① Give an example of a vector $v \in R^3$ which is a generalized eigenvector for $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $\lambda = 2$,

but it is not an eigenvector for A

② Suppose V is an n dimensional complex space, $T \in \mathcal{L}(V)$ and $\lambda = 0$ is The only eigenvalue of T. Prove that $T^n = 0$

③ Prove the Pythagorean th:
suppose u and v are orthogonal vectors in an inner product space V. Prove that $\|u+v\|^2 = \|u\|^2 + \|v\|^2$

(4) Prove $f((x,y),(z,t)) = 2xz + 3yt$ is a inner product on $\mathbb{R}^2$. Compute $\| (1,2) \|$, where $\| \ \|$ is the norm associated with the inner product f, that is $\|v\| = \sqrt{f(v,v)}$

(5) Consider $V = P^2(\mathbb{R})$, the set of degree ≤ 2 polynomials with real coefficients, with inner product $\langle p, q \rangle = \int_{-1}^{1} pq \, dx$

Calculate $\| 2x+3 \|$

Let $S = \{ p \in P^2(\mathbb{R}) \mid p'(1) = 0 \}$. Find an orthonormal basis for S.

(6) Use linear algebra to prove that

$$\left(\frac{a_1 + a_2 + \cdots + a_n}{n} \right)^2 \leq \frac{a_1^2 + \cdots + a_n^2}{n}$$

for all $a_1, \ldots, a_n$ in $\mathbb{R}$