

① Let $\varphi: \mathbb{R}^{\infty} \rightarrow \mathbb{R}^{\infty}$

$$\varphi(a_1, a_2, \dots, a_n, \dots) = (a_1, a_1 + a_2, a_1 + a_2 + a_3, \dots)$$

is φ a linear transformation? If it is find $\ker(\varphi)$ and $\text{R}(\varphi)$.

2) Prove that there is no linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ that is 1-1.

3) Let $T: V \rightarrow W$ a linear transformation and suppose that V is not finite dimensional. Prove that $\ker(T)$ and $\text{Range}(T)$ cannot be both finite dimensional. Give examples of linear transformations T_1 and T_2 where $\ker(T_1)$ is finite dimensional and $\text{Range}(T_2)$ is finite dimensional.

⑥ Let U, V, W be finite dimensional vector spaces on F
 $\text{Hom}(V, W) = \{ S: V \rightarrow W \mid S \text{ is a linear transf} \}$

$\text{Hom}(V, W)$ is a vector space on F

Let $T: V \rightarrow W$ be a linear transformation

Show that the set of real transformations

$\alpha: U \rightarrow V$ s.t. $T \circ \alpha = 0$ is a subspace of $\text{Hom}(U, V)$

and find its dimension