

1) Find a basis for  $\mathcal{L}(\mathbb{R}^2, \mathbb{R}^3)$ , the space of all linear transformations from  $\mathbb{R}^2$  to  $\mathbb{R}^3$ .

2) Let  $V$  and  $W$  be two vector spaces.

Recall  $\mathcal{L}(V, W)$  is the set containing all linear transformations from  $V$  to  $W$

Let  $v \neq 0, v \in V$ , and  $S = \{T : V \rightarrow W \mid T(v) = 0\}$

Prove that  $S$  is a subspace of  $\mathcal{L}(V, W)$

if  $\dim V = n$  and  $\dim W = m$ , what is  $\dim(S)$ ?

3) Is  $T: M_{2 \times 2}(\mathbb{R}) \rightarrow P_2(\mathbb{R})$  defined by

$$T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = a + 2bx + (c+d)x^2$$

invertible? If so find its inverse  $T^{-1}$ ,

that is find a formula

$$T^{-1}\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

4) Is  $T: M_{2 \times 2} \rightarrow M_{2 \times 2}$  defined by

$$T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \begin{pmatrix} a+tb & a \\ c & c+d \end{pmatrix}$$
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$$T^{-1}\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

5) Find the equation (in the  $x y$  plane) of the curve you get by rotating the parabola  $y = x^2$  counterclockwise of an angle of  $\pi/6$  rad.



6) Give an example of a linear transformation  $T: V \rightarrow V$  that is one to one but not invertible.

7) Give an example of a linear transformation  $T: V \rightarrow V$  that is onto but not invertible.

8)  $B_1 = \{1, x, x^2\}$  and  $B_2 = \{2x^2 - x, 3x^2 + 1, x^2\}$  are two bases for  $P_2(\mathbb{R})$ . Find the matrices  $I_{B_1}^{B_2}$ , of change of basis from  $B_1$  to  $B_2$  and  $I_{B_2}^{B_1}$ , of change of basis from  $B_2$  to  $B_1$ .