

① Suppose V is a real vector space and $T: V \rightarrow V$ is such that $T^2 = -I$. Show that T has no eigenvalues.

② Prove that AB has the same eigenvalues as BA , where A, B are $n \times n$ matrices. Does it have the same eigenvectors?

③ Two square matrices A and B are similar if there is an invertible matrix P s.t. $B = P^{-1} A P$. Do similar matrices have the same characteristic polynomial, eigenvalues, eigenvectors?

Show that $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are not similar but have the same characteristic polynomial.