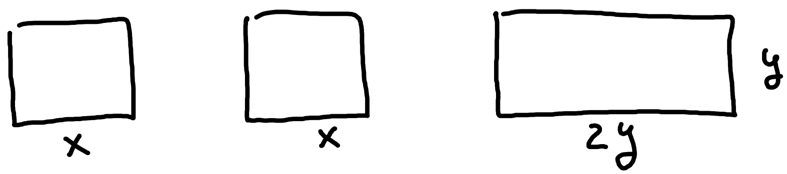
Lesson 12

Read Chapter 8

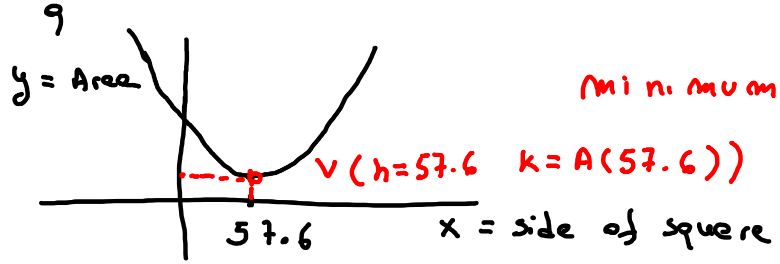
Composition

You have 720 m of fencing with which to build 3 enclosures. Two are identical squares and one is a rectangle that is twice as long as it is wide. What should the dimensions of the squares be, in order to to minimize the combined area of all three enclosures ? What should the dimensions of the squares be, in order to maximize the combined area of all three enclosures ?



From last time:

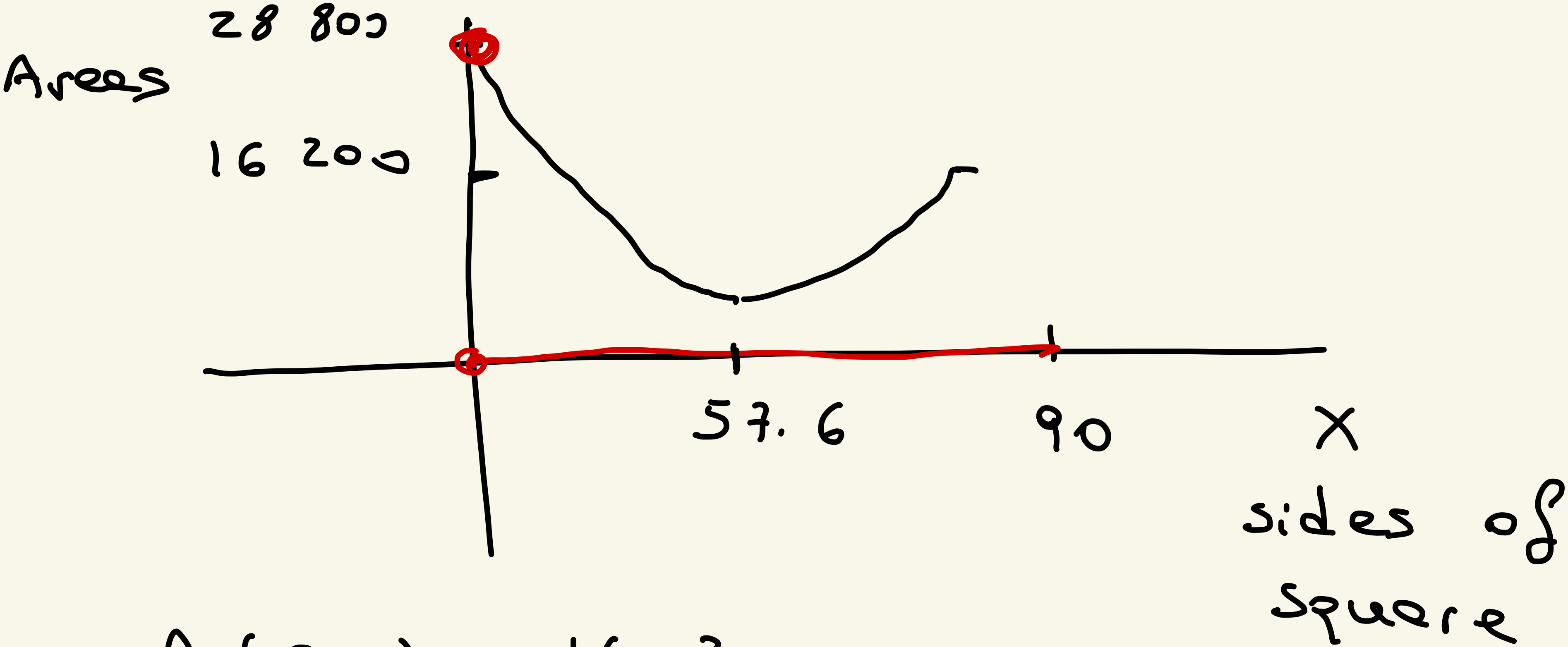
$A(x) = \frac{50}{9}x^2 - 640x + 28800$ $\qquad 720 = 8x + \cancel{6y}$



minimum area is A(57.6)

How about maximizing area?

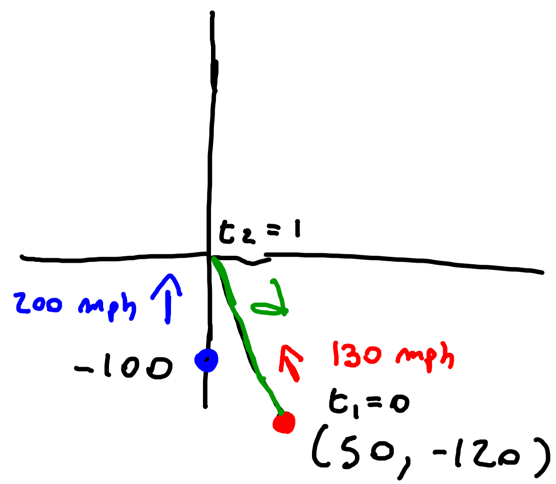
Domain of A(x) $\qquad 0 \leq x \leq \frac{720}{8} = 90$



$A(90) = 16\ 200$

Area is max when $x = 0$

An American Airlines plane is flying North at a speed of 200 mph. At time $t = 0$ it is located 100 mi South of a control tower. A United Airlines plane is flying in a straight line towards the control tower with a speed of 130 mi/hour. At time $t = 0$ it is located 50 mi East and 120 mi South of the control tower. When are the planes closest ? How close do they get ?



$d = \sqrt{50^2 + 120^2} = 130$

$t_2 = \frac{130}{130} = 1$

AA $\boxed{\begin{aligned} x(t) &= 0 \\ y(t) &= -100 + 200t \end{aligned}}$

UA : $\boxed{\begin{aligned} x(t) &= 50 + \frac{0-50}{1-0} t \\ y(t) &= -120 + \frac{0-(-120)}{1-0} t \end{aligned}}$

AA $(0, -100 + 200t)$ UA $= (50 - 50t, -120 + 120t)$

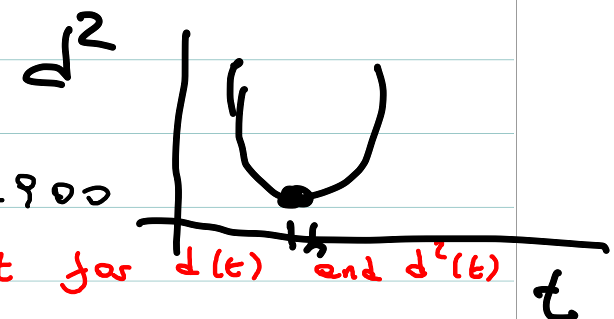
$d(t) = \sqrt{(50-50t)^2 + (-100+200t-(-120+120t))^2}$ Minimize $d(t)$

SIMPLIFY

$$d(t) = \sqrt{8900t^2 - 1800t + 2900}$$

1) For which value of t is $d(t)$ minimum?

2) What is minimum distance?



1) Trick look at $d^2(t) = 8900t^2 - 1800t + 2900$

min for $t = \frac{1800}{2 \cdot 8900} \approx 0.1$ $= -b/2a$ SAME t for $d(t)$ and $d^2(t)$

why? For positive values squaring does not change order

Ex:

t	$f(t)$	$f^2(t)$
1	5	25
2	2	4
3	1	1
4	7	49
5	-8	64

2) Compute $d(0.1) = \sqrt{8900(0.1)^2 - 1800 \cdot 0.1 + 2900} \simeq 53$ miles

Ch 8

What is a function ?

$$x \to \boxed{f} \to f(x) = y$$

g(f(x)) in pictures

composition

$$x \to \boxed{f} \to \underset{y}{f(x)} \to \boxed{g} \to \underset{\overset{\shortparallel}{z}}{g(f(x))}$$

Example $f(x) = x^2 + 1, \quad g(x) = \underline{2x + 3}$

$$g(f(x)) = g(x^2+1) = 2(x^2+1)+3 = 2x^2+5$$

$$\neq$$

$$f(\underbrace{g(x)}) = f(2x+3) = (2x+3)^2 + 1$$

Suppose $f(x)$ is the profit made by selling x barrels of apples and $g(x)$ is the number of barrels of apples produced by x trees. Explain in words the meaning of $f(g(x))$.

$$\underbrace{x}_{\text{Trees}} \longrightarrow \boxed{g} \longrightarrow g(x) \longrightarrow \boxed{f} \longrightarrow f(g(x))$$

$g(x)$: number of barrells of apples

$f(g(x))$: profit

f(# barrels produced by x trees)

= profit made by selling the barrels produced by x trees

Example $f(x) = \begin{cases} \boxed{x} + 1 & \text{if } \boxed{x} \leq 0 \\ 2\boxed{x}^2 + \boxed{x} + 1 & \text{if } \boxed{x} > 0 \end{cases}$ $\quad g(x) = 2x + 3$

$$g(f(x)) = \begin{cases} g(x+1) & \text{if } x \le 0 \\ g(2x^2 + x + 1) & \text{if } x > 0 \end{cases} = \begin{cases} 2(x+1)+3 & \text{if } x \le 0 \\ 2(2x^2+x+1)+3 & \text{if } x > 0 \end{cases}$$

$$f(g(x)) = \begin{cases} (2x+3)+1 & \text{if } (2x+3 \le 0) \quad \text{if } \cancel{2}x \le -3/2 \\ 2(2x+3)^2 + (2x+3)+1 & \text{if } (2x+3 > 0) \quad x > -\frac{3}{2} \end{cases}$$

Write the following functions as composition of two functions:

$e^{x^3} = f(g(x))$

$x \longrightarrow x^3 = y \longrightarrow e^{x^3} = e^{y}$

$\boxed{g(x) = x^3}$ $\quad f(y) = e^{y}$ $\quad \boxed{f(x) = e^{x}}$

$\sqrt{x^3+1} = f(g(x))$

$x \longrightarrow x^3 + 1 = y \longrightarrow \sqrt{y} = \sqrt{x^3+1}$

$g(x) = x^3+1$ $\quad f(y) = \sqrt{y}$ $\quad f(x) = \sqrt{x}$

$x \longrightarrow x^3 = y \longrightarrow \sqrt{y+1} = \sqrt{x^3+1}$

$\boxed{g(x) = x^3}$ $\quad f(y) = \sqrt{y+1}$ $\quad \boxed{f(x) = \sqrt{x+1}}$

$f(x) = ||1-|x||$ find a multipart formula for f (no | | there) and draw the graph of f.

Graphs

$y = 1 - |x|$

Algebra

$$f(x) = \begin{cases} 1-|x| & \text{if } 1-|x| \geq 0 \\ & \end{cases}$$

Third attempt

$$f(x) = \begin{cases} |1-x| & \text{if } x \geq 0 \\ |1+x| & \text{if } x < 0 \end{cases}$$

$$\begin{cases} 1-x & \text{if } 1-x \geq 0 \text{ and } x \geq 0 \\ -(1-x) & \text{if } 1-x < 0 \text{ and } x \geq 0 \end{cases}$$

$$\begin{cases} 1+x & \text{if } 1+x \geq 0 \text{ and } x < 0 \\ -(1+x) & \text{if } 1+x < 0,\ x < 0 \end{cases}$$