

① show  $(\mathbb{Z}, +)$  cannot be turned into a vector space over (any field)  $\mathbb{R}$ .

② Let  $W$  contain all vectors in  $\mathbb{R}^5$  having an odd number of entries equal to 0. For example  $(0, 1, 1, 1, 1) \in W$   
 $(0, 1, 1, 1, 0) \notin W$ . Is  $W$  a subspace of  $\mathbb{R}^5$ ?

③ Let  $U = \{(x, y, z) \mid x+y+z = 0\} \subseteq \mathbb{R}^3$   
 $V = \{(x, y, z) \mid x=y\} \subseteq \mathbb{R}^3$

What is  $U+V$

Is  $U+V$  a direct sum?

④ Prove the following

Th: If  $W_1, W_2, \dots, W_n$  are subspaces of  $V$  then

$W_1 + W_2 + \dots + W_n$  is a direct sum iff  $\forall i \leq n$

$$W_i \cap \sum_{j \neq i} W_j = \{0\}$$