① Let $V = C[-1, 1]$ $F = \mathbb{R}$ $\langle f, g \rangle = \int_{-1}^{1} f(x) g(x) \, dx$

Show $\langle \ \rangle$ is an inner product on V

② Let V be an inner product space, let

$v, v' \in V$ $v \neq v'$; show there is

$w \in V$ s.t $\langle v \ w \rangle \neq \langle v' \ w \rangle$

③ Let V be an inner product space and

$v, w \in V$ show $\|v\| = \|w\| \iff v+w \perp v-w$

④ QR decomposition: Given $A \in M_{k \times n}(F)$

of rank n (i.e the columns of A are linearly indep)

Show we can find a $k \times n$ matrix Q and

an $n \times n$ matrix R such that: the

columns of Q are orthonormal vectors,

R is non singular and upper triangular

and $A = Q \cdot R$