① Let $D \quad M_{n\times n} \longrightarrow \mathbb{R}$ $\quad D(A)=\sum_{j=1}^{n}(-1)^{1+j}\, a_{1j} \det(A_{1j})$

Prove that:

a) $D\begin{pmatrix} \vdots \\ u+v \\ \vdots \end{pmatrix} = D\begin{pmatrix} \vdots \\ u \\ \vdots \end{pmatrix} + D\begin{pmatrix} \vdots \\ v \\ \vdots \end{pmatrix}$

b) $D\begin{pmatrix} \vdots \\ kv \\ \vdots \end{pmatrix} = k\, D\begin{pmatrix} \vdots \\ v \\ \vdots \end{pmatrix}$

② Let $A=(a_{ij})$ be an $n\times n$ matrix. Prove

$$\det(A)=\sum_{j=1}^{n}(-1)^{1+j}\, a_{1j}\det(A_{1j}) = \sum_{j=1}^{n}(-1)^{k+j}\, a_{kj}\det(A_{kj})$$

for all $k=2,\dots n$

③ Prove that

$$D\begin{pmatrix} \vdots \\ r \\ \vdots \\ r \\ \vdots \end{pmatrix} = 0$$