}{\frac{2}{3}}\right)^{x} = \left(\frac{3}{2}\right)^{x}$$

a

Put $f(x) = \frac{5}{3^{2x-10}}$ in standard exponential form

$$f(x) = A_0 \cdot a^x$$

$$5 \cdot \frac{1}{3^{2x-10}}$$

$$5 \cdot \left(\frac{1}{3}\right)^{2x-10}$$

$$5 \cdot \left(\frac{1}{3}\right)^{2x} \cdot \left(\frac{1}{3}\right)^{-10}$$

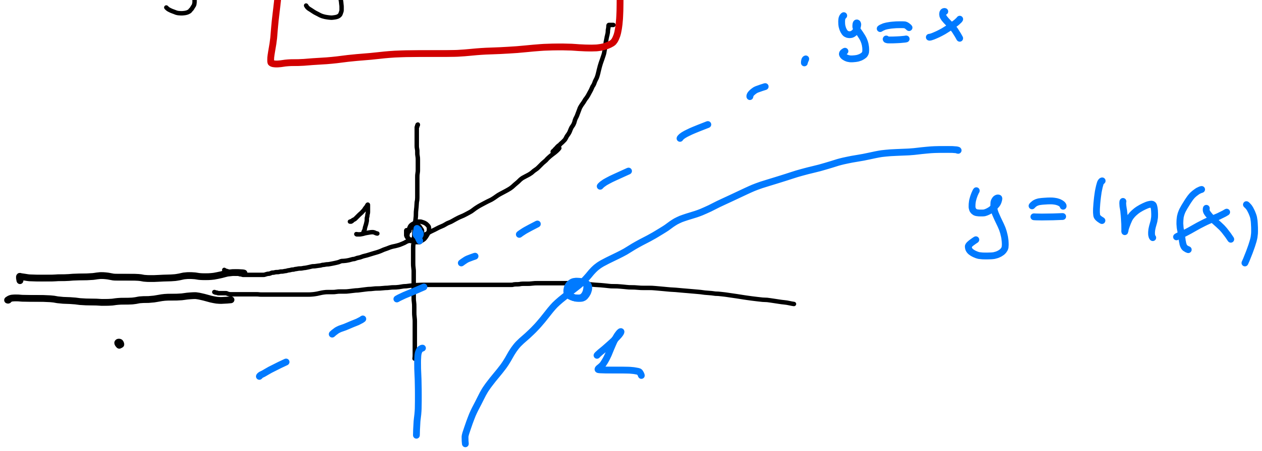
$$5 \cdot \left(\frac{1}{3}\right)^{-10} \quad \left(\frac{1}{3}\right)^{2x}$$

$$5 \quad 3^{10} \left(\left(\frac{1}{3}\right)^{2}\right)^{x}$$

$$5 \cdot 3^{10} \quad \left(\frac{1}{9}\right)^{x}$$

$$A_0 = 5 \cdot 3^{10} \qquad a = \frac{1}{9}$$

Graph of $\boxed{f(x) = e^x}$



Invertible ?

Domain of e^x is $(-\infty \quad +\infty)$

Range of e^x is $(0 \quad +\infty)$

Domain of $\ln x$ is $(0, +\infty)$

Range of $\ln x$ $(-\infty, +\infty)$

$\ln x$ is the inverse of e^x . This means

$$\ln(e^x) = x$$

$$e^{\ln y} = y$$

If $e^x = y$ then $x = \ln y$ and vice-versa