

① Prove that if M is an $n \times n$ matrix and can be written in the form

$$M = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix} \quad \text{where } A \text{ and } C \text{ are}$$

square matrices, then $\det(M) = \det(A) \cdot \det(C)$

you can use the column expansion formula for the determinant, even though I did not prove in the video it is equivalent to the row expansion formula. It follows from problem 3.

② Prove that if M is an $n \times n$ matrix and can be written in the form

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad \text{where } A, B, C, D \text{ are}$$

square matrices, then it is not necessarily true that $\det(M) = \det(A)\det(D) - \det(B)\det(C)$

③ Prove that for any $n \times n$ matrix M

$$\det(M) = \det(M^t) . \quad \text{Do not use}$$

column expansion formula for the determinant, since I did not prove it in the videos

(Hint: if M is invertible then you can transform I into M by performing a series of elementary operations) you can use

some facts from 308 : M is invertible iff
 M^t is and $(A \cdot B)^t = B^t \cdot A^t$