Homework 1

1) Show that the set V of all real valued $n \times m$ matrices with the usual matrix addition and scaler multiplication is a vector space over $\mathbb{R}$ for any $n \geq 1$, $m \geq 1$.

2) Define a new matrix addition for square matrices. $A + B = C$ is defined as follows:

if $A = (a_{ij})$ $B = (b_{ij})$ $C = (c_{ij})$ then $c_{ij} = a_{ij} + b_{ji}$.

so for example $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 1+5 & 2+7 \\ 3+6 & 4+8 \end{pmatrix}$

Is the set of $n \times n$ real matrices with this new addition and the usual scaler multiplication a vector space?

3) In this problem V is a vector space over F. $v, w \in V$, $\lambda, \alpha, \beta \in F$

Prove

i) $\lambda v = \lambda w \Rightarrow v = w$ whenever $\lambda \neq 0$

ii) $\alpha v = \beta v \Rightarrow \alpha = \beta$ whenever $v \neq 0$

iii) $\lambda \cdot v = 0 \Leftrightarrow \lambda = 0$ or $v = 0$

4.) Let S be the set of all real valued differentiable functions $f:(0\,1) \to R$
Prove that S is a subspace of $R^{(0,1)}$.

5) Give an example of a subset S of R^2 that contains $(0,0)$, is closed under vector addition but it is not a subspace of R^2.

6) Assume V is a vector space and S_x, $x \in \mathcal{J}$ is a family of subspaces of V. Prove that $\bigcap_{x \in \mathcal{J}} S_x$ is a subspace of V.

7) Assume V is a vector space and U_1, U_2, W are subspaces of V, such that $U_1 \oplus W = U_2 \oplus W$. Does it necesserily follow that $U_1 = U_2$? Justify.

8) Let $U_1 = \{(x, y, 0, 0) \mid x, y \in R\}$
$U_2 = \{(x, 0, z\ t) \mid x, y, t \in R\}$
U_1 and U_2 are subspaces of R^4 (you do not need to prove this).
Prove that $U_1 + U_2$ is not a direct sum.
Find $U_3 \leq R^4$ s.t $U_1 \oplus U_3 = U_1 + U_2$

9) Let S be the set of all complex valued sequences $\{x_n\}$ with the property that there is $k \in N$ such that for all $n \geq k$ $x_n = 0$. Prove S is a subspace of C^∞.