Homework 2

(1) Let $V = P(\mathbb{R})$, the space of polynomials with real coefficients, describe $\text{Span}\{1, x, x^2\}$

(2) Let $V = P(\mathbb{R})$

Is $x^3 - 3x + 5$ in $\text{Span}\{x^3 + 2x^2 - x + 1,\ x^3 + 3x^2 - 1\}$?

(3) Let $V = \mathbb{R}^{[0,1]}$, the space of all real functions defined on $[0,1]$. Let $S = \{f : [0,1] \to \mathbb{R} \mid \exists x, f(x) = 1\}$

Is S linearly independent?

(4) Suppose that V is a vector space and that $S_1 \subseteq S_2 \subseteq V$. Prove that if S_1 spans V then S_2 also spans V, and if S_2 is independent then S_1 is also independent

5) Let $U, V < \mathbb{R}^4$ $U = \text{Span}\{(1,1,0,0)\}$

$V = \text{Span}\{(0,0,1,0), (0,0,0,1)\}$

Verify $U + V$ is a direct sum and find a basis for $U \oplus V$

6) Verify that $S = \{x+1, x-1\}$ is independent in $P_3(\mathbb{R})$ and extend S to a basis for $P_3(\mathbb{R})$.

7) Let $U = \{ p \in P_4(\mathbb{R}) \mid \int_{-1}^{1} p(x)\,dx = 0 \}$

Prove U is a subspace of $P_4(\mathbb{R})$

Find a basis for U

Extend the basis you found to a basis for $P_4(\mathbb{R})$

Find $W \leq P_4(\mathbb{R})$ such that $P_4(\mathbb{R}) = U \oplus W$.

8) Prove $\mathbb{R}^\infty$ (The space of all sequences of real numbers) is not finite dimensional.