1. Consider $\mathbb{R}$ as a vector space over $\mathbb{Q}$. Does it have a finite basis? Does it have a countable basis?

2. Show that $\mathbb{R}^\infty$ does not have a countable basis

3. Are $x^2, x^2-1, x+1$ linearly independent in $P_2[\mathbb{R}]$?

4. Let W = set of all finite sequences of real numbers that is $\{x_n\} \in W$ iff there is $m \in \mathbb{N}$ s.t $x_n = 0 \ \forall n \geq m$

   Describe Span (W)

The Fibonacci numbers

Consider the sequence $(U_n) \in \mathbb{R}^\infty$ given by $U_1 = 1$, $U_2 = 1$, $U_{n+1} = U_n + U_{n-1}$ for $n > 1$

Consider the subset $W \subseteq \mathbb{R}^\infty$ containing all sequences $\{x_n\}$ defined by:

$x_1 = a$ a is some real number

$x_2 = b$ b is some real number

$x_{n+1} = x_n + x_{n-1}$ if $n+1 \geq 3$

1) Prove $W \leq \mathbb{R}^\infty$
2) Find a basis for W

3) Consider $B = \{\{z_n\}, \{v_n\}\}$ where

$\{z_n\} = (1, \alpha, \alpha^2, \dots \alpha^n \dots)$.
$\{v_n\} = (1, \beta, \beta^2, \dots, \beta^n \dots)$

$\{z_n\}$ is in W if $\alpha^2 = \alpha + 1$
$\{v_n\}$ is in W if $\beta^2 = \beta + 1$

Let α, β be the roots of $x^2 = x + 1$

Show that with this choice of α end β B is a basis for W

4) Find scelers λ_1 and λ_2 s.t

$\{u_n\} = (1, 1, 2, 3, 5, \dots \dots) = \lambda_1 \{z_n\} + \lambda_2 \{v_n\}$

and show $u_n = \dfrac{\alpha^n - \beta^n}{\sqrt{5}}$