Homework 3

1) Prove that the function $T: P_2(R) \to P_4(R)$ $T(p(x)) = p(x)\cdot(x^2+1)$ is a linear transformation. Describe $N(T)$. Is x^4 in $R(T)$?.

2) Is $T: P_2(R) \to P_4(R)$ $T(p) = p(x^2+1)$ (That is p composed with x^2+1) a linear transformation? Justify.

3) Let T_1 and T_2 be linear transformations from V to W. Prove that T_1+T_2 is a linear transformation.

4) Give an example of a linear transformation T s.t $\dim N(T)=3$ and $\dim R(T)=2$

5) Prove there is no linear transformation T $R^5 \to R^5$ with $R(T)=N(T)$

6) Find a linear transformation T $R^4 \to R^4$ s.t $R(T)=N(T)$.

7) Let V be a vector space (not necessarily finite dimensional) and $S_1 \subseteq S_2 \subseteq V$. Suppose S_1 is linearly independent and S_2 spans V. Prove there is a basis B for V with $S_1 \subseteq B \subseteq S_2$.