## Lesson 11

Chapter 7. Min/max problems

Parabola through three points
Find the equation of the parabola through $(1,2),(-1,1)$ and $(2,3)$
Start with standard form:

$$
y=a x^{2}+b x+c
$$

plug in all three points to get three equations.

$$
\begin{aligned}
& 2=a \cdot 1^{2}+b \cdot 1+c \\
& 1=a(-1)^{2}+b(-1)+c \\
& 3=a \cdot \mathbf{2}^{2}+b \cdot \mathbf{2}+c
\end{aligned}
$$

Solve a system

$$
\begin{aligned}
& \left\{\begin{array} { l } 
{ 2 = a + b + c } \\
{ 1 = a - b + c } \\
{ 3 = 4 a + 2 b + c }
\end{array} \quad \left\{\begin{array}{l}
a=2-b-c \\
1=(2-b-c)-b+c \\
3=4(2-b-c)+2 b+c
\end{array}\right.\right. \\
& \left\{\begin{array} { l } 
{ a = 2 - b - c } \\
{ 1 = 2 - 2 b } \\
{ 3 = 8 - 2 b - 3 c }
\end{array} \quad \left\{\begin{array}{l}
a=2-b-c \\
b=\frac{1}{2} \\
3=8-2\left(\frac{1}{2}\right)-3 c
\end{array}\right.\right. \\
& \left\{\begin{array} { l } 
{ a = 2 - b - c }
\end{array} \quad \left\{\begin{array}{l}
a=\frac{1}{2} \\
\frac{3-8+1}{3}=c \\
c=\frac{1}{2}
\end{array}\right.\right. \\
& \begin{array}{l}
a=\frac{4}{3} \\
e=2-\frac{1}{2}-\frac{4}{3}=\frac{1}{6}
\end{array} \\
& y=\frac{1}{6} x^{2}+\frac{1}{2} \times+\frac{4}{3}
\end{aligned}
$$

Current Status
Epidemiologic Curves
Cumulative Counts
Demographics
Testing
CoVID-like Illness
Healthcare System
Vaccinations

Cases
Hospitalizations
Deaths
R-effective Estimates
Count
Rate

COVID-19 IN WASHINGTON STATE Cases and Deaths by Specimen Collection Date, and Hospitalizations by Admission Date
DATA AS OF 10/12/2021 11:59PM PT
This chart shows the progression of the COVID-19 outbreak in Washington by cases, hospitalizations and deaths over time and is known as an epidemiologic curve. The epidemiologic curve is the curve referred to in the phrase, "flatten the curve." Learn More

SELECT COUNTIES
O Search
Select all
Adams County Asotin County Benton County Chelan County Clallam County Clark County Columbia County Cowlitz County Douglas County Ferry County Franklin County Garfield County Grant County Grays Harbor County

CASE COUNTS
$\bullet$ Probable Cases $*$ Confirmed Cases $\otimes$ Incomplete (Probable Cases) Incomplete (Confirmed Cases) — Total Cases (7 day avg.) -- - Incomplete (7 day avg.)


1,787 of 689,484 cases do not have an assigned county. Cases from the last 8 days may yet not be reported.

Summary Data Tables

Find equation of prabole with vertex at $(5,400)$ and through $(5.25,1000)$
$\qquad$

$$
1000=a(5.25-5)^{2}+400
$$

$$
\begin{aligned}
& 1000=a \frac{1}{16}+400 \\
& 600=a \cdot \frac{1}{16} \\
& 600 \times 16=a \\
& y=600 \times 16(x-5)^{2}+400
\end{aligned}
$$

A min/max problem is a modeling problem where you need to minimize/maximize a quantity $q$.
In this class $q=q(x)=a x^{2}+b x+c-y=a x^{2}+b x+C$
The min / max usually is at the vertex of the parabola


To solve a min/max problem

- Choose your variables and find a formula for $\mathrm{q} . \mathrm{q}=\mathrm{q}(\mathrm{x})$.
- In $120 \mathrm{q}(\mathrm{x})$ should involve a quadratic function. Usually you find $\max / \mathrm{min}$ by finding the vertex.
- Pay attention whether the problem is asking for an $\times$ value (h) or a $q$ value (k) or both.


## Issues-tricks

- The quantity is a distance given by a formula : $q=\sqrt{\cdots x \cdots}$.
- q depends on more than one variable. $\mathrm{q}=\mathrm{q}(\mathrm{x}, \mathrm{y})$.
- Min/max not at vertex.

Rosalie is organizing a circus performance to raise money for a charity. She is trying to decide how much to charge for tickets. From past experience she knows that the number of tickets sold is a linear function of the price. If she charges 5 dollars per ticket, she can sell 1000 tickets, if she charges 7 dollars she can only sell 900 tickets. How much should she charge per ticket to make the most money ?
I need to maximize $q$ : the amount of money Roselie makes $q(x)=$ quadratic formula Look at the graph of $y=p(x)$ and find mex
$q(x)$ : money Roselir makes.
$\downarrow$ price of ticket

$$
q(x)=n(x) \cdot x
$$

$n(x)=$ the number of tickets sold
(5 1000) (7,900)

$$
\begin{aligned}
& y=y_{0}+m\left(x-x_{0}\right) \\
& y=1000+\frac{1000-900}{5-7}(x-5) \\
& y=1000-50(x-5)
\end{aligned}
$$



$$
q(x)=(1000-50(x-5)) \cdot x
$$

Find which value of $x$ makes $q(x)$ maximum.

$$
\begin{aligned}
& q(x)=1000 x-50 x(x-5) \\
& q(x)=1000 x-50 x^{2}+250 x \\
& q(x)=-50 x^{2}+1250 x
\end{aligned}
$$



Problem went a ticket price so $h \quad h=-\frac{b}{2 a}=-\frac{1250}{-100}=12.50$

You have 720 m of fencing with which to build 3 enclosures. Two are identical squares and one is a rectangle that is twice as long as it is wide. What should the dimensions of the squares be, in order to to minimize the combined area of all three enclosures? What should the dimensions of the squares be, in order to maximize the combined area of all three enclosures?


$$
\begin{aligned}
& 720=4 x+4 x+2 y+y+2 y+y \\
& 720=8 x+6 y
\end{aligned}
$$

$$
\frac{720-8 x}{6}=y
$$

Area: $A(x)=2 x^{2}+2\left(\frac{720-8 x}{6}\right)^{2}$ went to find $x$ velue thet makes Aree smellest. Do elgebre...

$$
A(x)=\frac{50}{9} x^{2}-640 x+28800
$$

Ares


At the vertex cree is smellesl

$$
h=-\frac{b}{2 a}=\frac{640}{2 \cdot \frac{50}{9}}=57.6
$$

If problem asked for minimum Area, what should I celculete?

In this case I need $k$

1) Use formula

$$
\begin{aligned}
& \text { 2) } k=A(h)=A(57.6) \\
& k=\frac{50}{9}(57.6)^{2}-640 \cdot(57.6)+ \\
& 28800
\end{aligned}
$$

An American Airlines plane is flying North at a speed of 200 mph . At time $t=0$ it is located 100 mi South of a control tower. A United Airlines plane is flying in a straight line towards the control tower with a speed of $130 \mathrm{mi} /$ hour. At time $t=0$ it is located 50 mi East and 100 mi South of the control tower. When are the planes closest? How close do they get?


$$
\begin{aligned}
& d=\sqrt{50^{2}+120^{2}}=130 \\
& t_{2}=\frac{130}{130}=1
\end{aligned}
$$

$A A \quad \begin{aligned} & x(t)=0 \\ & y(t)=-100+200 t\end{aligned}$
UN: $x(t)=50+\frac{0-50}{1-0} t$
$y(t)=-120+\frac{0-(-120)}{1-0} t$

$$
\begin{aligned}
& A A(0,-100+200 t) \quad U A=(50-50 t,-120+120 t) \\
& d(t)=\sqrt{(50-50 t)^{2}+(-100+200 t-(-120+120 t))^{2}} \quad \text { Minimize } d(t)
\end{aligned}
$$

Simplify

$$
d(t)=\sqrt{8900 t^{2}-1800 t+2900}
$$

