Lesson 11

Chapter 7. Min/max problems

Parabola through three points

Find the equation of the parabola through (1,2),(-1,1) and (2,3)

Start with standard form:

$$y = ax^2 + bx + c$$

plug in all three points to get three equations.

$$2 = a\cdot 1^2 + b\cdot 1 + c$$
$$1 = a(-1)^2 + b(-1) + c$$
$$3 = a\cdot 2^2 + b\cdot 2 + c$$

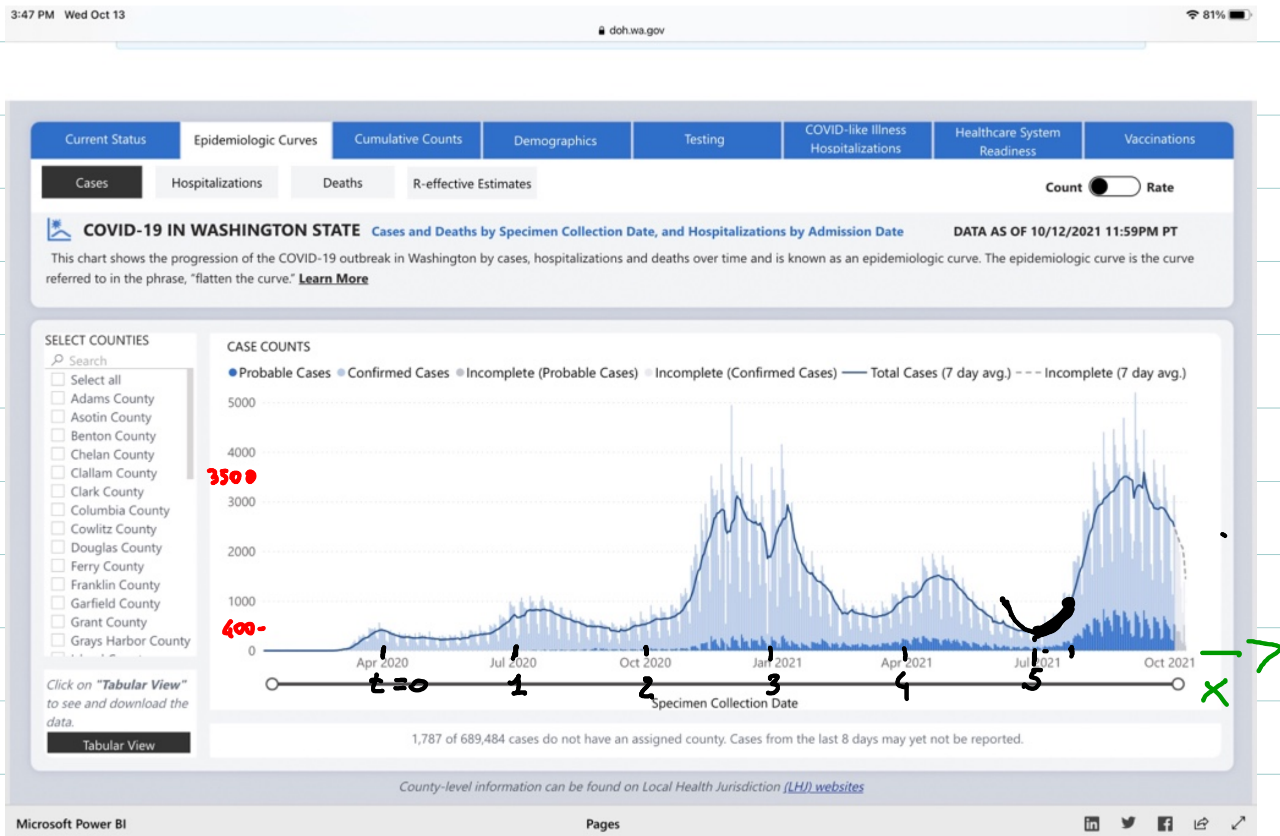
Solve a system

$$\begin{cases} 2 = a + b + c \\ 1 = a - b + c \\ 3 = 4a + 2b + c \end{cases} \qquad \begin{cases} a = 2 - b - c \\ 1 = (2 - b - c) - b + c \\ 3 = 4(2 - b - c) + 2b + c \end{cases}$$

$$\begin{cases} a = 2 - b - c \\ 1 = 2 - 2b \\ 3 = 8 - 2b - 3c \end{cases} \qquad \begin{cases} a = 2 - b - c \\ b = \frac{1}{2} \\ 3 = 8 - 2\left(\frac{1}{2}\right) - 3c \end{cases}$$

$$\cdot \begin{cases} a = 2 - b - c \\ b = \frac{1}{2} \\ \frac{3 - 8 + 1}{3} = c \end{cases} \qquad \begin{cases} b = \frac{1}{2} \\ c = \frac{4}{3} \\ a = 2 - \frac{1}{2} - \frac{4}{3} = \frac{1}{6} \end{cases}$$

$$y = \frac{1}{6} x^2 + \frac{1}{2} x + \frac{4}{3}$$



Summary Data Tables

Find equation of parabola with vertex at (5, 400)

and through (5.25, 1000)

$$y = a(x-h)^2 + K$$

$$y = a(x-5)^2 + 400$$

$$1000 = a(5.25-5)^2 + 400$$

Solve for a

$$1000 = a \frac{1}{16} + 400$$

$$600 = a \cdot \frac{1}{16}$$

$$600 \times 16 = a$$

$$\boxed{y = 600 \times 16 \, (x - 5)^2 + 400}$$

A min/max problem is a modeling problem where you need to minimize/maximize a quantity q.

In this class $q = q(x) = ax^2 + bx + c$ — $y = ax^2 + bx + c$

The min / max usually is at the vertex of the parabola

To solve a min/max problem

- Choose your variables and find a formula for q. q=q(x).
- In 120 q(x) should involve a quadratic function. Usually you find max/min by finding the vertex.
- Pay attention whether the problem is asking for an x value (h) or a q value (k) or both.

Issues–tricks

- The quantity is a distance given by a formula : $q = \sqrt{\cdots x \cdots}$.
- q depends on more than one variable. q=q(x,y).
- Min/max not at vertex.

Rosalie is organizing a circus performance to raise money for a charity. She is trying to decide how much to charge for tickets. From past experience she knows that the number of tickets sold is a linear function of the price. If she charges 5 dollars per ticket , she can sell 1000 tickets, if she charges 7 dollars she can only sell 900 tickets. How much should she charge per ticket to make the most money ?

I need to maximize q : the amount of money Rosalie makes

$q(x)$ = quadratic formula

Look at the graph of $y = q(x)$ and find max

$q(x)$: money Roselie makes.

$x \rightarrow$ price of ticket

$q(x) = n(x) \cdot x$

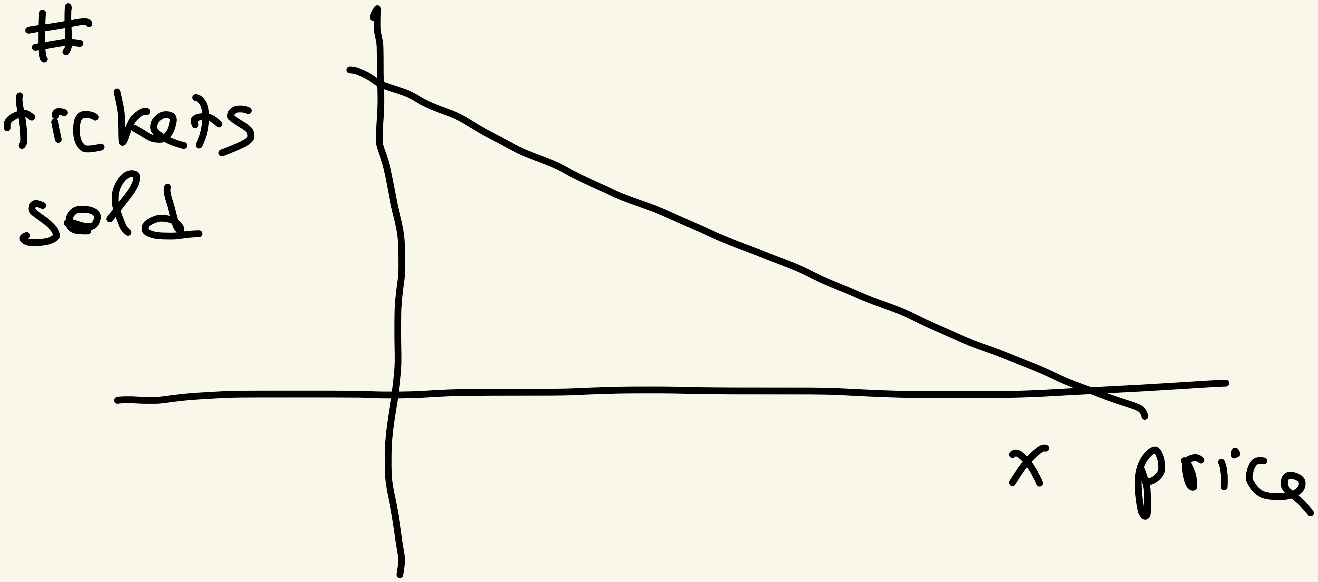
$n(x) =$ the number of tickets sold

$(5 \quad 1000) \qquad (7, \ 900)$

$$y = y_0 + m(x - x_0)$$

$$y = 1000 + \frac{1000 - 900}{5 - 7}(x - 5)$$

$$y = 1000 - 50(x - 5)$$



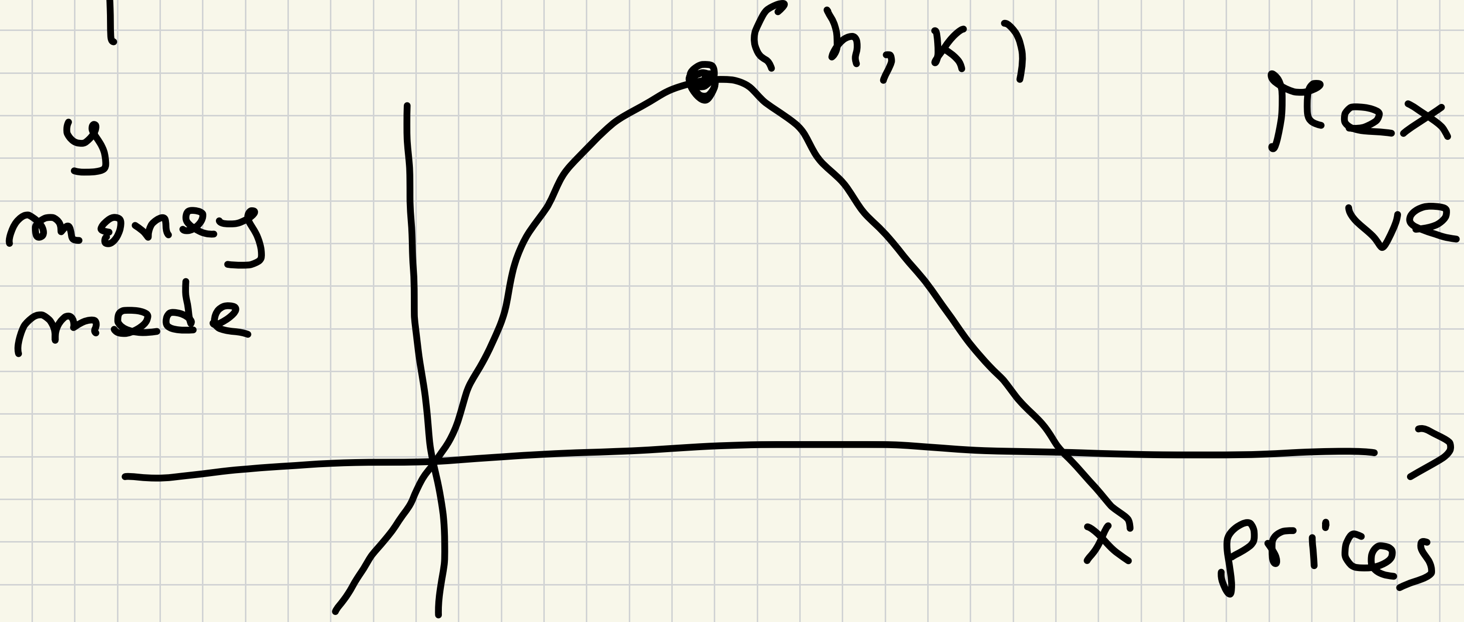
$$q(x) = \left(1000 - 50(x-5)\right) \cdot x$$

Find which value of x makes q(x) maximum.

$$q(x) = 1000x - 50x(x-5)$$

$$q(x) = 1000x - 50x^2 + 250x$$

$$q(x) = -50x^2 + 1250x$$



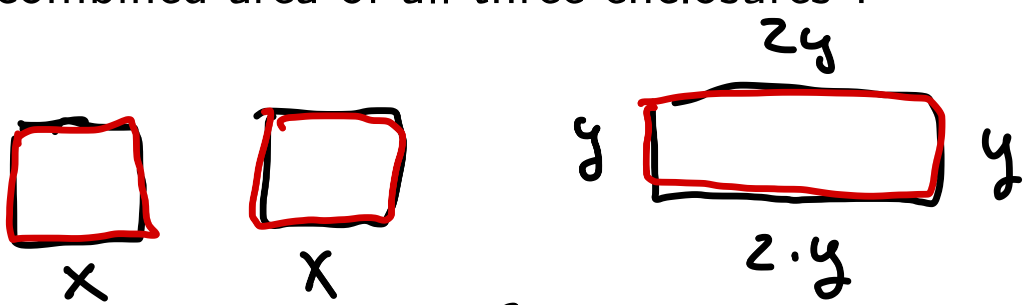
max is at vertex

Problem want a ticket price

so h $\quad h = -\frac{b}{2a} = -\frac{1250}{-100} = \boxed{12.50}$

You have 720 m of fencing with which to build 3 enclosures. Two are identical squares and one is a rectangle that is twice as long as it is wide. What should the dimensions of the squares be, in order to to minimize the combined area of all three enclosures ? What should the dimensions of the squares be, in order to maximize the combined area of all three enclosures ?

x



$$\text{Area}: \quad x^2 + x^2 + 2y\cdot y = 2x^2 + 2y^2$$

$$720 = 4x + 4x + 2y + y + 2y + y$$

$$720 = 8x + 6y$$

$$\frac{720-8x}{6} = y$$

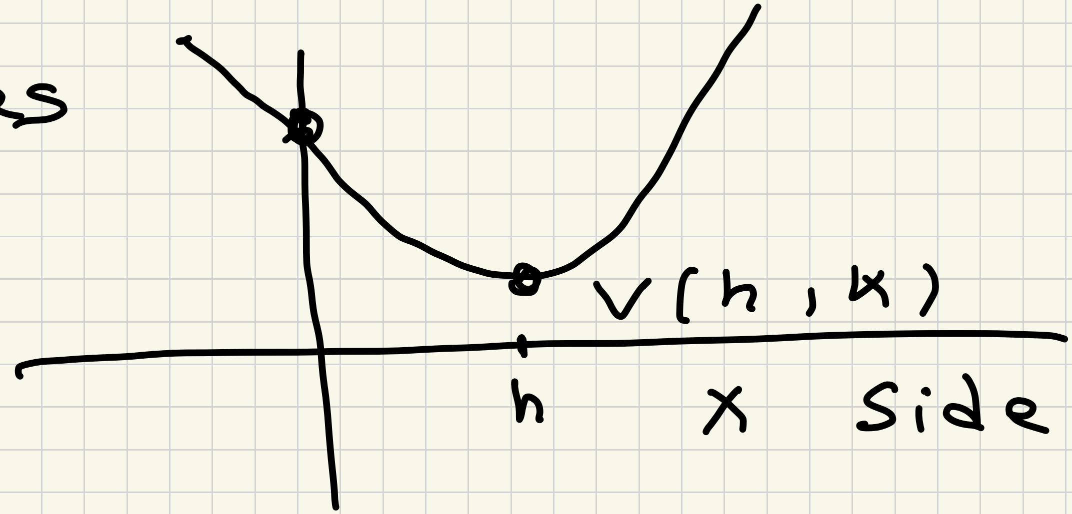
Area : $A(x) = 2x^2 + 2\left(\frac{720-8x}{6}\right)^2$

want to find x value that makes Area smallest.

Do algebra ...

$$A(x) = \frac{50}{9}x^2 - 640x + 28800$$

Areas



side of square

At the vertex area is smallest

$$h = -\frac{b}{2a} = \frac{640}{2 \cdot \frac{50}{9}} = \boxed{57.6}$$

If problem asked for minimum Area, what should I calculate?

In this case I need K

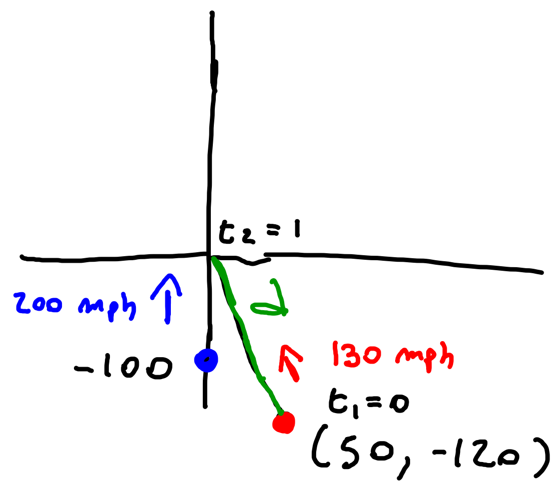
1) Use formula

2) $K = A(h) = A(57.6)$

$$K = \frac{50}{9}(57.6)^2 - 640 \cdot (57.6) +$$

28800

An American Airlines plane is flying North at a speed of 200 mph. At time $t = 0$ it is located 100 mi South of a control tower. A United Airlines plane is flying in a straight line towards the control tower with a speed of 130 mi/hour. At time $t = 0$ it is located 50 mi East and 100 mi South of the control tower. When are the planes closest ? How close do they get ?



$d = \sqrt{50^2 + 120^2} = 130$

$t_2 = \frac{130}{130} = 1$

AA $\boxed{\begin{aligned} x(t) &= 0 \\ y(t) &= -100 + 200t \end{aligned}}$

UA : $\boxed{\begin{aligned} x(t) &= 50 + \frac{0-50}{1-0}\, t \\ y(t) &= -120 + \frac{0-(-120)}{1-0}\, t \end{aligned}}$

AA $(0, -100 + 200t)$ UA $= (50 - 50t, -120 + 120t)$

$d(t) = \sqrt{(50 - 50t)^2 + (-100 + 200t - (-120 + 120t))^2}$ Minimize $d(t)$

SIMPLIFY

$$d(t) = \sqrt{8900t^2 - 1800t + 2900}$$