Lesson 13

Read Chapter 9

Inverse function

$$|1-|x|| = \begin{cases} |1-x| & \text{if } x \geq 0 \\ |1+x| & \text{if } x < 0 \end{cases}$$

$$|1-x| = \begin{cases} 1-x & \text{if } 1-x \geq 0 \text{ and } x \geq 0 \\ -(1-x) & \text{if } 1-x \leq 0 \text{ and } x \geq 0 \end{cases}$$

$1 \geq x$

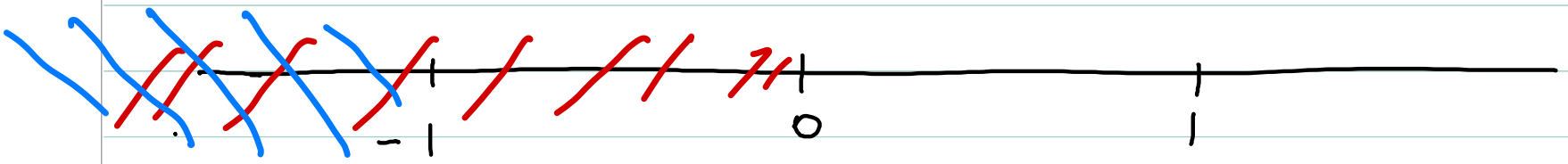
$1 \leq x$

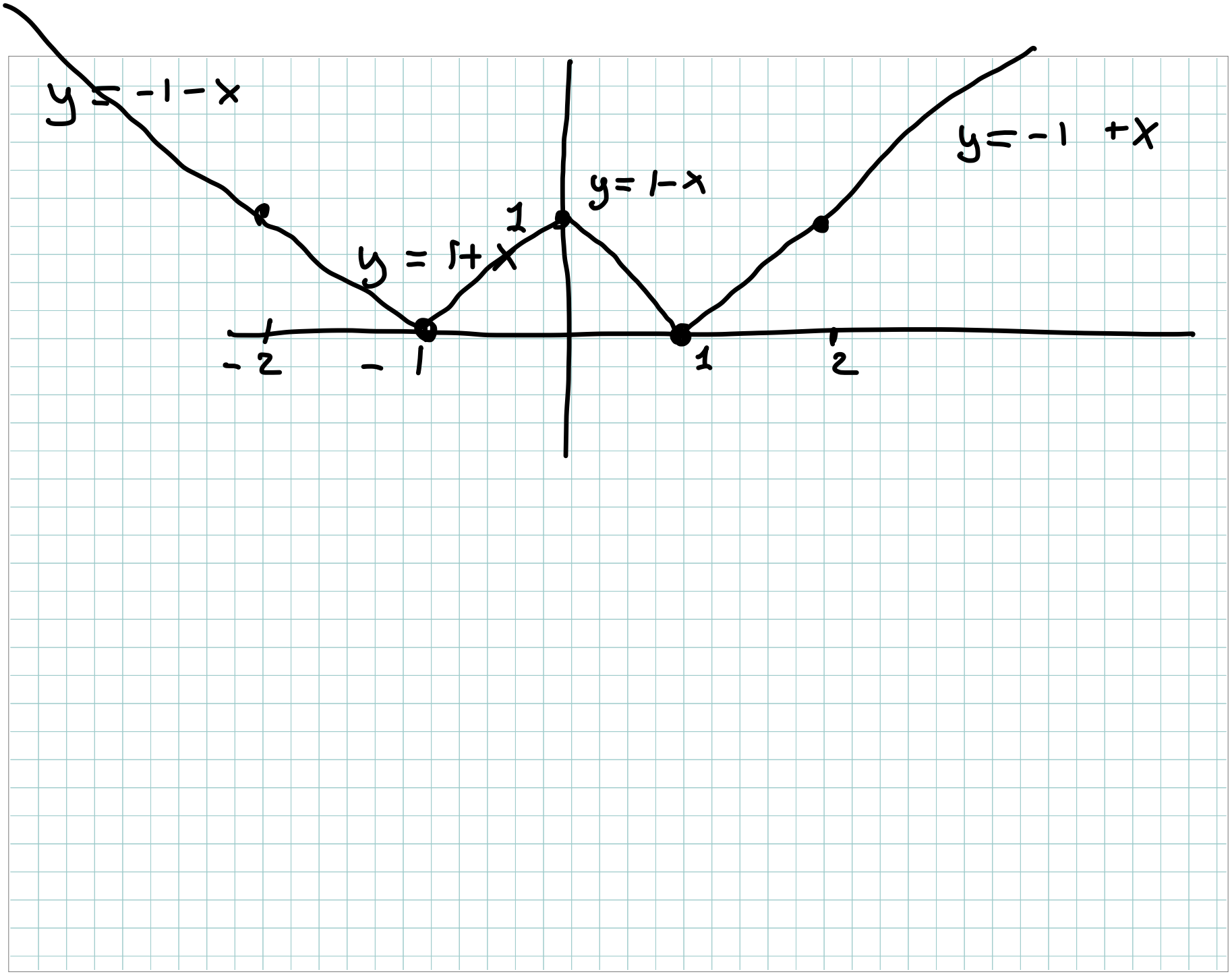
$$|1+x| = \begin{cases} 1+x & \text{if } 1+x \geq 0 \text{ and } x < 0 \\ -(1+x) & \text{if } 1+x < 0 \text{ and } x < 0 \end{cases}$$

$x \geq -1$

$x < -1$

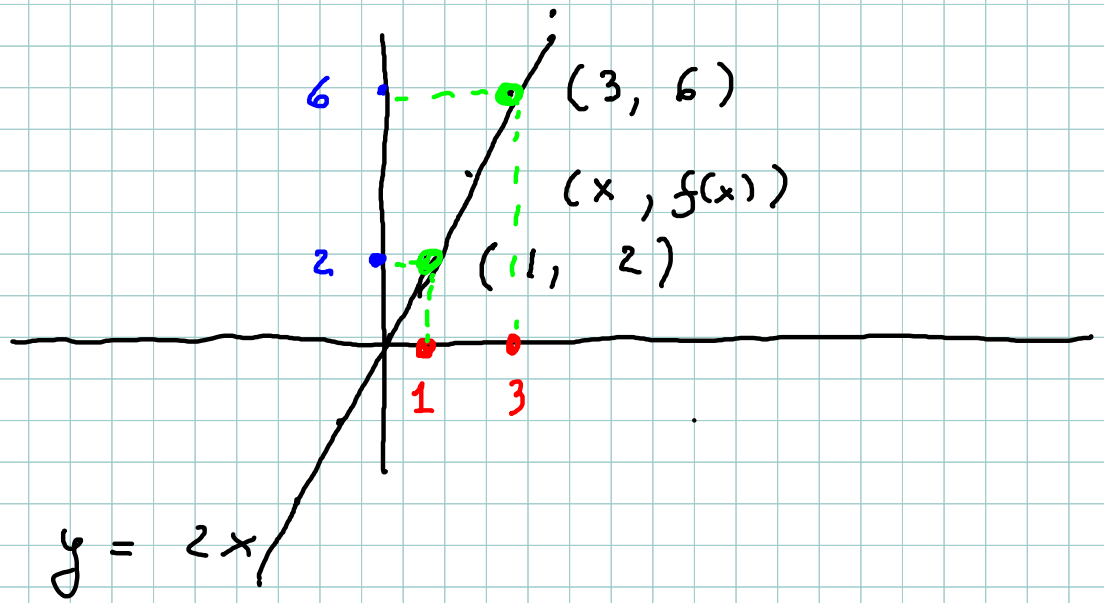
$$|1-|x|| = \begin{cases} 1-x & \text{if } 0 \leq x \leq 1 \\ -1+x & \text{if } x > 1 \\ 1+x & \text{if } -1 \leq x < 0 \\ -1-x & \text{if } x < -1 \end{cases}$$




y = -1 - x
y = 1 + x
y = 1 - x
y = -1 + x
1
-2
-1
1
2

Ch 9 Inverse functions

$f(x) = 2x$



x	$f^{-1}(x)$
6	3
2	1
4	2
x	x/2

x	f(x)
1	2
3	6
x	2x

Formula for $f(x)$ $f(x) = 2x$

Formula for $f^{-1}(y)$

$y = 2x$ solve for x

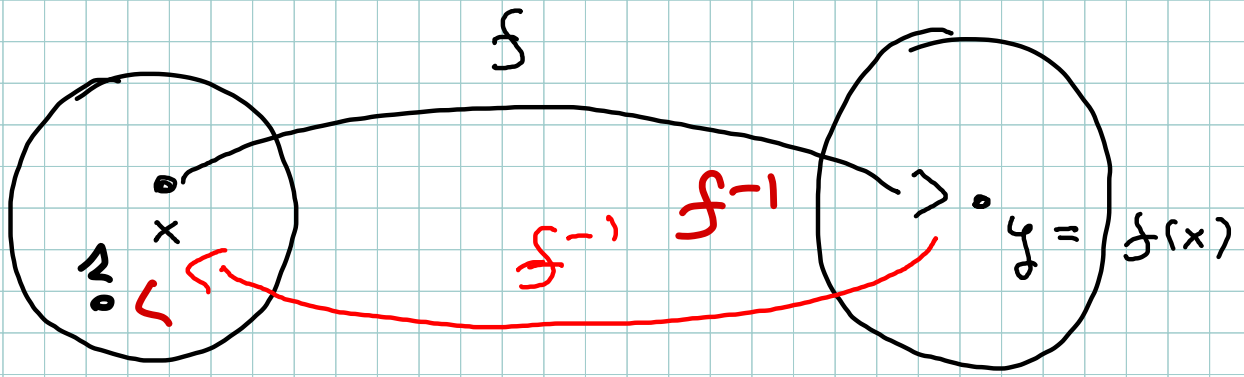
$\frac{y}{2} = x$

$$f^{-1}(y) = \frac{y}{2}$$

$$f^{-1}(x) = \frac{x}{2}$$

} formulas for inverse

Inverse function



Domain of f

f

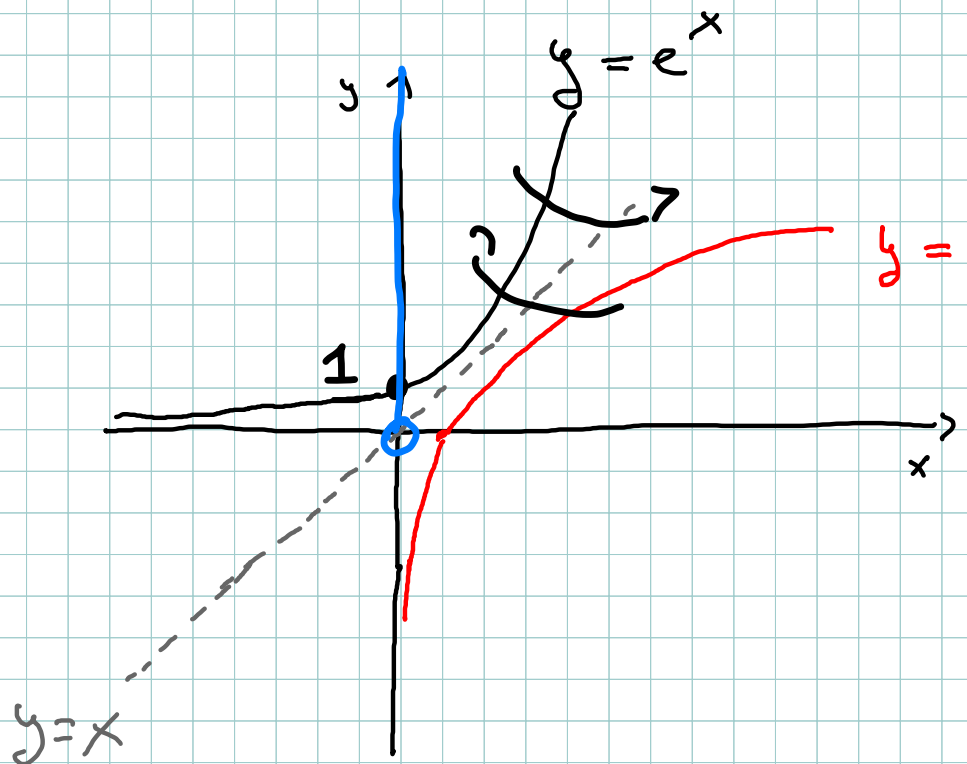
Range of f

Range of f^{-1}

Domain of f^{-1}

f^{-1} : notation for inverse of f

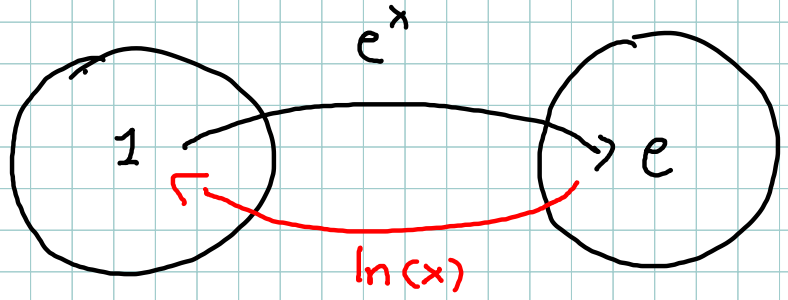
- The graph of $f^{-1}(y)$ is the graph of $f(x)$ flipped around the line $y = x$
- Domain f^{-1}= Range f. Range f^{-1} = Domain f
- To find a formula for f^{-1} set $y = f(x)$ and solve for x



Domain of $f(x) = e^x = (-\infty, +\infty)$

Range of $f(x) = e^x$

$(0, +\infty)$



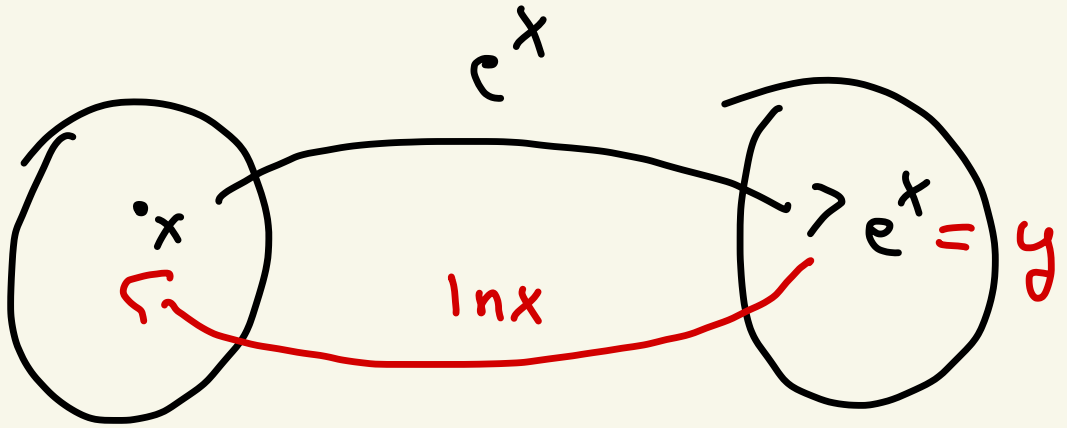
$f^{-1}(f(x)) = x$

$f(f^{-1}(y)) = y$

Remember

Inverse of e^x is $\ln x$

$f(x) = e^x$ $\quad$ $f^{-1}(x) = \ln x$



Domain of e^x
$(-\infty, +\infty)$

Range of e^x $(0, +\infty)$
Domain of $\ln x$

Range of
$\ln x$

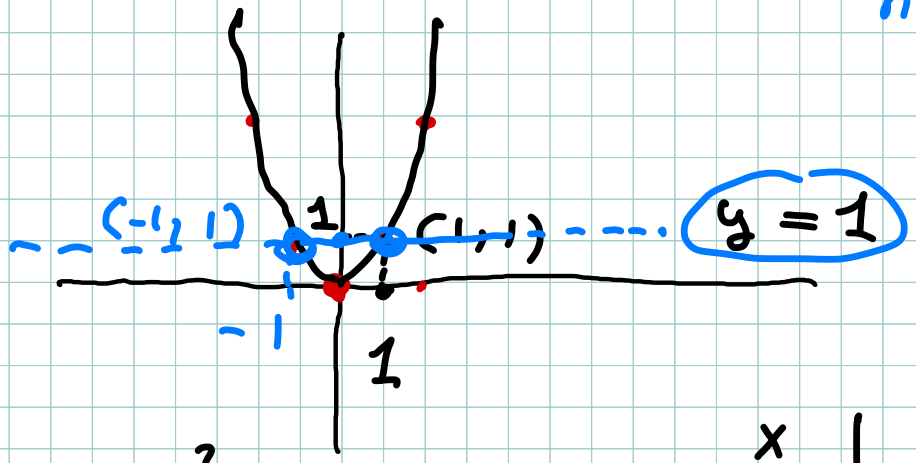
$\ln(e^x) = x$ $\quad$ $e^{\ln(y)} = y$

Does $f(x) = x^2$ have an inverse function?

Domain:

Range:

Does not satisfy horizontal line test.



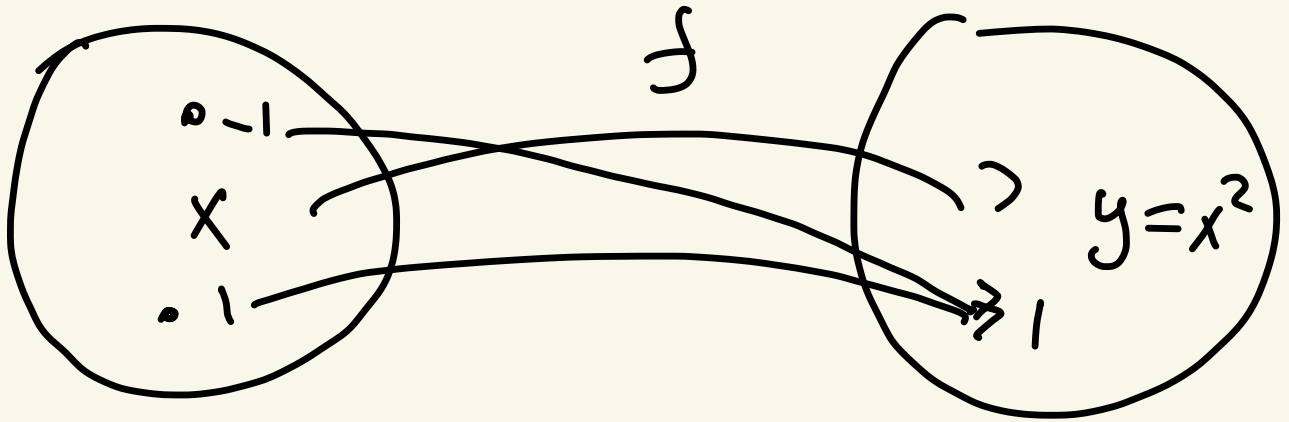
1) $y = x^2$

2)

x	f(x)
-1	1
0	0
1	1
2	4

x	$f^{-1}(x)$
1	-1
0	0
1	1
4	2

NOT A FUNCTION



To find formula for f^{-1}

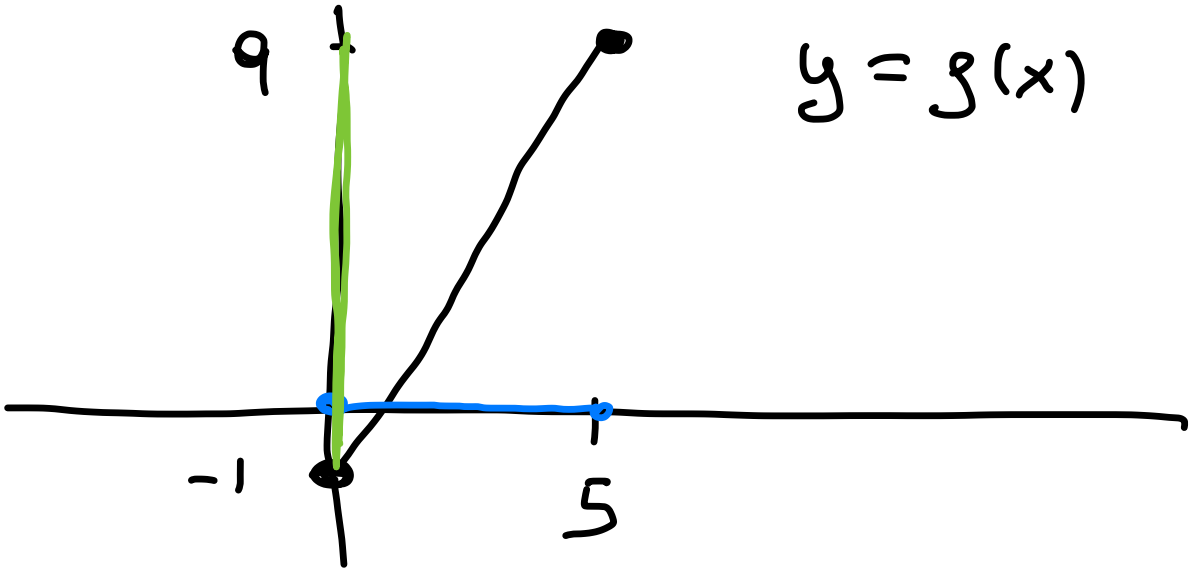
look at $y = x^2$ solve for x

$$\pm\sqrt{y} = x$$

but $f(y) = \pm\sqrt{y}$ NOT

Formula for a function.

Given $g(x) = 2x - 1$ on the domain $0 \leq x \leq 5$. Is g invertible ? If it is find the inverse , its domain and its range



g is invertible because it satisfies horizontal line test

Domain of g^{-1} is range of g : $[-1, 9]$

Range of g^{-1} is domain of g : $[0, 5]$

To find a formula for f^{-1}

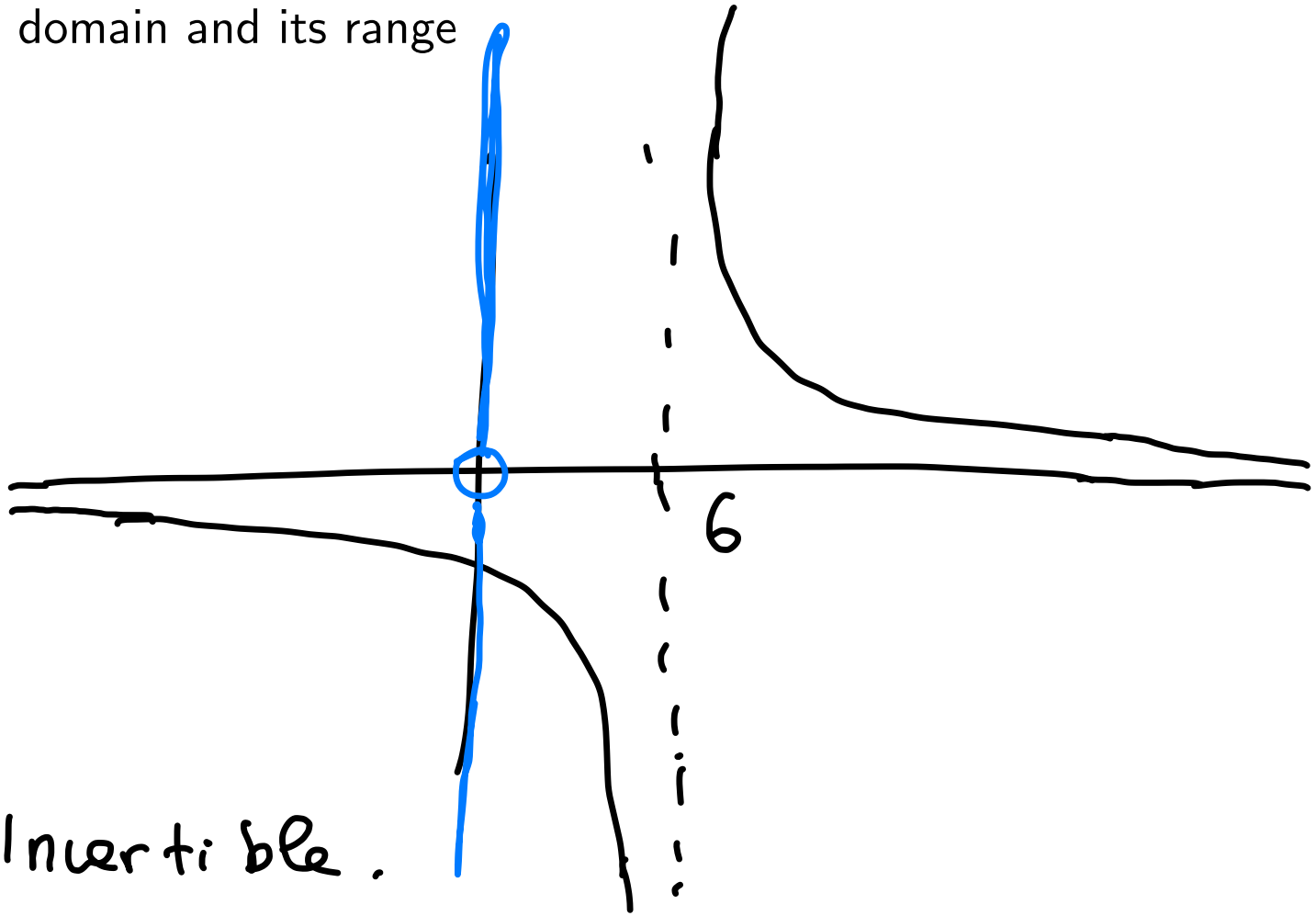
set $y = 2x - 1$ solve for x

$$y + 1 = 2x$$

$$\frac{y+1}{2} = x$$

$$f^{-1}(y) = \frac{y+1}{2} \quad \text{or} \quad f^{-1}(x) = \frac{x+1}{2}$$

Given $f(x) = \frac{2}{x-6}$. Is g invertible ? If it is find the inverse , its domain and its range



Invertible.

Domain of f^{-1} = Range of f =

$(-\infty \; 0) \cup (0 \; +\infty)$

Range of f^{-1} = Domain of f =

$(-\infty\ 6) \cup (6\ +\infty)$

Formula for f^{-1}:

$y = \frac{2}{x-6}$ solve for x

$y \cdot (x-6) = 2$

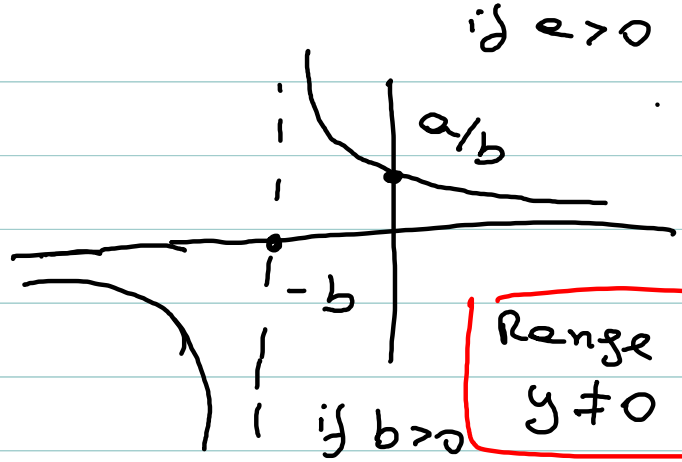
$x - 6 = \frac{2}{y}$

$x = 6 + \frac{2}{y}$

$f^{-1}(y) = 6 + 2/y$

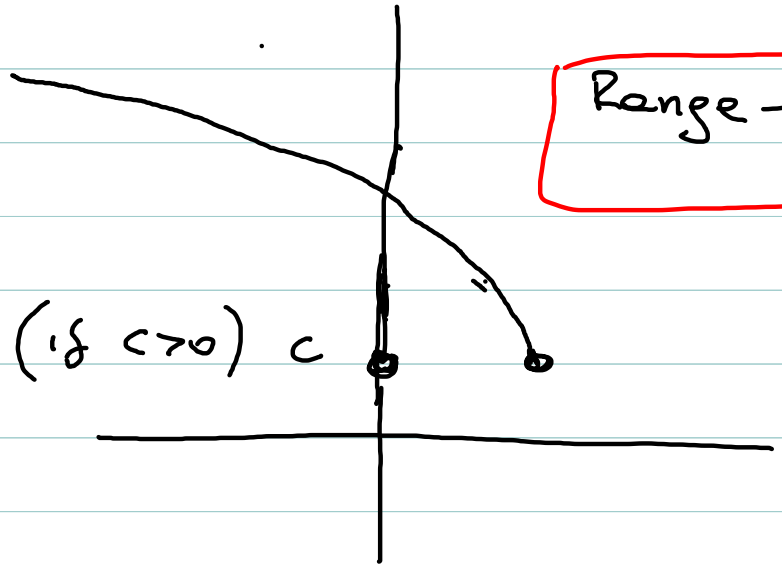
Graphs for Ch 9 problem 1

b) $h(x) = \dfrac{a}{x+b}$



Range $y \neq 0$

c) $g(x) = a\sqrt{b-x} + c$



Range - $y \geq c$