

① Find 3×3 matrices A, B, C having only $\lambda = 2$ for eigenvalue s.t

1) For A , $\dim E_\lambda = 3$. What is $\dim K_\lambda$?

2) For B , $\dim E_\lambda = 2$. What is $\dim K_\lambda$?

3) For C , $\dim E_\lambda = 1$. What is $\dim K_\lambda$?

Are A, B, C diagonalizable?

② Let $\alpha: V \rightarrow V$ be a linear transformation. $v \in V$.

Then $W = \text{span} \{ \alpha^l(v) \mid l \geq 0 \}$ is α invariant and

it is the intersection of all invariant subspaces containing v .

③ Suppose $\dim V = n$; we say that

$\alpha \in \mathcal{L}(V)$ is nilpotent if there is k s.t $\alpha^k = 0$.

The smallest such k is called the index of nilpotence of α . If α is nilpotent with index k there is w in V s.t $W = \text{span} \{ \alpha^l(w) \mid 0 \leq l \leq k-1 \}$ has dimension k .

$(T-\lambda I)^{p-1}x, (T-\lambda I)^{p-2}x, \dots, (T-\lambda I)x, x$ is

called a cycle of generalized eigenvectors of λ .

Prove that K_λ has a basis consisting of a union of disjoint cycles of generalized eigenvectors of λ .

$\alpha: V \rightarrow V$ $\dim V = n$ α nilpotent with index k

There are numbers $k = k_1 \cdots k_t$ with $n = k_1 + \cdots + k_t$

and vectors v_1, \dots, v_t s.t

$\{v_1, \alpha v_1, \dots, \alpha^{k_1} v_1, v_2, \alpha v_2, \dots, \alpha^{k_2} v_2, \dots\}$ is a basis for V

Let $\dim V < +\infty$ $\alpha: V \rightarrow V$
 $\beta: V \rightarrow V$

be nilpotent

if $T_B^B(\alpha)$ and $T_B^B(\beta)$ are similar

must the index of nilpotency for α and β be the same?

Look at diff eq book definition of e^A

Eigenvalues / vector of stochastic matrices