

1) Find a basis for $\mathcal{L}(\mathbb{R}^2, \mathbb{R}^3)$, the space of all linear transformations from \mathbb{R}^2 to \mathbb{R}^3 .

2) Let V and W be two vector spaces.

Recall $\mathcal{L}(V, W)$ is the set containing all linear transformations from V to W

Let $v \neq 0, v \in V$, and $S = \{T : V \rightarrow W \mid T(v) = 0\}$

Prove that S is a subspace of $\mathcal{L}(V, W)$

if $\dim V = n$ and $\dim W = m$, what is $\dim(S)$?

3) Is $T: M_{2 \times 2}(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ defined by

$$T \begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + bx + (c+dx)x^2$$

invertible? If so find its inverse T^{-1} ,

that is find a formula

$$T^{-1} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

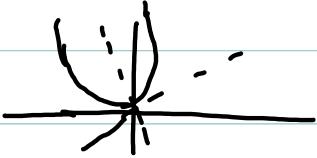
4) Is $T: M_{2 \times 2} \rightarrow M_{2 \times 2}$ defined by

$$T \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+b & a \\ c & c+d \end{pmatrix}$$

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$$T^{-1} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}$$

5) Find the equation (in the x y plane) of the curve you get by rotating the parabola $y = x^2$ counterclockwise of an angle of $\pi/6$ rad.



6) Give an example of a linear transformation $T: V \rightarrow V$ that is one to one but not invertible.

7) Give an example of a linear transformation $T: V \rightarrow V$ that is onto but not invertible.

8) $B_1 = \{1, x, x^2\}$ and $B_2 = \{2x^2 - x, 3x^2 + 1, x^2\}$ are two bases for $P_2(\mathbb{R})$. Find the matrices $I_{B_1}^{B_2}$, of change of basis from B_1 to B_2 and $I_{B_2}^{B_1}$, of change of basis from B_2 to B_1 .